

# SEM over time

Changes in Structure, changes in Means

# Measuring at two time points

- Is the structure the same
- Do the means change (is there growth)

# Create the data

```
x.model <- sim.congeneric(c(.  
8,.7,.6,.8,.7,.6),N=200,short=FALSE)  
x <- x.model$observed  
x[,4:6] <- x[,4:6] + .5  
  
fx <-  
structure.list(6,list(post=c(4,5,6),pre=c(1,2,3)),item.labels=colnames(x))  
mod <- structure.graph(fx,matrix(c(1,"r",0,1),2))  
  
describe(x)  
pairs.panels(x)
```

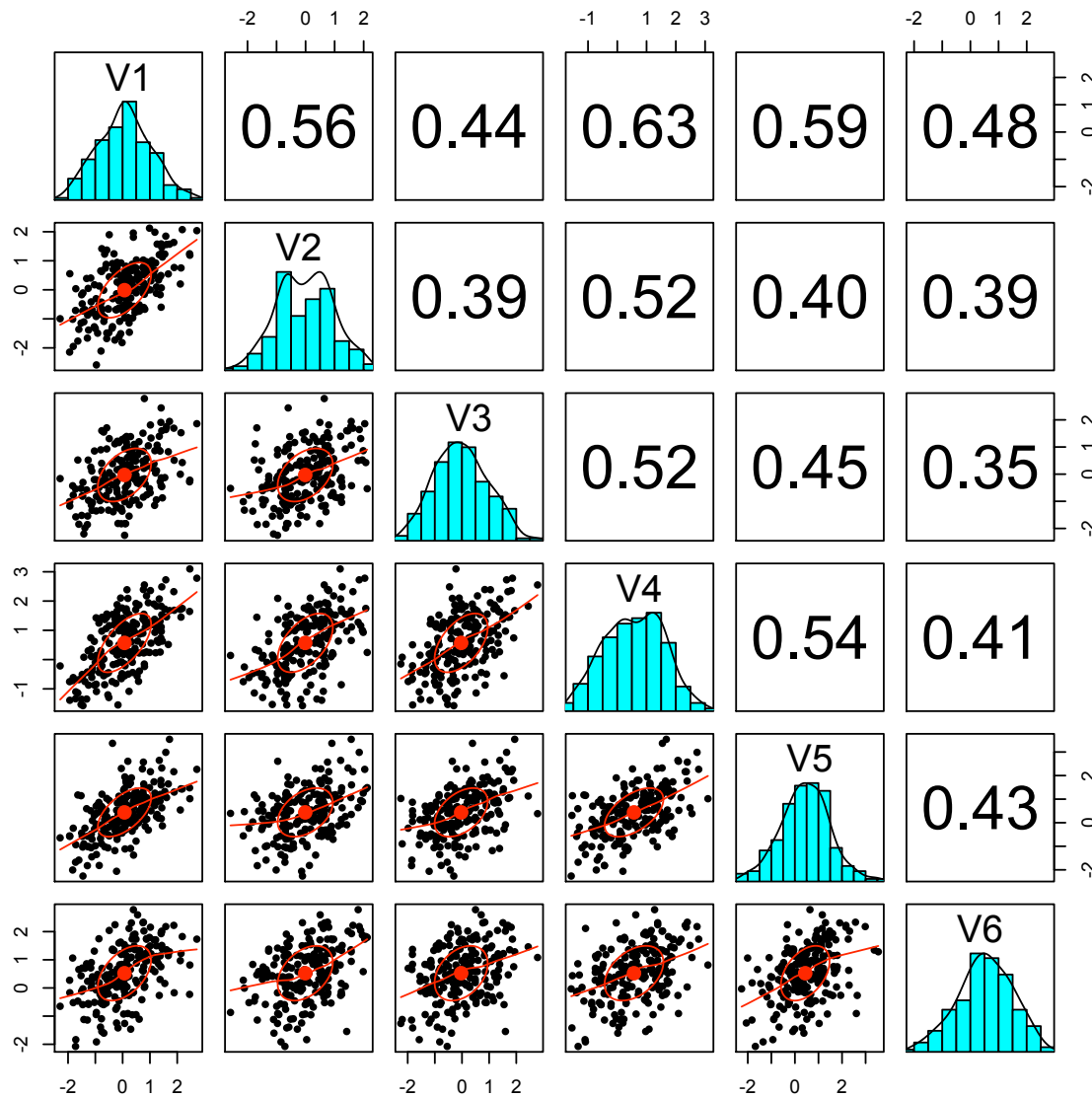
# Descriptive statistics

```
> describe(x)
```

|    | var | n   | mean  | sd   | median | trimmed | mad  | min   | max  | range | skew  | kurtosis | se   |
|----|-----|-----|-------|------|--------|---------|------|-------|------|-------|-------|----------|------|
| V1 | 1   | 200 | 0.07  | 0.98 | 0.08   | 0.06    | 1.01 | -2.29 | 2.72 | 5.01  | 0.11  | -0.30    | 0.07 |
| V2 | 2   | 200 | -0.01 | 0.95 | 0.00   | -0.01   | 1.00 | -2.59 | 2.12 | 4.71  | -0.01 | -0.44    | 0.07 |
| V3 | 3   | 200 | -0.03 | 0.98 | -0.11  | -0.05   | 1.01 | -2.25 | 2.78 | 5.04  | 0.16  | -0.41    | 0.07 |
| V4 | 4   | 200 | 0.57  | 1.01 | 0.60   | 0.58    | 1.14 | -1.57 | 3.10 | 4.67  | -0.04 | -0.67    | 0.07 |
| V5 | 5   | 200 | 0.44  | 1.03 | 0.48   | 0.44    | 0.99 | -2.26 | 3.54 | 5.80  | 0.05  | 0.20     | 0.07 |
| V6 | 6   | 200 | 0.52  | 0.97 | 0.57   | 0.55    | 0.96 | -2.08 | 2.78 | 4.86  | -0.22 | -0.31    | 0.07 |

# SPLOM

`pairs.panels(x)`



# Multiple ways to specify the data

- Ignoring time and ignoring structure
  - does a congeneric model fit?
- Ignoring time, considering structure
  - does a two factor congeneric model fit?
  - what about same loadings across factors
- Considering time, ignoring growth
- Considering time, considering change

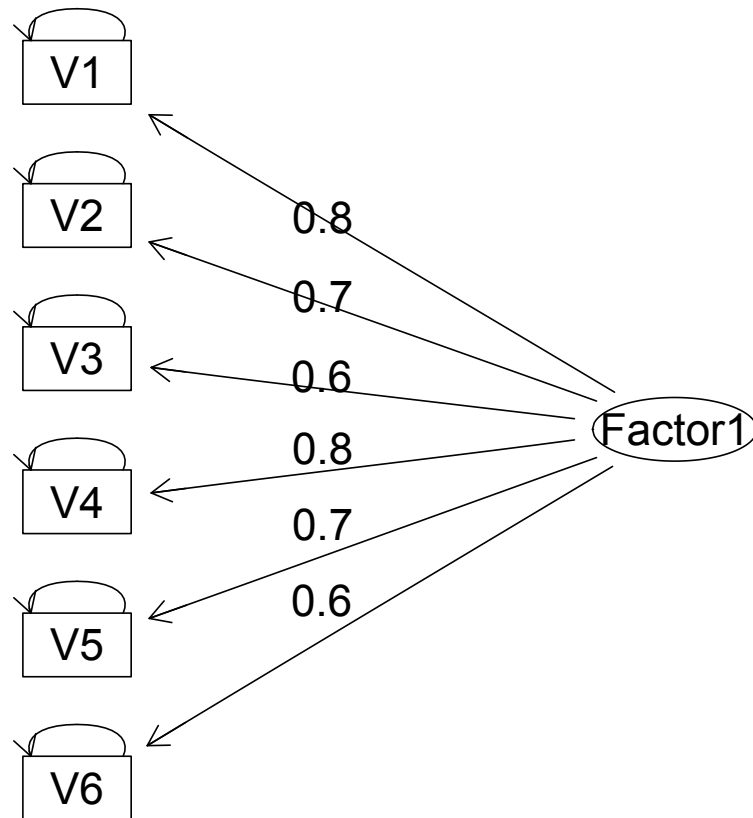
# Ignoring time and structure

```
> f1 <- factanal(x,1)
> f1
Call:
factanal(x = x, factors = 1)
Uniquenesses:
      V1      V2      V3      V4      V5      V6
0.334 0.565 0.635 0.388 0.511 0.669
Loadings:
      Factor1
V1 0.816
V2 0.660
V3 0.604
V4 0.782
V5 0.699
V6 0.575

              Factor1
SS loadings      2.898
Proportion Var   0.483
```

Test of the hypothesis that 1 factor is sufficient.  
The chi square statistic is 11.09 on 9 degrees of freedom.  
The p-value is 0.27

Structural model



# one factor model



# One factor model

```
> mod.1 <- structure.graph(f1)
> sem.1 <- sem(mod.1,cor(x),200)
> summary(sem.1)
```

```
Model Chisquare = 11.286    Df = 9 Pr(>Chisq) = 0.25664
Chisquare (null model) = 438.18    Df = 15
Goodness-of-fit index = 0.98188
Adjusted goodness-of-fit index = 0.95773
RMSEA index = 0.035723    90% CI: (NA, 0.091827)
Bentler-Bonnett NFI = 0.97424
Tucker-Lewis NNFI = 0.991
Bentler CFI = 0.9946
SRMR = 0.025748
BIC = -36.399
```

# One factor parameters

## Parameter Estimates

|     | Estimate | Std Error | z value | Pr(> z )   |                 |
|-----|----------|-----------|---------|------------|-----------------|
| V1  | 0.81604  | 0.062213  | 13.1169 | 0.0000e+00 | V1 <--- Factor1 |
| V2  | 0.65969  | 0.067024  | 9.8426  | 0.0000e+00 | V2 <--- Factor1 |
| V3  | 0.60441  | 0.068884  | 8.7743  | 0.0000e+00 | V3 <--- Factor1 |
| V4  | 0.78213  | 0.063323  | 12.3515 | 0.0000e+00 | V4 <--- Factor1 |
| V5  | 0.69948  | 0.065860  | 10.6208 | 0.0000e+00 | V5 <--- Factor1 |
| V6  | 0.57539  | 0.069365  | 8.2950  | 0.0000e+00 | V6 <--- Factor1 |
| x1e | 0.33408  | 0.050009  | 6.6804  | 2.3830e-11 | V1 <--> V1      |
| x2e | 0.56481  | 0.064672  | 8.7335  | 0.0000e+00 | V2 <--> V2      |
| x3e | 0.63469  | 0.070582  | 8.9921  | 0.0000e+00 | V3 <--> V3      |
| x4e | 0.38827  | 0.052822  | 7.3506  | 1.9740e-13 | V4 <--> V4      |
| x5e | 0.51073  | 0.060677  | 8.4171  | 0.0000e+00 | V5 <--> V5      |
| x6e | 0.66893  | 0.072922  | 9.1733  | 0.0000e+00 | V6 <--> V6      |

# Two factor model

```
> f2 <- factanal(x,2)
```

```
> f2
```

```
Call:
```

```
factanal(x = x, factors = 2)
```

```
Uniquenesses:
```

|  | V1    | V2    | V3    | V4    | V5    | V6    |
|--|-------|-------|-------|-------|-------|-------|
|  | 0.233 | 0.573 | 0.407 | 0.398 | 0.520 | 0.676 |

```
Loadings:
```

|    | Factor1 | Factor2 |
|----|---------|---------|
| V1 | 0.833   | 0.272   |
| V2 | 0.570   | 0.320   |
| V3 | 0.294   | 0.712   |
| V4 | 0.605   | 0.486   |
| V5 | 0.569   | 0.395   |
| V6 | 0.493   | 0.284   |

|                | Factor1 | Factor2 |
|----------------|---------|---------|
| SS loadings    | 2.037   | 1.156   |
| Proportion Var | 0.340   | 0.193   |
| Cumulative Var | 0.340   | 0.532   |

```
Test of the hypothesis that 2 factors are sufficient.
```

```
The chi square statistic is 4 on 4 degrees of freedom.
```

```
The p-value is 0.406
```

# Correlated factors?

```
> f2p <- Promax(f2)
```

```
> f2p
```

|    | V | Factor1 | Factor2 |
|----|---|---------|---------|
| V1 | 1 | 1.01    |         |
| V2 | 2 | 0.57    |         |
| V3 | 3 |         | 0.917   |
| V4 | 4 | 0.48    | 0.333   |
| V5 | 5 | 0.51    |         |
| V6 | 6 | 0.49    |         |

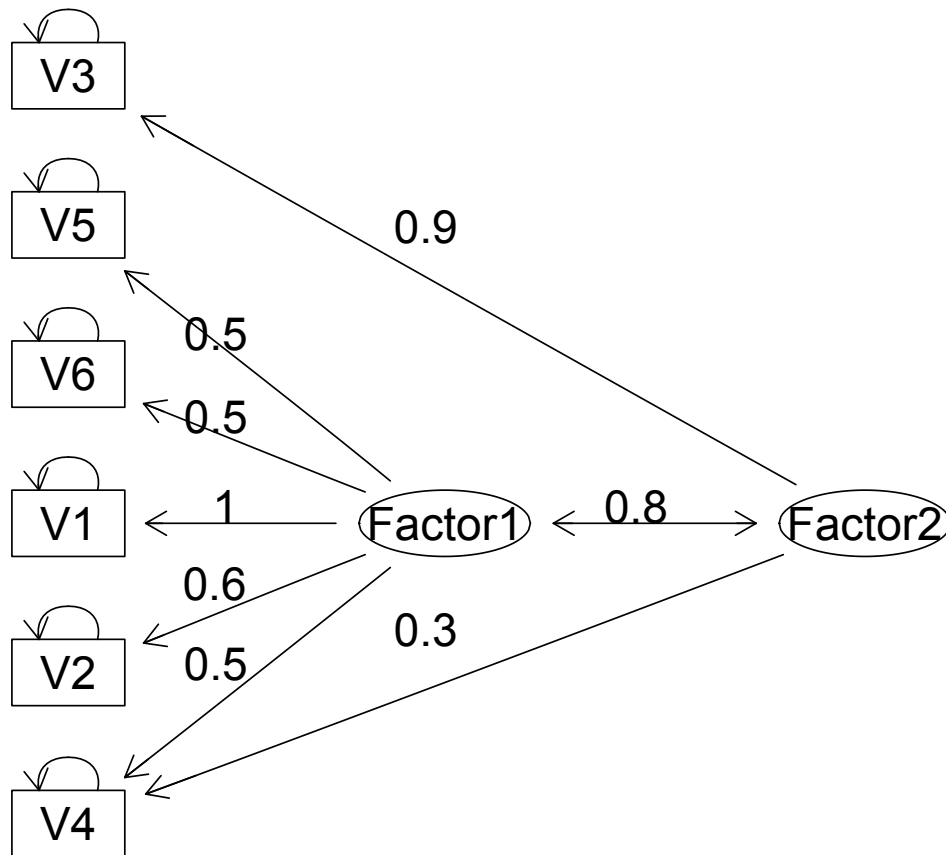
|                | Factor1 | Factor2 |
|----------------|---------|---------|
| SS loadings    | 2.11    | 1.05    |
| Proportion Var | 0.35    | 0.17    |
| Cumulative Var | 0.35    | 0.53    |

With factor correlations of

|         | Factor1 | Factor2 |
|---------|---------|---------|
| Factor1 | 1.00    | 0.82    |
| Factor2 | 0.82    | 1.00    |

# Completely wrong

Structural model



# Good fit!

Model Chisquare = 7.8826    Df = 7    Pr(>Chisq) = 0.34306  
Chisquare (null model) = 438.18    Df = 15  
Goodness-of-fit index = 0.98738  
Adjusted goodness-of-fit index = 0.96214  
RMSEA index = 0.025171    90% CI: (NA, 0.093204)  
Bentler-Bonnett NFI = 0.98201  
Tucker-Lewis NNFI = 0.99553  
Bentler CFI = 0.99791  
SRMR = 0.022587  
BIC = -29.206

# Reasonable parameters

## Parameter Estimates

|       | Estimate | Std Error | z value   | Pr(> z )   |         |      |         |
|-------|----------|-----------|-----------|------------|---------|------|---------|
| F1V1  | 0.83043  | 0.062363  | 13.316023 | 0.0000e+00 | V1      | <--- | Factor1 |
| F1V2  | 0.66169  | 0.067201  | 9.846410  | 0.0000e+00 | V2      | <--- | Factor1 |
| F2V3  | 0.67075  | 1.290004  | 0.519961  | 6.0309e-01 | V3      | <--- | Factor2 |
| F1V4  | 0.35921  | 5.726878  | 0.062723  | 9.4999e-01 | V4      | <--- | Factor1 |
| F2V4  | 0.47312  | 5.798517  | 0.081594  | 9.3497e-01 | V4      | <--- | Factor2 |
| F1V5  | 0.69998  | 0.066129  | 10.584958 | 0.0000e+00 | V5      | <--- | Factor1 |
| F1V6  | 0.57951  | 0.069458  | 8.343389  | 0.0000e+00 | V6      | <--- | Factor1 |
| x1e   | 0.31039  | 0.051109  | 6.073115  | 1.2545e-09 | V1      | <--> | V1      |
| x2e   | 0.56217  | 0.064898  | 8.662383  | 0.0000e+00 | V2      | <--> | V2      |
| x3e   | 0.55009  | 1.729548  | 0.318055  | 7.5044e-01 | V3      | <--> | V3      |
| x4e   | 0.35689  | 0.861580  | 0.414228  | 6.7871e-01 | V4      | <--> | V4      |
| x5e   | 0.51004  | 0.061232  | 8.329626  | 0.0000e+00 | V5      | <--> | V5      |
| x6e   | 0.66416  | 0.072793  | 9.124044  | 0.0000e+00 | V6      | <--> | V6      |
| rF2F1 | 0.85389  | 1.639772  | 0.520735  | 6.0255e-01 | Factor1 | <--> | Factor2 |

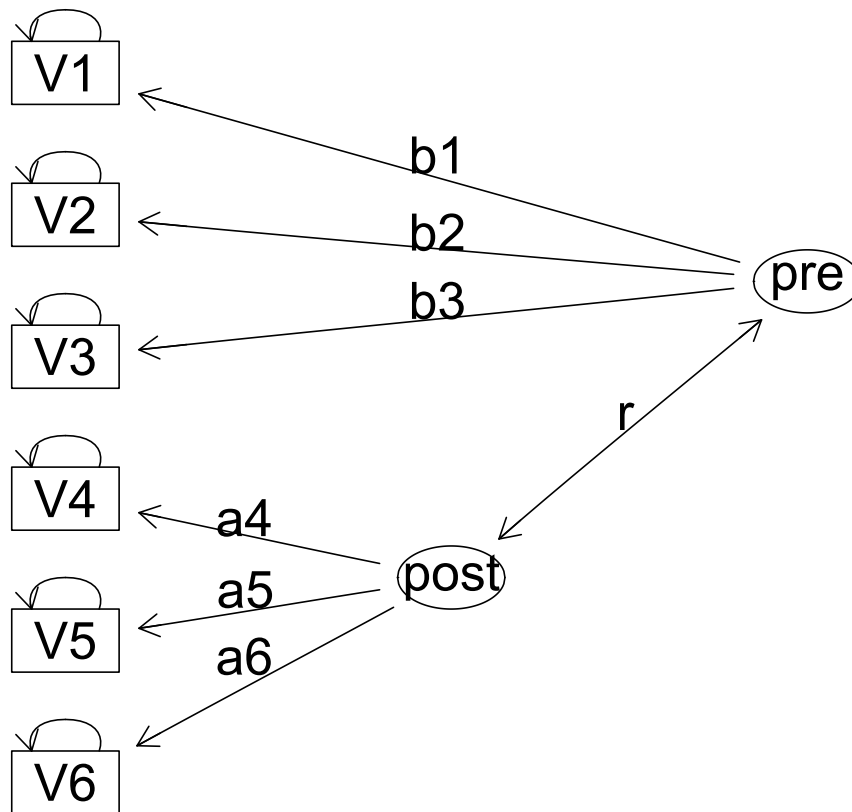
# Force the structure we want

```
fx <-  
structure(list(6,list(post=c(4,5,6),pre=c(1,2,3)),item.labels=colnames(x))  
mod <- structure.graph(fx,"r")
```



Incorrect  
specification  
does not  
show  
temporal  
effect

Structural model



# But fits the data

```
> mod
```

|       | Path          | Parameter | Value |
|-------|---------------|-----------|-------|
| [1,]  | "pre->V1"     | "b1"      | NA    |
| [2,]  | "pre->V2"     | "b2"      | NA    |
| [3,]  | "pre->V3"     | "b3"      | NA    |
| [4,]  | "post->V4"    | "a4"      | NA    |
| [5,]  | "post->V5"    | "a5"      | NA    |
| [6,]  | "post->V6"    | "a6"      | NA    |
| [7,]  | "V1<->V1"     | "x1e"     | NA    |
| [8,]  | "V2<->V2"     | "x2e"     | NA    |
| [9,]  | "V3<->V3"     | "x3e"     | NA    |
| [10,] | "V4<->V4"     | "x4e"     | NA    |
| [11,] | "V5<->V5"     | "x5e"     | NA    |
| [12,] | "V6<->V6"     | "x6e"     | NA    |
| [13,] | "pre<->post"  | "rF2F1"   | NA    |
| [14,] | "post<->post" | NA        | "1"   |
| [15,] | "pre<->pre"   | NA        | "1"   |

# wrong model can be estimated

```
> sem.1 <- sem(mod,cor(x),200)
> summary(sem.1)
```

```
Model Chisquare = 7.8388    Df = 8 Pr(>Chisq) = 0.44937
Chisquare (null model) = 433.37    Df = 15
Goodness-of-fit index = 0.98704
Adjusted goodness-of-fit index = 0.96599
RMSEA index = 0    90% CI: (NA, 0.082016)
Bentler-Bonnett NFI = 0.98191
Tucker-Lewis NNFI = 1.0007
Bentler CFI = 1
SRMR = 0.02138
BIC = -34.548
```

# Shows one factor

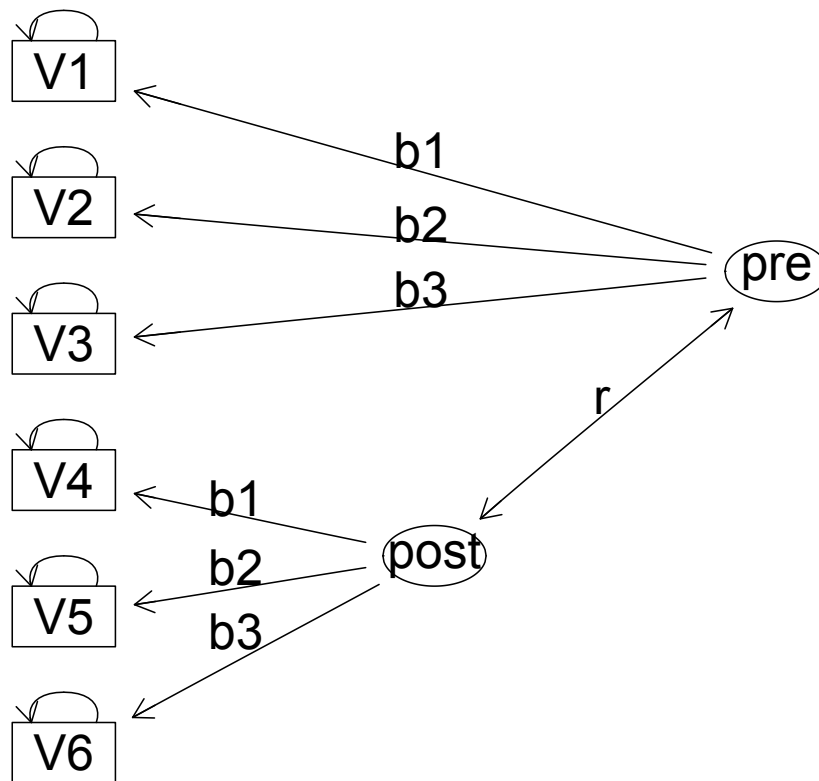
|       | Estimate | Std Error | z value | Pr(> z )   |      |      |      |
|-------|----------|-----------|---------|------------|------|------|------|
| b1    | 0.78543  | 0.065310  | 12.0262 | 0.0000e+00 | V1   | <--- | pre  |
| b2    | 0.65922  | 0.068519  | 9.6209  | 0.0000e+00 | V2   | <--- | pre  |
| b3    | 0.65924  | 0.068072  | 9.6844  | 0.0000e+00 | V3   | <--- | pre  |
| a4    | 0.78090  | 0.064855  | 12.0407 | 0.0000e+00 | V4   | <--- | post |
| a5    | 0.69054  | 0.067171  | 10.2804 | 0.0000e+00 | V5   | <--- | post |
| a6    | 0.65704  | 0.068334  | 9.6152  | 0.0000e+00 | V6   | <--- | post |
| x1e   | 0.38310  | 0.058511  | 6.5475  | 5.8523e-11 | V1   | <--> | V1   |
| x2e   | 0.56543  | 0.067364  | 8.3938  | 0.0000e+00 | V2   | <--> | V2   |
| x3e   | 0.56541  | 0.066572  | 8.4932  | 0.0000e+00 | V3   | <--> | V3   |
| x4e   | 0.39020  | 0.057171  | 6.8251  | 8.7865e-12 | V4   | <--> | V4   |
| x5e   | 0.52316  | 0.063697  | 8.2132  | 2.2204e-16 | V5   | <--> | V5   |
| x6e   | 0.56829  | 0.067123  | 8.4664  | 0.0000e+00 | V6   | <--> | V6   |
| rF2F1 | 0.95411  | 0.040315  | 23.6664 | 0.0000e+00 | post | <--> | pre  |

# Test for equality of loadings across factors

```
> fx[4:6,1] <- fx[1:3,2]
> fx
      post pre
v1 "0"    "b1"
v2 "0"    "b2"
v3 "0"    "b3"
v4 "b1"   "0"
v5 "b2"   "0"
v6 "b3"   "0"
> mod.eq <- structure.graph(fx,"r")
```

# Are the loadings equal?

Structural model



```
> mod.eq <- structure.graph(fx,"r")
> sem.eq <- sem(mod.eq,cor(x),200)
> summary(sem.eq)
```

```
Model Chisquare = 7.9861    Df = 11 Pr(>Chisq) = 0.71455
Chisquare (null model) = 433.37    Df = 15
Goodness-of-fit index = 0.9868
Adjusted goodness-of-fit index = 0.9748
RMSEA index = 0    90% CI: (NA, 0.05639)
Bentler-Bonnett NFI = 0.98157
Tucker-Lewis NNFI = 1.0098
Bentler CFI = 1
SRMR = 0.023802
BIC = -50.295
```

# Parameter estimates

## Parameter Estimates

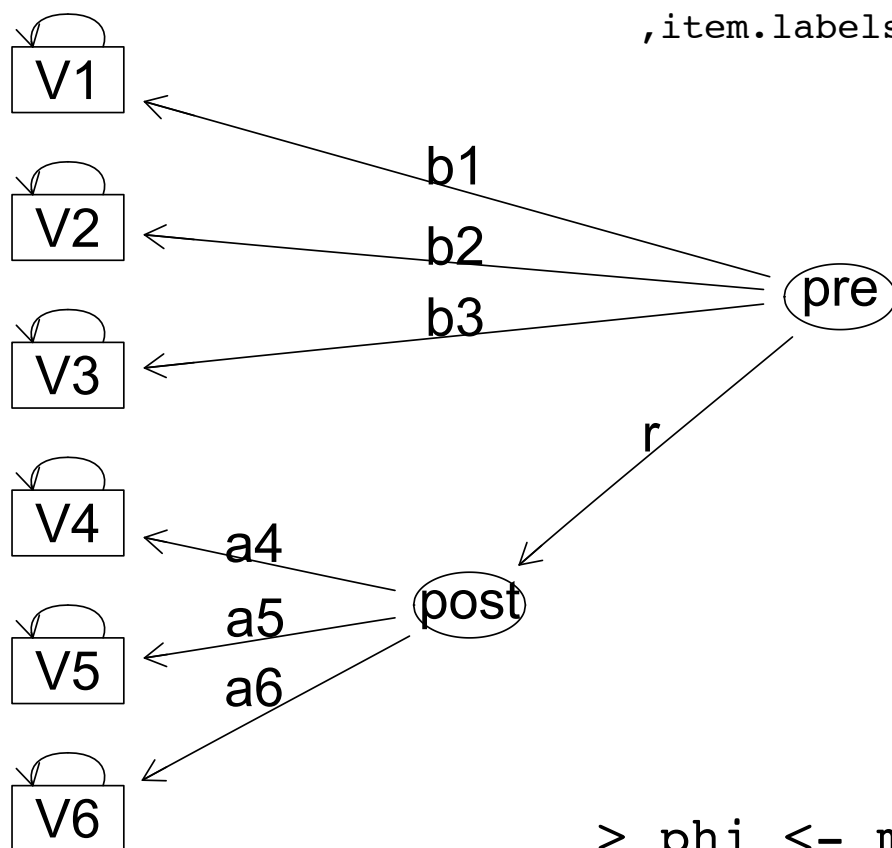
|       | Estimate | Std Error | z value | Pr(> z )    |      |          |
|-------|----------|-----------|---------|-------------|------|----------|
| b1    | 0.78368  | 0.053317  | 14.6984 | 0.00000e+00 | V1   | <--- pre |
| b2    | 0.67525  | 0.053392  | 12.6472 | 0.00000e+00 | V2   | <--- pre |
| b3    | 0.65819  | 0.053113  | 12.3924 | 0.00000e+00 | V3   | <--- pre |
| x1e   | 0.38328  | 0.056023  | 6.8414  | 7.8404e-12  | V1   | <--> V1  |
| x2e   | 0.56170  | 0.066368  | 8.4635  | 0.00000e+00 | V2   | <--> V2  |
| x3e   | 0.56590  | 0.065594  | 8.6272  | 0.00000e+00 | V3   | <--> V3  |
| x4e   | 0.38846  | 0.055627  | 6.9832  | 2.8861e-12  | V4   | <--> V4  |
| x5e   | 0.52741  | 0.062902  | 8.3847  | 0.00000e+00 | V5   | <--> V5  |
| x6e   | 0.56767  | 0.066215  | 8.5731  | 0.00000e+00 | V6   | <--> V6  |
| rF2F1 | 0.95308  | 0.040080  | 23.7795 | 0.00000e+00 | post | <--> pre |



# Measures over time

## Structural model

```
fx <-  
structure.list(6,list(post=c(4,5,6),pre=c(1,2,3))  
,item.labels=colnames(x))
```



```
> fx  
      post pre  
V1 "0"  "b1"  
V2 "0"  "b2"  
V3 "0"  "b3"  
V4 "a4"  "0"  
V5 "a5"  "0"  
V6 "a6"  "0"
```

```
> phi <- matrix(c(1,"r",0,1),ncol=2)  
> mod.t <- structure.graph(fx,phi)
```

# Model

```
> mod.t
```

|       | Path          | Parameter | Value |
|-------|---------------|-----------|-------|
| [1,]  | "pre->V1"     | "b1"      | NA    |
| [2,]  | "pre->V2"     | "b2"      | NA    |
| [3,]  | "pre->V3"     | "b3"      | NA    |
| [4,]  | "post->V4"    | "a4"      | NA    |
| [5,]  | "post->V5"    | "a5"      | NA    |
| [6,]  | "post->V6"    | "a6"      | NA    |
| [7,]  | "V1<->V1"     | "x1e"     | NA    |
| [8,]  | "V2<->V2"     | "x2e"     | NA    |
| [9,]  | "V3<->V3"     | "x3e"     | NA    |
| [10,] | "V4<->V4"     | "x4e"     | NA    |
| [11,] | "V5<->V5"     | "x5e"     | NA    |
| [12,] | "V6<->V6"     | "x6e"     | NA    |
| [13,] | "pre ->post"  | "rF2F1"   | NA    |
| [14,] | "post<->post" | NA        | "1"   |
| [15,] | "pre<->pre"   | NA        | "1"   |

```
> sem.t <- sem(mod.t,cor(x),200)
> summary(sem.t)
```

```
Model Chisquare = 7.8388    Df = 8 Pr(>Chisq) = 0.44937
Chisquare (null model) = 433.37    Df = 15
Goodness-of-fit index = 0.98704
Adjusted goodness-of-fit index = 0.96599
RMSEA index = 0    90% CI: (NA, 0.082016)
Bentler-Bonnett NFI = 0.98191
Tucker-Lewis NNFI = 1.0007
Bentler CFI = 1
SRMR = 0.02138
BIC = -34.548
```

# Parameters are weird

## Parameter Estimates

|       | Estimate | Std Error | z value | Pr(> z )   |      |           |
|-------|----------|-----------|---------|------------|------|-----------|
| b1    | 0.78543  | 0.065411  | 12.0076 | 0.0000e+00 | V1   | <--- pre  |
| b2    | 0.65922  | 0.068566  | 9.6143  | 0.0000e+00 | V2   | <--- pre  |
| b3    | 0.65924  | 0.068072  | 9.6845  | 0.0000e+00 | V3   | <--- pre  |
| a4    | 0.23385  | 0.108465  | 2.1560  | 3.1085e-02 | V4   | <--- post |
| a5    | 0.20679  | 0.095101  | 2.1744  | 2.9674e-02 | V5   | <--- post |
| a6    | 0.19676  | 0.092176  | 2.1346  | 3.2793e-02 | V6   | <--- post |
| x1e   | 0.38310  | 0.058824  | 6.5126  | 7.3840e-11 | V1   | <--> V1   |
| x2e   | 0.56544  | 0.067459  | 8.3820  | 0.0000e+00 | V2   | <--> V2   |
| x3e   | 0.56541  | 0.066572  | 8.4931  | 0.0000e+00 | V3   | <--> V3   |
| x4e   | 0.39020  | 0.057310  | 6.8085  | 9.8617e-12 | V4   | <--> V4   |
| x5e   | 0.52316  | 0.063706  | 8.2121  | 2.2204e-16 | V5   | <--> V5   |
| x6e   | 0.56829  | 0.067187  | 8.4584  | 0.0000e+00 | V6   | <--> V6   |
| rF2F1 | 3.18610  | 1.573934  | 2.0243  | 4.2940e-02 | post | <--- pre  |

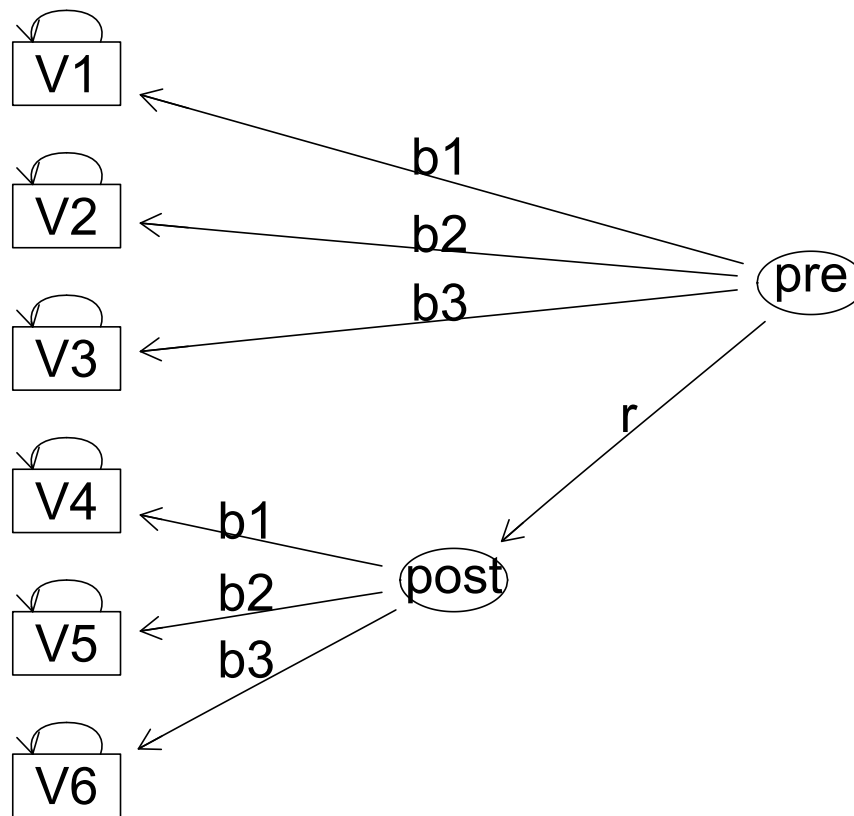
# But standardized make sense

```
> std.coef(sem.t)
```

|       |       | Std. Estimate |               |
|-------|-------|---------------|---------------|
| b1    | b1    | 0.78543       | V1 <--- pre   |
| b2    | b2    | 0.65922       | V2 <--- pre   |
| b3    | b3    | 0.65924       | V3 <--- pre   |
| a4    | a4    | 0.78090       | V4 <--- post  |
| a5    | a5    | 0.69054       | V5 <--- post  |
| a6    | a6    | 0.65704       | V6 <--- post  |
| rF2F1 | rF2F1 | 0.95411       | post <--- pre |

# Do matching variables match

Structural model



# Test for equality of paths

```
> fx
      post pre
v1 "0"    "b1"
v2 "0"    "b2"
v3 "0"    "b3"
v4 "a4"    "0"
v5 "a5"    "0"
v6 "a6"    "0"
> fx[4:6,1] <- fx[1:3,2]
> fx
      post pre
v1 "0"    "b1"
v2 "0"    "b2"
v3 "0"    "b3"
v4 "b1"    "0"
v5 "b2"    "0"
v6 "b3"    "0"
> mod.eq.t <- structure.graph(fx,phi)
> mod.eq.t
```

# Solution is terrible

```
> sem.eq.t <- sem(mod.eq.t,cor(x),200)
> summary(sem.eq.t)
```

```
Model Chisquare = 49.567    Df = 11 Pr(>Chisq) =
7.4868e-07
Chisquare (null model) = 433.37    Df = 15
Goodness-of-fit index = 0.93538
Adjusted goodness-of-fit index = 0.87663
RMSEA index = 0.13273    90% CI: (0.096497, 0.17138)
Bentler-Bonnett NFI = 0.88563
Tucker-Lewis NNFI = 0.8743
Bentler CFI = 0.90782
SRMR = 0.15295
BIC = -8.7144
```

## Normalized Residuals

| Min.   | 1st Qu. | Median | Mean  | 3rd Qu. | Max.  |
|--------|---------|--------|-------|---------|-------|
| -1.500 | 0.991   | 2.030  | 1.530 | 2.680   | 3.240 |



# Parameters are ok

## Parameter Estimates

|       | Estimate | Std Error | z value | Pr(> z )    |      |      |     |
|-------|----------|-----------|---------|-------------|------|------|-----|
| b1    | 0.64060  | 0.045464  | 14.0902 | 0.00000e+00 | V1   | <--- | pre |
| b2    | 0.54379  | 0.043997  | 12.3597 | 0.00000e+00 | V2   | <--- | pre |
| b3    | 0.53090  | 0.042779  | 12.4105 | 0.00000e+00 | V3   | <--- | pre |
| x1e   | 0.38292  | 0.062663  | 6.1107  | 9.9165e-10  | V1   | <--> | V1  |
| x2e   | 0.55525  | 0.068724  | 8.0795  | 6.6613e-16  | V2   | <--> | V2  |
| x3e   | 0.58544  | 0.070487  | 8.3056  | 0.00000e+00 | V3   | <--> | V3  |
| x4e   | 0.33702  | 0.064857  | 5.1963  | 2.0329e-07  | V4   | <--> | V4  |
| x5e   | 0.52752  | 0.069694  | 7.5690  | 3.7526e-14  | V5   | <--> | V5  |
| x6e   | 0.53894  | 0.068200  | 7.9024  | 2.6645e-15  | V6   | <--> | V6  |
| rF2F1 | 0.93228  | 0.097306  | 9.5809  | 0.00000e+00 | post | <--- | pre |

# Perhaps should fix a path?

```
> mod.eq.r <- edit(mod.eq)
```

```
> mod.eq.r
```

|       | Path          | Parameter | Value |
|-------|---------------|-----------|-------|
| [1,]  | "pre->V1"     | "b1"      | NA    |
| [2,]  | "pre->V2"     | "b2"      | NA    |
| [3,]  | "pre->V3"     | "b3"      | NA    |
| [4,]  | "post->V4"    | NA        | "1"   |
| [5,]  | "post->V5"    | "b2"      | NA    |
| [6,]  | "post->V6"    | "b3"      | NA    |
| [7,]  | "V1<->V1"     | "x1e"     | NA    |
| [8,]  | "V2<->V2"     | "x2e"     | NA    |
| [9,]  | "V3<->V3"     | "x3e"     | NA    |
| [10,] | "V4<->V4"     | "x4e"     | NA    |
| [11,] | "V5<->V5"     | "x5e"     | NA    |
| [12,] | "V6<->V6"     | "x6e"     | NA    |
| [13,] | "pre<->post"  | "rF2F1"   | NA    |
| [14,] | "post<->post" | NA        | "1"   |
| [15,] | "pre<->pre"   | NA        | "1"   |

# Much better solution

```
> sem.eq.r <- sem(mod.eq.r,cor(x),200)
> summary(sem.eq.r)
```

```
Model Chisquare = 17.855    Df = 11 Pr(>Chisq) = 0.085017
Chisquare (null model) = 433.37    Df = 15
Goodness-of-fit index = 0.97141
Adjusted goodness-of-fit index = 0.94542
RMSEA index = 0.055959    90% CI: (NA, 0.10145)
Bentler-Bonnett NFI = 0.9588
Tucker-Lewis NNFI = 0.97766
Bentler CFI = 0.98362
SRMR = 0.16442
BIC = -40.427
```

# Parameter Estimates

|       | Estimate | Std Error | z value | Pr(> z )   |      |      |     |
|-------|----------|-----------|---------|------------|------|------|-----|
| b1    | 0.87501  | 0.066894  | 13.0805 | 0.0000e+00 | V1   | <--- | pre |
| b2    | 0.74747  | 0.055550  | 13.4558 | 0.0000e+00 | V2   | <--- | pre |
| b3    | 0.73428  | 0.054620  | 13.4434 | 0.0000e+00 | V3   | <--- | pre |
| x1e   | 0.37949  | 0.057995  | 6.5434  | 6.0122e-11 | V1   | <--> | V1  |
| x2e   | 0.56565  | 0.066766  | 8.4721  | 0.0000e+00 | V2   | <--> | V2  |
| x3e   | 0.56486  | 0.065718  | 8.5952  | 0.0000e+00 | V3   | <--> | V3  |
| x4e   | 0.35267  | 0.057993  | 6.0813  | 1.1922e-09 | V4   | <--> | V4  |
| x5e   | 0.53886  | 0.063523  | 8.4829  | 0.0000e+00 | V5   | <--> | V5  |
| x6e   | 0.57011  | 0.065567  | 8.6952  | 0.0000e+00 | V6   | <--> | V6  |
| rF2F1 | 0.95070  | 0.032892  | 28.9036 | 0.0000e+00 | post | <--> | pre |

# Analyzing change

- McArdle (2009) Latent variable modeling of differences and changes with longitudinal data. Annual Review of Psychology, 60, 577-605.
- Use moments rather than covariances

# Using the moments matrix

- Means can be estimated as part of the model
- Explicitly model the means and changes in means
- `one <- rep(1,100)`
- `x1 <- cbind(x,one)`
- `mom.x1 <- t(x1)%*% x1/100`
- `mom.x1`

# Moments capture means

```
> describe(x1)
```

|     | var | n   | mean | sd   | median | trimmed | mad  | min   | max  | range | skew  | kurtosis | se   |
|-----|-----|-----|------|------|--------|---------|------|-------|------|-------|-------|----------|------|
| V1  | 1   | 200 | 0.07 | 0.94 | 0.02   | 0.05    | 0.98 | -2.62 | 3.29 | 5.92  | 0.21  | 0.52     | 0.07 |
| V2  | 2   | 200 | 0.02 | 1.00 | 0.01   | 0.00    | 1.08 | -3.12 | 2.94 | 6.06  | 0.11  | -0.15    | 0.07 |
| V3  | 3   | 200 | 0.00 | 1.04 | -0.02  | 0.00    | 1.03 | -3.00 | 2.68 | 5.68  | 0.03  | -0.10    | 0.07 |
| V4  | 4   | 200 | 0.51 | 0.98 | 0.41   | 0.48    | 0.86 | -1.86 | 3.09 | 4.95  | 0.29  | 0.03     | 0.07 |
| V5  | 5   | 200 | 0.41 | 1.00 | 0.39   | 0.39    | 1.14 | -2.01 | 3.28 | 5.29  | 0.20  | -0.21    | 0.07 |
| V6  | 6   | 200 | 0.58 | 0.95 | 0.53   | 0.59    | 1.04 | -1.88 | 3.66 | 5.54  | -0.02 | -0.08    | 0.07 |
| one | 7   | 200 | 1.00 | 0.00 | 1.00   | 1.00    | 0.00 | 1.00  | 1.00 | 0.00  | NaN   | NaN      | 0.00 |

```
> round(mom.x1,2)
```

|     | V1   | V2   | V3   | V4   | V5   | V6   | one  |
|-----|------|------|------|------|------|------|------|
| V1  | 0.89 | 0.52 | 0.48 | 0.58 | 0.51 | 0.45 | 0.07 |
| V2  | 0.52 | 1.00 | 0.43 | 0.44 | 0.47 | 0.39 | 0.02 |
| V3  | 0.48 | 0.43 | 1.07 | 0.53 | 0.46 | 0.42 | 0.00 |
| V4  | 0.58 | 0.44 | 0.53 | 1.21 | 0.71 | 0.79 | 0.51 |
| V5  | 0.51 | 0.47 | 0.46 | 0.71 | 1.16 | 0.68 | 0.41 |
| V6  | 0.45 | 0.39 | 0.42 | 0.79 | 0.68 | 1.24 | 0.58 |
| one | 0.07 | 0.02 | 0.00 | 0.51 | 0.41 | 0.58 | 1.00 |

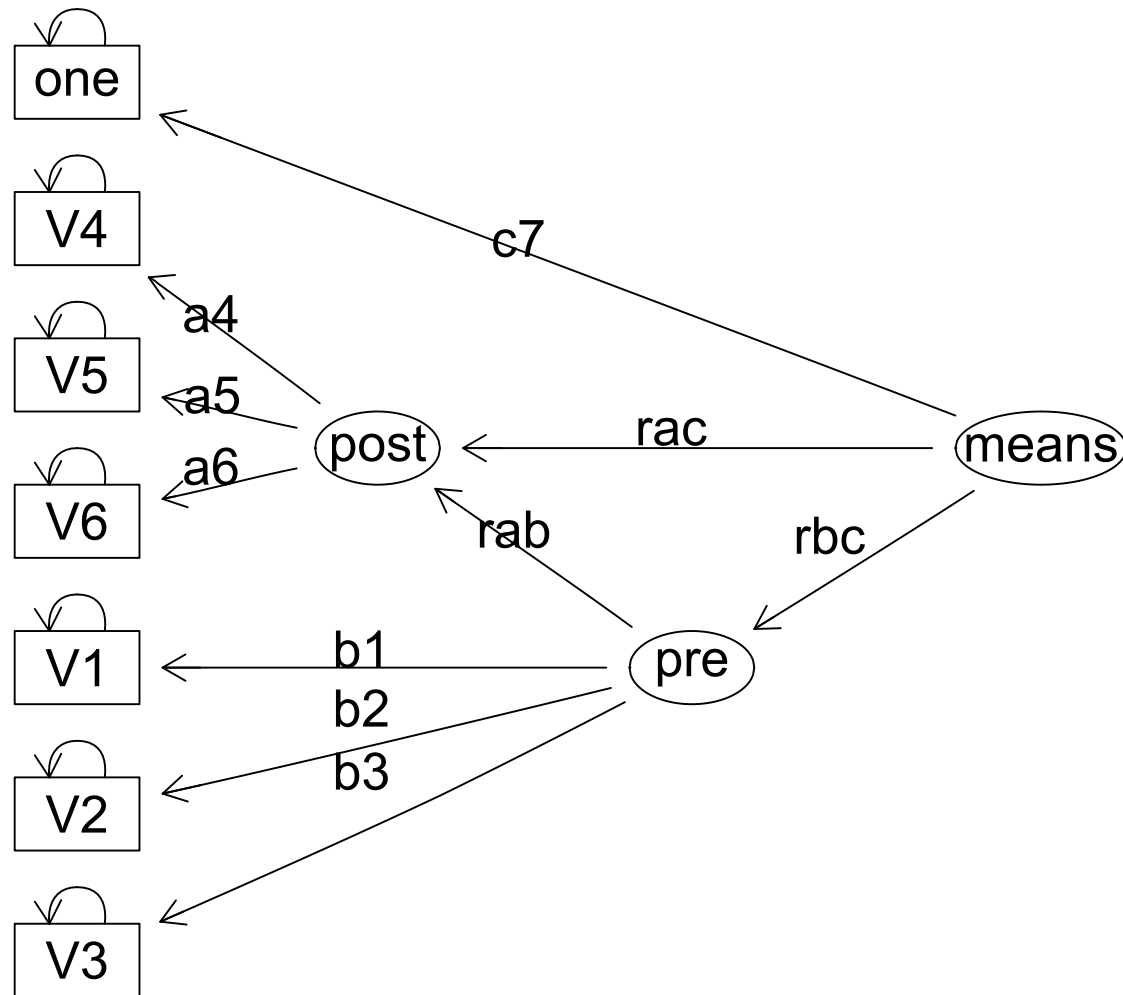
# Make a structure model

```
> fxg <-  
structure.list(7,list(post=c(4:6),pre=c(1:3),means=c(7))  
  ,item.labels=colnames(mom.x1))  
> phi <- phi.list(3,list(post=c(2,3),pre=c(3),means=NULL))  
  
> mod.mom <- structure.graph(fxg,phi)
```



## Structural model

# Moments model



```
> mod.mom1
```

|       | Path            | Parameter | Value |
|-------|-----------------|-----------|-------|
| [1,]  | "pre->V1"       | "b1"      | NA    |
| [2,]  | "pre->V2"       | "b2"      | NA    |
| [3,]  | "pre->V3"       | "b3"      | NA    |
| [4,]  | "post->V4"      | NA        | "1"   |
| [5,]  | "post->V5"      | "a5"      | NA    |
| [6,]  | "post->V6"      | "a6"      | NA    |
| [7,]  | "means->one"    | NA        | "1"   |
| [8,]  | "V1<->V1"       | "x1e"     | NA    |
| [9,]  | "V2<->V2"       | "x2e"     | NA    |
| [10,] | "V3<->V3"       | "x3e"     | NA    |
| [11,] | "V4<->V4"       | "x4e"     | NA    |
| [12,] | "V5<->V5"       | "x5e"     | NA    |
| [13,] | "V6<->V6"       | "x6e"     | NA    |
| [14,] | "one<->one"     | NA        | "1"   |
| [15,] | "pre ->post"    | "rF2F1"   | NA    |
| [16,] | "means ->post"  | "rF3F1"   | NA    |
| [17,] | "means ->pre"   | "rF3F2"   | NA    |
| [18,] | "post<->post"   | NA        | "1"   |
| [19,] | "pre<->pre"     | NA        | "1"   |
| [20,] | "means<->means" | NA        | "1"   |

Have to  
fix some  
paths

# Nested models may be compared

```
> sem.mom1 <- sem(mod.mom1,mom.x1,raw=TRUE,N=200)
> summary(sem.mom1)
Model fit to raw moment matrix.
Model Chisquare = 178.37 Df = 14 Pr(>Chisq) = 0
BIC = 104.19
Normalized Residuals
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-5.0000 -0.9240 -0.0472 -0.4270  0.3720  1.6300
Parameter Estimates
      Estimate Std Error z value Pr(>|z|)
b1      0.78185   0.069191 11.2999 0.0000e+00 V1 <--- pre
b2      0.66118   0.073036  9.0529 0.0000e+00 V2 <--- pre
b3      0.62730   0.076927  8.1545 4.4409e-16 V3 <--- pre
a5      0.69496   0.062795 11.0671 0.0000e+00 V5 <--- post
a6      0.75713   0.062689 12.0775 0.0000e+00 V6 <--- post
x1e     0.26151   0.072182  3.6229 2.9127e-04 V1 <--> V1
x2e     0.55385   0.074857  7.3988 1.3745e-13 V2 <--> V2
x3e     0.66610   0.083545  7.9729 1.5543e-15 V3 <--> V3
x4e     0.24793   0.083129  2.9825 2.8590e-03 V4 <--> V4
x5e     0.58902   0.078117  7.5403 4.6851e-14 V5 <--> V5
x6e     0.55749   0.079343  7.0263 2.1205e-12 V6 <--> V6
rF2F1  0.61758   0.099501  6.2068 5.4064e-10 post <--- pre
rF3F1  0.46968   0.105924  4.4341 9.2438e-06 post <--- means
rF3F2  0.17493   0.154160  1.1347 2.5649e-01 pre <--- means
```

# Fix pre to 0

```
> mod.mom3
```

|       | Path            | Parameter | Value |
|-------|-----------------|-----------|-------|
| [1,]  | "pre->V1"       | "b1"      | NA    |
| [2,]  | "pre->V2"       | "b2"      | NA    |
| [3,]  | "pre->V3"       | "b3"      | NA    |
| [4,]  | "post->V4"      | NA        | "1"   |
| [5,]  | "post->V5"      | "a5"      | NA    |
| [6,]  | "post->V6"      | "a6"      | NA    |
| [7,]  | "means->one"    | NA        | "1"   |
| [8,]  | "V1<->V1"       | "x1e"     | NA    |
| [9,]  | "V2<->V2"       | "x2e"     | NA    |
| [10,] | "V3<->V3"       | "x3e"     | NA    |
| [11,] | "V4<->V4"       | "x4e"     | NA    |
| [12,] | "V5<->V5"       | "x5e"     | NA    |
| [13,] | "V6<->V6"       | "x6e"     | NA    |
| [14,] | "one<->one"     | NA        | "1"   |
| [15,] | "pre ->post"    | "rF2F1"   | NA    |
| [16,] | "means ->post"  | "rF3F1"   | NA    |
| [17,] | "means ->pre"   | NA        | "0"   |
| [18,] | "post<->post"   | NA        | "1"   |
| [19,] | "pre<->pre"     | NA        | "1"   |
| [20,] | "means<->means" | NA        | "1"   |

# Reduced model

```
> sem.mom3 <- sem(mod.mom3,mom.x1,row=TRUE,N=200)
```

```
> summary(sem.mom3)
```

Model fit to raw moment matrix.

Model Chisquare = 179.70 Df = 15 Pr(>Chisq) = 0

BIC = 100.23

Normalized Residuals

|  | Min.    | 1st Qu. | Median | Mean    | 3rd Qu. | Max.   |
|--|---------|---------|--------|---------|---------|--------|
|  | -5.0000 | -0.1160 | 0.2250 | -0.0487 | 0.6980  | 1.9200 |

Parameter Estimates

|       | Estimate | Std Error | z value | Pr(> z )   |                 |
|-------|----------|-----------|---------|------------|-----------------|
| b1    | 0.79114  | 0.068841  | 11.4924 | 0.0000e+00 | V1 <--- pre     |
| b2    | 0.67023  | 0.073061  | 9.1736  | 0.0000e+00 | V2 <--- pre     |
| b3    | 0.63609  | 0.077123  | 8.2477  | 2.2204e-16 | V3 <--- pre     |
| a5    | 0.69510  | 0.062919  | 11.0475 | 0.0000e+00 | V5 <--- post    |
| a6    | 0.75731  | 0.062821  | 12.0550 | 0.0000e+00 | V6 <--- post    |
| x1e   | 0.26139  | 0.073213  | 3.5702  | 3.5666e-04 | V1 <--> V1      |
| x2e   | 0.55217  | 0.075254  | 7.3374  | 2.1783e-13 | V2 <--> V2      |
| x3e   | 0.66433  | 0.083838  | 7.9240  | 2.2204e-15 | V3 <--> V3      |
| x4e   | 0.24762  | 0.083321  | 2.9719  | 2.9594e-03 | V4 <--> V4      |
| x5e   | 0.58930  | 0.078220  | 7.5338  | 4.9294e-14 | V5 <--> V5      |
| x6e   | 0.55778  | 0.079468  | 7.0189  | 2.2362e-12 | V6 <--> V6      |
| rF2F1 | 0.65973  | 0.090926  | 7.2556  | 3.9990e-13 | post <--- pre   |
| rF3F1 | 0.47800  | 0.104520  | 4.5733  | 4.8005e-06 | post <--- means |

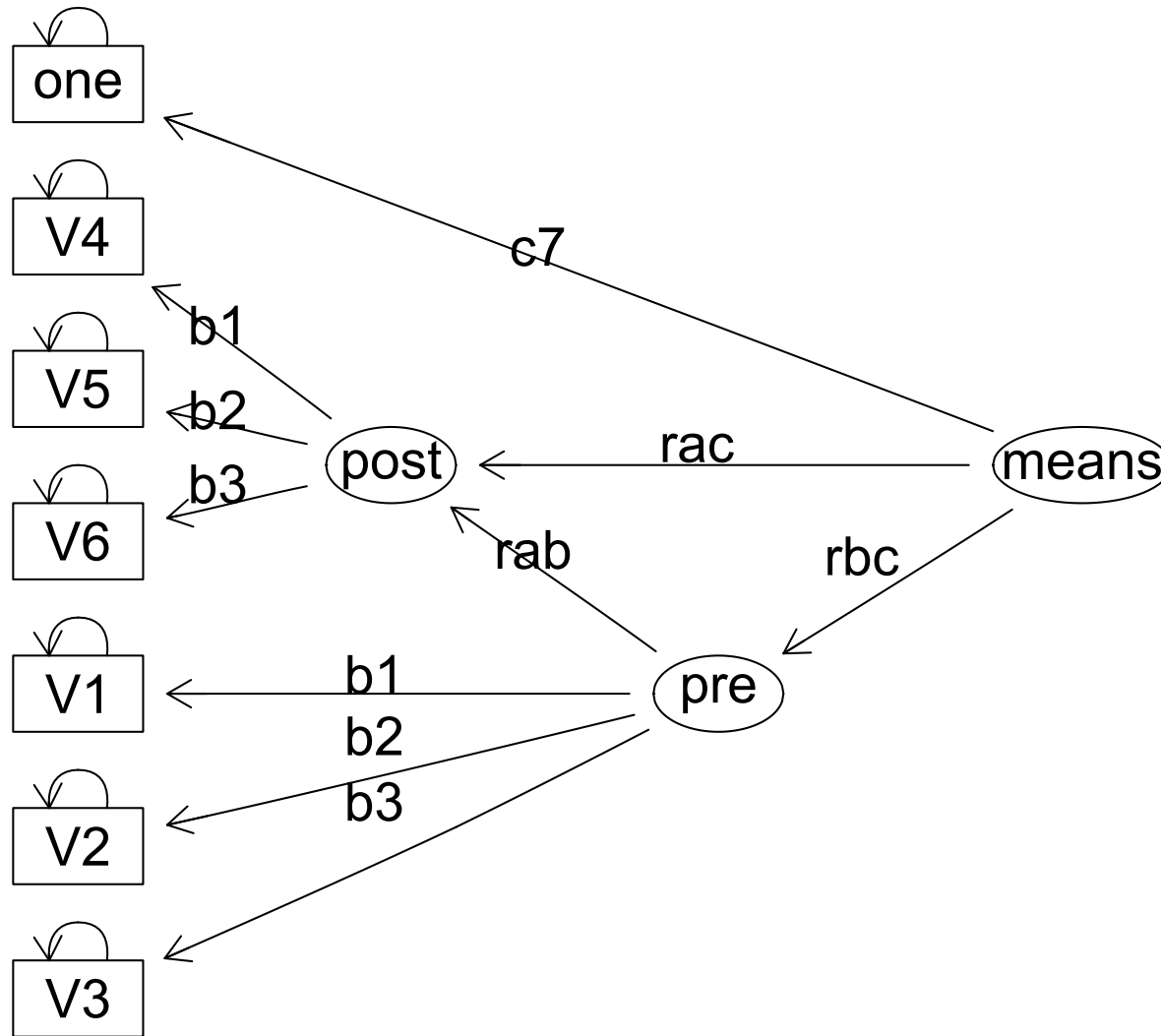
# Now test for equivalence of structure

```
> fxg.c <- fxg
> fxg.c[4:6,1] <- fxg.c[1:3,2]
> fxg.c
```

|     | post | pre  | means |
|-----|------|------|-------|
| v1  | "0"  | "b1" | "0"   |
| v2  | "0"  | "b2" | "0"   |
| v3  | "0"  | "b3" | "0"   |
| v4  | "b1" | "0"  | "0"   |
| v5  | "b2" | "0"  | "0"   |
| v6  | "b3" | "0"  | "0"   |
| one | "0"  | "0"  | "c7"  |

```
>
```

### Structural model



Are  
natching  
loadings  
the  
same?

# Fix some paths

```
> mod.mom4 <- edit(mod.mom4)
> mod.mom4
```

|       | Path            | Parameter | Value |
|-------|-----------------|-----------|-------|
| [1,]  | "pre->V1"       | "b1"      | NA    |
| [2,]  | "pre->V2"       | "b2"      | NA    |
| [3,]  | "pre->V3"       | "b3"      | NA    |
| [4,]  | "post->V4"      | NA        | "1"   |
| [5,]  | "post->V5"      | "b2"      | NA    |
| [6,]  | "post->V6"      | "b3"      | NA    |
| [7,]  | "means->one"    | NA        | "1"   |
| [8,]  | "V1<->V1"       | "x1e"     | NA    |
| [9,]  | "V2<->V2"       | "x2e"     | NA    |
| [10,] | "V3<->V3"       | "x3e"     | NA    |
| [11,] | "V4<->V4"       | "x4e"     | NA    |
| [12,] | "V5<->V5"       | "x5e"     | NA    |
| [13,] | "V6<->V6"       | "x6e"     | NA    |
| [14,] | "one<->one"     | NA        | "1"   |
| [15,] | "pre ->post"    | "rF2F1"   | NA    |
| [16,] | "means ->post"  | "rF3F1"   | NA    |
| [17,] | "means ->pre"   | NA        | "0"   |
| [18,] | "post<->post"   | NA        | "1"   |
| [19,] | "pre<->pre"     | NA        | "1"   |
| [20,] | "means<->means" | NA        | "1"   |



# equal paths

```
> sem.mom4 <- sem(mod.mom4,mom.x1,raw=TRUE,N=200)
> summary(sem.mom4)
Model fit to raw moment matrix.
Model Chisquare = 181.17 Df = 17 Pr(>Chisq) = 0
BIC = 91.103
Normalized Residuals
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-5.000 -0.664   0.201  -0.174   0.710   1.970
Parameter Estimates
      Estimate Std Error z value Pr(>|z|)
b1      0.79616  0.068934 11.5496 0.0000e+00 V1 <--- pre
b2      0.68428  0.048510 14.1062 0.0000e+00 V2 <--- pre
b3      0.70959  0.049762 14.2597 0.0000e+00 V3 <--- pre
x1e     0.27927  0.067373  4.1451 3.3966e-05 V1 <--> V1
x2e     0.54952  0.070740  7.7682 7.9936e-15 V2 <--> V2
x3e     0.64142  0.081540  7.8663 3.5527e-15 V3 <--> V3
x4e     0.22377  0.084627  2.6443 8.1871e-03 V4 <--> V4
x5e     0.60145  0.079888  7.5287 5.1292e-14 V5 <--> V5
x6e     0.58136  0.079303  7.3309 2.2848e-13 V6 <--> V6
rF2F1  0.68146  0.090654  7.5171 5.5955e-14 post <--- pre
rF3F1  0.48132  0.104495  4.6061 4.1027e-06 post <--- means
> anova(sem.mom3,sem.mom4)
LR Test for Difference Between Models
      Model Df Model Chisq Df LR Chisq Pr(>Chisq)
Model 1      15      179.703
Model 2      17      181.175  2      1.472      0.4791
```