

Psychology 454: Latent Variable Modeling

Advanced modeling with lavaan

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Outline

- 1 Regression and Path Analysis
 - A simple regression
 - A more complicated regression
- 2 CFA and SEM
 - Example 5.1 6 variables
 - Example 5.6 12 variables, 4 correlated factors
 - Consider a higher order structure
 - Latent variables with covariates – Multiple Indicators - Multiple Causes (MIMIC)
 - More interesting SEMs
 - MIMIC with multiple groups
- 3 Growth Models
 - Linear growth
 - A quadratic growth model
 - Piecewise growth

More examples from MPlus manual

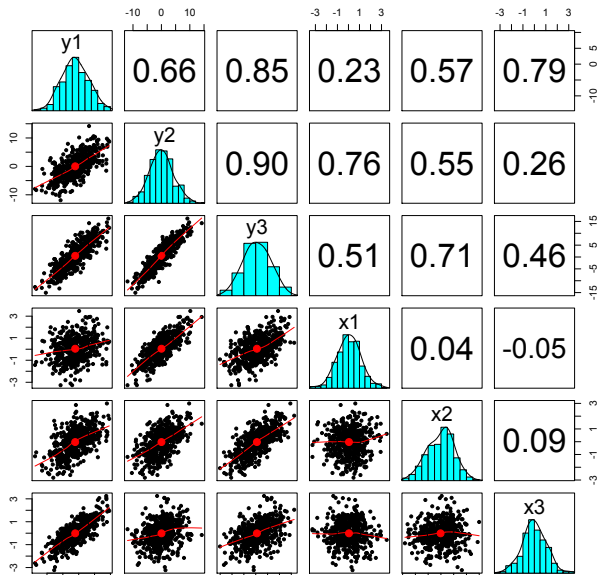
- The *lavaan* introduction <http://users.ugent.be/~yrosseel/lavaan/lavaanIntroduction.pdf> gives examples from the MPlus User's guide
<http://www.statmodel.com/ugexcerpts.shtml>
 - Regression and Path analysis
 - Growth Modeling
 - CFA and SEM
- The data examples may be downloaded and stored in a local directory.
- Then, set the working directory to that folder/directory, e.g.
 - `dir <- "/Volumes/WR/bill/Downloads/chapter3"`
 - `setwd(dir)`

Example A.1: Regression and Path Analysis

```
dir <- "/Volumes/WR/bill/Downloads/chapter3"
setwd(dir)
# ex3.1
Data <- read.table("ex3.1.dat")
names(Data) <- c("y1", "x1", "x2")
model.ex3.1 <- ' y1 ~ x1 + x2 '
fit <- sem(model.ex3.1, data=Data)
summary(fit, standardized=TRUE, fit.measures=TRUE)

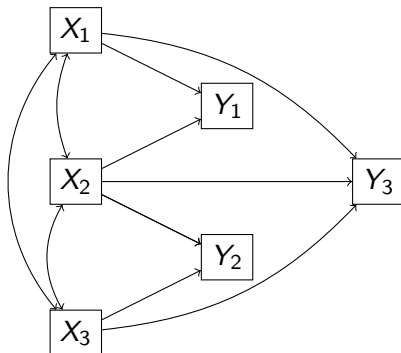
                                Estimate  Std.err  Z-value  P(>|z|)
Regressions:
  y1 ~
      x1          0.969    0.042   23.357    0.000
      x2          0.649    0.044   14.627    0.000
Variances:
      y1          0.941    0.060   15.811    0.000
#compare to linear regression
> lm(y1~x1+x2,data=Data)
Call:
lm(formula = y1 ~ x1 + x2, data = Data)
Coefficients:
(Intercept)          x1          x2
    0.5110      0.9695      0.6490
```

Example 3.11 – 3 criteria, 3 predictors



More complicated regression - 3 dependent variables

```
# ex3.11
Data <- read.table("ex3.11.dat")
names(Data) <- c("y1", "y2", "y3",
  "x1", "x2", "x3")
model.ex3.11 <- ' y1 + y2 + y3 ~ x1 + x2 + x3 '
fit <- sem(model.ex3.11, data=Data)
summary(fit, standardized=TRUE)
```



3 criteria, 3 predictors

	Estimate	Std.err	Z-value	P(> z)	Std.lv	Std.all
Regressions:						
y1 ~						
x1	0.992	0.043	22.979	0.000	0.992	0.254
x2	2.001	0.045	44.618	0.000	2.001	0.495
x3	3.052	0.045	68.274	0.000	3.052	0.758
y2 ~						
x1	2.935	0.050	59.002	0.000	2.935	0.759
x2	1.992	0.052	38.556	0.000	1.992	0.497
x3	1.023	0.051	19.869	0.000	1.023	0.256
y3 ~						
x1	2.672	0.071	37.819	0.000	2.672	0.507
x2	3.544	0.073	48.301	0.000	3.544	0.649
x3	2.318	0.073	31.686	0.000	2.318	0.426
Covariances:						
y1 ~~						
y2	0.002	0.055	0.032	0.975	0.002	0.001
y3	0.529	0.081	6.511	0.000	0.529	0.304
y2 ~~						
y3	1.104	0.102	10.803	0.000	1.104	0.552
Variances:						
y1	1.061	0.067	15.811	0.000	1.061	0.061
y2	1.408	0.089	15.811	0.000	1.408	0.082
y3	2.842	0.180	15.811	0.000	2.842	0.089

Compare to regression

```
> mat.regress(Data,c("x1","x2","x3"),c("y1","y2","y3"))
```

```
Call: mat.regress(m = Data, x = c("x1", "x2", "x3"), y = c("y1", "y2", "y3"))
```

Multiple Regression from matrix input

Beta weights

	y1	y2	y3
x1	0.99	2.93	2.67
x2	2.00	1.99	3.54
x3	3.05	1.02	2.32

Multiple R

y1	y2	y3
0.97	0.96	0.95

Multiple R2

y1	y2	y3
0.94	0.92	0.91

SE of Beta weights

	y1	y2	y3
x1	0.04	0.05	0.07
x2	0.05	0.05	0.07
x3	0.04	0.05	0.07

t of Beta Weights

	y1	y2	y3
x1	22.89	58.77	37.67
x2	44.44	38.40	48.11
x3	68.00	19.79	31.56

Probability of $t <$

	y1	y2	y3
x1	0	0	0
x2	0	0	0
x3	0	0	0

Shrunken R2

y1	y2	y3
0.94	0.92	0.91

Standard Error of R2

y1	y2	y3
0.0052	0.0070	0.0076

F

y1	y2	y3
2550.11	1843.07	1682.39

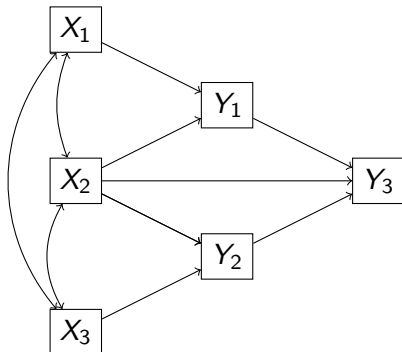
Probability of $F <$

y1	y2	y3
0	0	0

degrees of freedom of regression

[1] 3 496


```
# ex3.11
Data <- read.table("ex3.11.dat")
names(Data) <- c("y1", "y2", "y3",
  "x1", "x2", "x3")
model.ex3.11 <- ' y1 + y2 ~ x1 + x2 + x3
y3 ~ y1 + y2 + x2 '
fit <- sem(model.ex3.11, data=Data)
summary(fit, standardized=TRUE, fit.measures=TRUE)
```



A more complicated regression

ex3.11

Lavaan (0.4-7) converged normally after 46 iterations

Number of observations 500

Estimator ML

Minimum Function Chi-square 0.757

Degrees of freedom 3

P-value 0.860

Chi-square test baseline model:

Regressions:

Estimate Std.err Z-value P(>|z|)

Minimum Function Chi-square 4107.449

Degrees of freedom 12

P-value 0.000

y1 ~

x1

0.992 0.043 22.979 0.000

x2

2.001 0.045 44.618 0.000

x3

3.052 0.045 68.274 0.000

y2 ~

x1

2.935 0.050 59.002 0.000

x2

1.992 0.052 38.556 0.000

x3

1.023 0.051 19.869 0.000

y3 ~

y1

0.507 0.020 25.494 0.000

y2

0.746 0.020 37.918 0.000

x2

1.046 0.072 14.539 0.000

Full model versus baseline model:

Comparative Fit Index (CFI) 1.000

Tucker-Lewis Index (TLI) 1.002

Loglikelihood and Information Criteria:

Loglikelihood user model (H0) -4556.552

Loglikelihood unrestricted model (H1) -4556.174

Number of free parameters 12

Akaike (AIC) 9137.105

Bayesian (BIC) 9187.680

Sample-size adjusted Bayesian (BIC) 9149.591

Variances:

y1

1.061 0.067 15.811 0.000

y2

1.408 0.089 15.811 0.000

y3

1.717 0.109 15.811 0.000

Root Mean Square Error of Approximation:

RMSEA 0.000

90 Percent Confidence Interval 0.000 0.040

P-value RMSEA <= 0.05 0.972

y1	y2
2550.11	1843.07

y3
2892.53

Example 5.1.6 variables

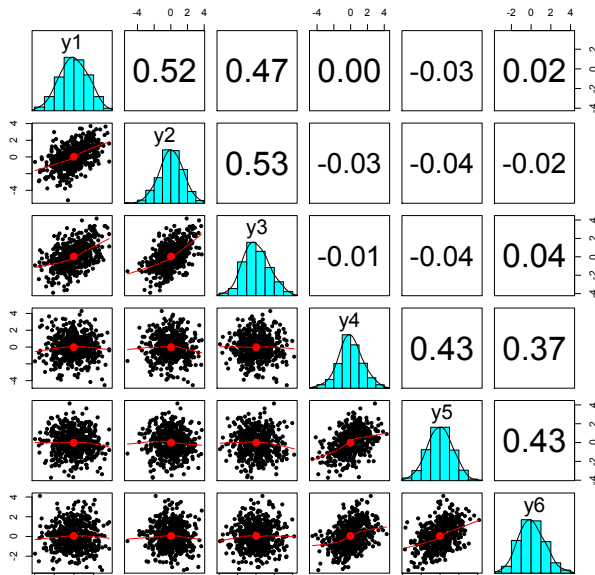
First, get the examples

```
#In browser, download Chapter 5 examples from
      \url{http://www.statmodel.com/ugexcerpts.shtml}
#Set working directory to new folder using the Misc menu or

dir    <- "/Volumes/WR/bill/Downloads/chapter5"
setwd(dir)
#show it
getwd()
[1] "/Volumes/WR/bill/Downloads/chapter5"
# ex5.1
Data <- read.table("ex5.1.dat")
names(Data) <- paste("y", 1:6, sep="")
describe(Data)
```

	var	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
y1	1	500	-0.02	1.41	-0.04	-0.01	1.49	-3.96	3.59	7.55	-0.05	-0.35	0.06
y2	2	500	0.03	1.40	0.04	0.04	1.35	-5.19	3.70	8.90	-0.14	0.12	0.06
y3	3	500	0.03	1.40	-0.06	0.00	1.41	-3.91	4.16	8.07	0.17	-0.12	0.06
y4	4	500	-0.02	1.43	-0.07	-0.03	1.33	-4.55	4.29	8.83	-0.02	0.26	0.06
y5	5	500	-0.02	1.31	-0.01	-0.01	1.34	-3.73	4.15	7.88	-0.04	-0.08	0.06
y6	6	500	0.05	1.30	0.02	0.02	1.34	-3.42	4.11	7.53	0.21	-0.02	0.06

Graph example 5.1



Example 5.1 6 variables

Exploratory factor analysis

```
> fa(Data,2)
```

```
Loading required package: GPArotation
```

```
Factor Analysis using method = minres
```

```
Call: fa(r = Data, nfactors = 2)
```

```
Standardized loadings based upon correlation matrix
```

	MR1	MR2	h2	u2
y1	0.68	0.01	0.46	0.54
y2	0.77	-0.02	0.59	0.41
y3	0.70	0.01	0.49	0.51
y4	0.00	0.61	0.37	0.63
y5	-0.02	0.70	0.50	0.50
y6	0.04	0.61	0.37	0.63

	MR1	MR2
SS loadings	1.54	1.24
Proportion Var	0.26	0.21
Cumulative Var	0.26	0.46

```
With factor correlations of
```

	MR1	MR2
MR1	1.00	-0.04
MR2	-0.04	1.00

```
Test of the hypothesis that 2 factors are sufficient.
```

```
The degrees of freedom for the null model are 15 and the
objective function was 1.19 with Chi Square of 15.00
```

```
The degrees of freedom for the model are 4 and the
objective function was 0.00
```

```
The root mean square of the residuals is 0.01
```

```
The df corrected root mean square of the residuals is 0.02
```

```
The number of observations was 500 with
```

```
Chi Square = 2.18 with prob < 0.7
```

```
Tucker Lewis Index of factoring reliability = 1.012
```

```
RMSEA index = 0 and the 90 % confidence intervals are 0.00 0.02
```

```
BIC = -22.68
```

```
Fit based upon off diagonal values = 1
```

```
Measures of factor score adequacy
```

	MR1	MR2
Correlation of scores with factors	0.87	0.83
Multiple R square of scores with factors	0.76	0.68
Minimum correlation of possible factor scores	0.53	0.37

Example 5.1 6 variables

Confirmatory Factor Analysis is almost identical to EFA because cross loadings are trivial

```
model.ex5.1 <- ' f1 =~ y1 + y2 + y3
f2 =~ y4 + y5 + y6 '
fit <- cfa(model.ex5.1, data=Data,std.ov=TRUE,std.lv=TRUE)
summary(fit, standardized=TRUE, fit.measures=TRUE)
```

Lavaan (0.4-7) converged normally after 16 iterations

			Estimate	Std.err	Z-value	P(> z)
Number of observations	500					
Estimator	ML	Latent variables:				
Minimum Function Chi-square	3.896	f1 =~				
Degrees of freedom	8	y1	0.678	0.047	14.508	0.000
P-value	0.866	y2	0.768	0.047	16.244	0.000
		y3	0.694	0.047	14.832	0.000
Chi-square test baseline model:		f2 =~				
Minimum Function Chi-square	596.921	y4	0.608	0.053	11.481	0.000
Degrees of freedom	15	y5	0.706	0.056	12.700	0.000
P-value	0.000	y6	0.603	0.053	11.411	0.000
Full model versus baseline model:						
Comparative Fit Index (CFI)	1.000					
Tucker-Lewis Index (TLI)	1.013	Covariances:				
Loglikelihood and Information Criteria:		f1 ~~				
Loglikelihood user model (H0)	-3957.300	f2	-0.036	0.062	-0.585	0.559
Loglikelihood unrestricted model (H1)	-3955.352					
Number of free parameters	13	Variances:				
Akaike (AIC)	7940.600	y1	0.539	0.048	11.124	0.000
Bayesian (BIC)	7995.390	y2	0.408	0.051	7.981	0.000
Sample-size adjusted Bayesian (BIC)	7954.127	y3	0.516	0.049	10.600	0.000
Root Mean Square Error of Approximation:		y4	0.628	0.058	10.880	0.000
RMSEA	0.000	y5	0.500	0.065	7.736	0.000
90 Percent Confidence Interval	0.000	y6	0.634	0.057	11.035	0.000
P-value RMSEA <= 0.05	0.995	f1	1.000			
Standardized Root Mean Square Residual:		f2	1.000			

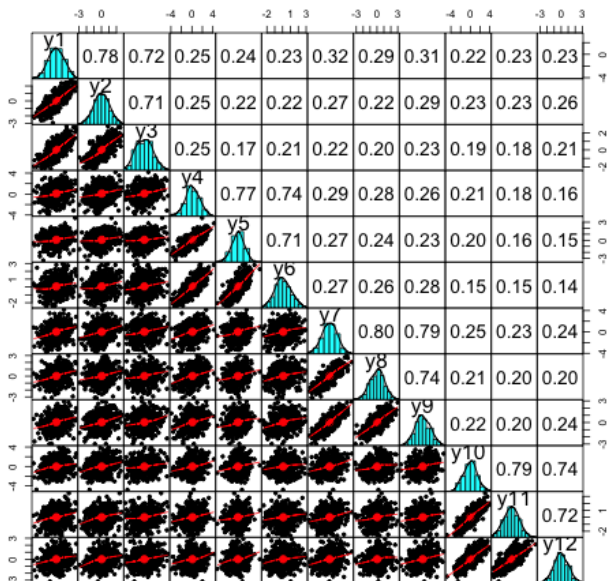
Example 5.6: 12 variables, 4 correlated factors

```
Data <- read.table("ex5.6.dat")
names(Data) <- paste("y", 1:12, sep="")
describe(Data)
pairs.panels(Data,gap=0)
```

	var	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
y1	1	500	0.01	1.31	0.02	0.01	1.38	-3.87	3.39	7.27	-0.04	-0.25	0.06
y2	2	500	0.03	1.02	0.01	0.03	1.01	-2.93	2.75	5.68	0.00	-0.17	0.05
y3	3	500	0.00	0.97	0.01	-0.02	1.03	-2.22	2.79	5.01	0.16	-0.36	0.04
y4	4	500	0.10	1.35	0.04	0.11	1.26	-3.91	4.19	8.11	-0.07	0.04	0.06
y5	5	500	0.08	1.01	0.11	0.09	1.00	-3.34	3.70	7.04	-0.12	0.18	0.05
y6	6	500	0.08	1.02	0.02	0.06	1.04	-2.46	2.95	5.41	0.16	-0.20	0.05
y7	7	500	0.02	1.38	0.05	0.04	1.37	-3.78	4.16	7.94	-0.06	-0.12	0.06
y8	8	500	0.03	1.04	0.08	0.03	1.04	-3.17	2.99	6.16	-0.11	-0.03	0.05
y9	9	500	0.03	1.03	-0.03	0.03	1.05	-2.98	2.92	5.90	0.04	-0.19	0.05
y10	10	500	-0.02	1.38	0.05	-0.02	1.31	-4.79	3.95	8.74	0.01	-0.04	0.06
y11	11	500	0.01	1.06	0.01	0.01	1.07	-2.62	3.46	6.08	-0.01	-0.16	0.05
y12	12	500	0.01	1.01	-0.07	0.00	1.01	-3.18	2.88	6.06	0.06	-0.07	0.05

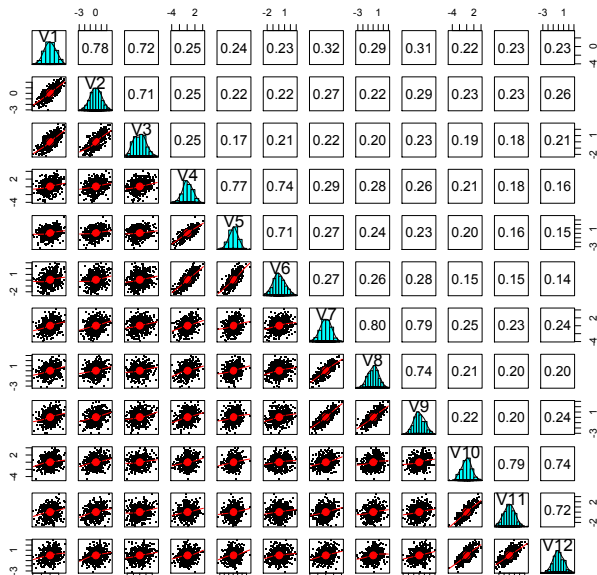
Example 5.6: 12 variables, 4 correlated factors

Example 5.6: 12 variables, ? factors



Example 5.6: 12 variables, 4 correlated factors

Example 5.6: 12 variables, ? factors



Better yet, just show the correlations

```
> pairs.panels(ex.5.6, pch=".")
> lowerCor(ex.5.6)
```

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12
V1	1.00											
V2	0.78	1.00										
V3	0.72	0.71	1.00									
V4	0.25	0.25	0.25	1.00								
V5	0.24	0.22	0.17	0.77	1.00							
V6	0.23	0.22	0.21	0.74	0.71	1.00						
V7	0.32	0.27	0.22	0.29	0.27	0.27	1.00					
V8	0.29	0.22	0.20	0.28	0.24	0.26	0.80	1.00				
V9	0.31	0.29	0.23	0.26	0.23	0.28	0.79	0.74	1.00			
V10	0.22	0.23	0.19	0.21	0.20	0.15	0.25	0.21	0.22	1.00		
V11	0.23	0.23	0.18	0.18	0.16	0.15	0.23	0.20	0.20	0.79	1.00	
V12	0.23	0.26	0.21	0.16	0.15	0.14	0.24	0.20	0.24	0.74	0.72	1.00

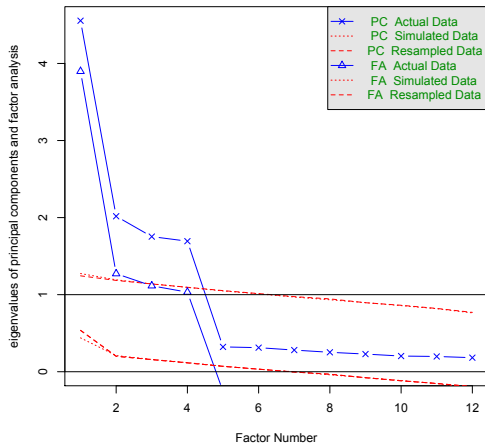
Example 3.6 12 variables, 4 correlated factors

Parallel Analysis

```
fa.parallel(Data)
```

Parallel analysis suggests that the number of factors = 4 and the number of components = 4

Parallel Analysis Scree Plots



Example 3.6 12 variables, 4 correlated factors

Extract 4 factors using EFA

```
fa(Data,4)
```

```
Factor Analysis using method = minres
```

```
Call: fa(r = Data, nfactors = 4)
```

```
Standardized loadings based upon correlation matrix
```

	MR1	MR2	MR4	MR3	h2	u2
y1	0.05	-0.01	0.87	0.00	0.79	0.21
y2	-0.01	0.02	0.87	0.00	0.77	0.23
y3	-0.04	-0.01	0.83	0.01	0.67	0.33
y4	0.00	0.01	0.01	0.88	0.80	0.20
y5	-0.02	0.01	-0.01	0.87	0.74	0.26
y6	0.02	-0.03	0.00	0.82	0.69	0.31
y7	0.92	0.01	-0.01	0.00	0.86	0.14
y8	0.87	-0.01	-0.02	0.01	0.74	0.26
y9	0.85	0.00	0.03	-0.01	0.74	0.26
y10	0.00	0.90	-0.03	0.02	0.81	0.19
y11	-0.01	0.88	0.00	0.00	0.76	0.24
y12	0.02	0.82	0.04	-0.03	0.68	0.32

	MR1	MR2	MR4	MR3
SS loadings	2.34	2.26	2.23	2.22
Proportion Var	0.19	0.19	0.19	0.19
Cumulative Var	0.19	0.38	0.57	0.75

```
With factor correlations of
```

	MR1	MR2	MR4	MR3
MR1	1.00	0.29	0.34	0.35
MR2	0.29	1.00	0.29	0.22
MR4	0.34	0.29	1.00	0.30
MR3	0.35	0.22	0.30	1.00

```
Test of the hypothesis that 4 factors are sufficient.
```

```
The degrees of freedom for the null model are 66 and the
objective function was 8.02 with Chi Square of 3965.23
The degrees of freedom for the model are 24 and the
objective function was 0.06
```

```
The root mean square of the residuals is 0.01
The df corrected root mean square of the residuals is 0.01
The number of observations was 500 with
Chi Square = 27.97 with prob < 0.26
```

```
Tucker Lewis Index of factoring reliability = 0.997
```

```
RMSEA index = 0.019 and the
```

```
90 % confidence intervals are 0.019 0.023
```

```
BIC = -121.18
```

```
Fit based upon off diagonal values = 1
```

```
Measures of factor score adequacy
```

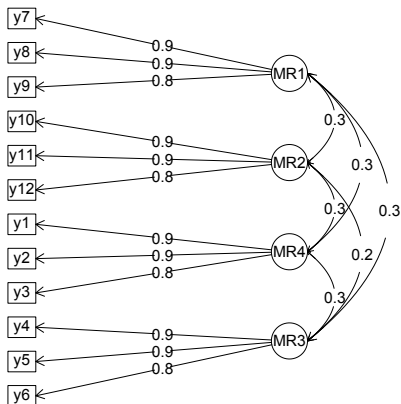
	MR1	MR2	MR4	MR3
Correlation of scores with factors	0.96	0.95	0.95	0.95
Multiple R square of scores with factors	0.92	0.91	0.90	0.90
Minimum correlation of possible factor scores	0.84	0.82	0.80	0.80

Example 3.6: 12 variables, 4 correlated factors

Show the 4 factor exploratory solution

```
fa.diagram(fa(Data,4),cut=.2)
```

Factor Analysis



Example 5.6 12 variables, 4 correlated factors

4 factors using CFA

```

Data <- read.table("ex5.6.dat")
names(Data) <- paste("y", 1:12, sep="")
model.ex5.6 <- ' f1 =~ y1 + y2 + y3
f2 =~ y4 + y5 + y6
f3 =~ y7 + y8 + y9
f4 =~ y10 + y11 + y12'
fit <- cfa(model.ex5.6, data=Data, estimator="ML",
std.ov=TRUE, std.lv=TRUE)
summary(fit, standardized=TRUE, fit.measures=TRUE)

```

		Estimate	Std.err	Z-value	P(> z)
Latent variables:					
	f1 =~				
	y1	0.892	0.037	24.413	0.000
	y2	0.872	0.037	23.576	0.000
	y3	0.811	0.038	21.238	0.000
	f2 =~				
	y4	0.893	0.037	24.421	0.000
	y5	0.857	0.037	22.997	0.000
	y6	0.827	0.038	21.846	0.000
	f3 =~				
	y7	0.924	0.035	26.284	0.000
	y8	0.859	0.037	23.425	0.000
	y9	0.857	0.037	23.367	0.000
	f4 =~				
	y10	0.899	0.036	24.819	0.000
	y11	0.872	0.037	23.721	0.000
	y12	0.826	0.038	21.896	0.000
Covariances:					
	f1 ~~				
	f2	0.310	0.045	6.817	0.000
	f3	0.351	0.044	8.050	0.000
	f4	0.292	0.046	6.374	0.000
	f2 ~~				
	f3	0.348	0.044	7.952	0.000
	f4	0.226	0.047	4.790	0.000
	f3 ~~				
	f4	0.292	0.045	6.456	0.000
Variances:					
	f4	0.292	0.045	6.456	0.000

```

Number of observations      500
Estimator                   ML
Minimum Function Chi-square 45.780
Degrees of freedom          48
P-value                     0.564
Chi-square test baseline model:
  Minimum Function Chi-square 4012.035
  Degrees of freedom          66
  P-value                     0.000
Full model versus baseline model:
  Comparative Fit Index (CFI) 1.000
  Tucker-Lewis Index (TLI)   1.001
...
Root Mean Square Error of Approximation:
  RMSEA 0.000
  90 Percent Confidence Interval 0.000 0.027
  P-value RMSEA <= 0.05 1.000

```

Omega solution

```
om.5_6 <- omega(Data,4,sl=FALSE)
```

Omega

Call: omega(m = Data, nfactors = 4)

Alpha: 0.85

G.6: 0.92

Omega Hierarchical: 0.6

Omega H asymptotic: 0.63

Omega Total 0.95

Schmid Leiman Factor loadings greater than 0.2

	g	F1*	F2*	F3*	F4*	h2	u2	p2
y1	0.53				0.72	0.79	0.21	0.35
y2	0.50				0.71	0.77	0.23	0.33
y3	0.45				0.68	0.67	0.33	0.30
y4	0.48			0.75		0.80	0.20	0.29
y5	0.45			0.74		0.74	0.26	0.27
y6	0.44			0.70		0.68	0.32	0.28
y7	0.57	0.72				0.86	0.14	0.39
y8	0.53	0.68				0.74	0.26	0.38
y9	0.54	0.67				0.74	0.26	0.40
y10	0.42		0.80			0.82	0.18	0.21
y11	0.40		0.78			0.76	0.24	0.21
y12	0.40		0.72			0.68	0.32	0.23

With eigenvalues of:

g F1* F2* F3* F4*

2.8 1.4 1.8 1.6 1.5

general/max 1.56 max/min = 1.23

mean percent general = 0.3 with sd = 0.07 and cv of 0.22

The degrees of freedom are 24 and the fit is 0.06

The number of observations was 500 with Chi Square = 27.97

with prob < 0.26

The root mean square of the residuals is 0.01

The df corrected root mean square of the residuals is 0.01

RMSEA index = 0.019 and the 90 %

confidence intervals are 0.019 0.023

BIC = -121.18

Compare this with the adequacy of just a general factor and no group factors

The degrees of freedom for just the general factor are 54 and the fit is 5.33

The number of observations was 500 with Chi Square = 2630.39 with prob < 0

The root mean square of the residuals is 0.16

The df corrected root mean square of the residuals is 0.25

RMSEA index = 0.311 and the 90 % confidence intervals are 0.3

BIC = 2294.8

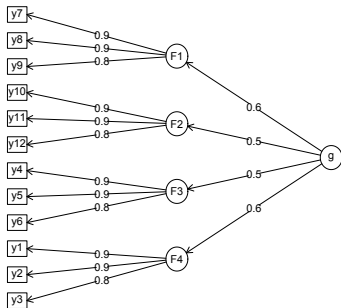
Measures of factor score adequacy

	g	F1*	F2*	F3*
Correlation of scores with factors	0.78	0.83	0.9	0.87
Multiple R square of scores with factors	0.61	0.69	0.8	0.75
Minimum correlation of factor score estimates	0.22	0.38	0.6	0.50

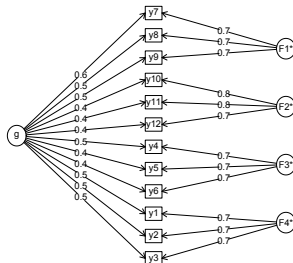
Consider a higher order structure

Two omega solutions

Omega



Omega



Consider a higher order structure

Higher order model with CFA

```

Data <- read.table("ex5.6.dat")
names(Data) <- paste("y", 1:12, sep="")
model.ex5.6 <- ' f1 =~ y1 + y2 + y3
f2 =~ y4 + y5 + y6
f3 =~ y7 + y8 + y9
f4 =~ y10 + y11 + y12
f5 =~ f1 + f2 + f3 + f4'
fit <- cfa(model.ex5.6, data=Data, estimator="ML",
           std.ov=TRUE, std.lv=TRUE)
summary(fit, standardized=TRUE, fit.measures=TRUE)

```

		Estimate	Std.err	Z-value	P(> z)
Latent variables:					
	f1 =~				
	y1	0.727	0.041	17.731	0.000
	y2	0.709	0.041	17.433	0.000
	y3	0.660	0.040	16.439	0.000
	f2 =~				
	y4	0.755	0.039	19.212	0.000
	y5	0.725	0.039	18.553	0.000
	y6	0.699	0.039	17.932	0.000
	f3 =~				
	y7	0.721	0.044	16.557	0.000
	y8	0.670	0.042	15.845	0.000
	y9	0.669	0.042	15.829	0.000
	f4 =~				
	y10	0.795	0.038	21.154	0.000
	y11	0.771	0.038	20.491	0.000
	y12	0.730	0.038	19.283	0.000
	f5 =~				
	f1	0.713	0.098	7.270	0.000
	f2	0.632	0.088	7.192	0.000
	f3	0.802	0.111	7.214	0.000
	f4	0.528	0.078	6.785	0.000
Variances:					
	y1	0.201	0.024	8.252	0.000
	y2	0.239	0.025	9.570	0.000
	y3	0.341	0.028	12.265	0.000
	y4	0.201	0.024	8.221	0.000
	y5	0.263	0.026	10.298	0.000
Root Mean Square Error of Approximation:					
RMSEA	0.000				
90 Percent Confidence Interval	0.000	0.026			
P-value RMSEA <= 0.05	1.000				

...

Consider a higher order structure

Try a Schmid Leiman model

```
Data <- read.table("ex5.6.dat")
names(Data) <- paste("y", 1:12, sep="")
model.ex5.6 <- ' f1 =~ y1 + y2 + y3
```

```
f2 =~ y4 + y5 + y6
```

```
f3 =~ y7 + y8 + y9
```

```
f4 =~ y10 + y11 + y12
```

```
g =~ y1 + y2 + y3 + y4 + y5 + y6 + y7 +
      y8 + y9 + y10 + y11 + y12'
```

```
fit <- cfa(model.ex5.6, data=Data, estimator="ML",
  std.ov=TRUE, std.lv=TRUE, orthogonal=TRUE)
```

```
Number of observations
```

```
500
```

```
Estimator
```

```
ML
```

```
Minimum Function Chi-square
```

```
41.640
```

```
Degrees of freedom
```

```
42
```

```
P-value
```

```
0.487
```

```
Chi-square test baseline model:
```

```
Minimum Function Chi-square
```

```
4012.035
```

```
Degrees of freedom
```

```
66
```

```
P-value
```

```
0.000
```

```
Full model versus baseline model:
```

```
Comparative Fit Index (CFI)
```

```
1.000
```

```
Tucker-Lewis Index (TLI)
```

```
1.000
```

```
Loglikelihood and Information Criteria:
```

```
Latent variables:
```

```
f1 =~
```

```
y1
```

```
0.704
```

```
0.045
```

```
15.768
```

```
0.000
```

```
y2
```

```
0.709
```

```
0.045
```

```
15.887
```

```
0.000
```

```
y3
```

```
0.694
```

```
0.045
```

```
15.314
```

```
0.000
```

```
f2 =~
```

```
y4
```

```
0.752
```

```
0.042
```

```
18.112
```

```
0.000
```

```
y5
```

```
0.742
```

```
0.042
```

```
17.667
```

```
0.000
```

```
y6
```

```
0.705
```

```
0.043
```

```
16.453
```

```
0.000
```

```
f3 =~
```

```
y7
```

```
0.713
```

```
0.047
```

```
15.248
```

```
0.000
```

```
y8
```

```
0.679
```

```
0.048
```

```
14.207
```

```
0.000
```

```
y9
```

```
0.646
```

```
0.049
```

```
13.211
```

```
0.000
```

```
f4 =~
```

```
y10
```

```
0.798
```

```
0.039
```

```
20.266
```

```
0.000
```

```
y11
```

```
0.781
```

```
0.040
```

```
19.546
```

```
0.000
```

```
y12
```

```
0.718
```

```
0.041
```

```
17.473
```

```
0.000
```

```
g =~
```

```
y1
```

```
0.543
```

```
0.057
```

```
9.490
```

```
0.000
```

```
y2
```

```
0.508
```

```
0.058
```

```
8.798
```

```
0.000
```

```
y3
```

```
0.431
```

```
0.059
```

```
7.356
```

```
0.000
```

```
y4
```

```
0.476
```

```
0.056
```

```
8.448
```

```
0.000
```

```
y5
```

```
0.435
```

```
0.057
```

```
7.644
```

```
0.000
```

```
y6
```

```
0.433
```

```
0.057
```

```
7.618
```

```
0.000
```

```
y7
```

```
0.588
```

```
0.059
```

```
10.039
```

```
0.000
```

```
y8
```

```
0.528
```

```
0.059
```

```
8.898
```

```
0.000
```

```
y9
```

```
0.563
```

```
0.059
```

```
9.565
```

```
0.000
```

```
y10
```

```
0.414
```

```
0.056
```

```
7.374
```

```
0.000
```

```
y11
```

```
0.392
```

```
0.056
```

```
6.944
```

```
0.000
```

```
27 / 49
```

Consider a higher order structure

Compare the omega solution with cfa higher order model

Schmid Leiman Factor loadings greater than 0.2

	g	F1*	F2*	F3*	F4*	h2	u2	p2
y1	0.53				0.72	0.79	0.21	0.35
y2	0.50				0.71	0.77	0.23	0.33
y3	0.45				0.68	0.67	0.33	0.30
y4	0.48			0.75		0.80	0.20	0.29
y5	0.45			0.74		0.74	0.26	0.27
y6	0.44			0.70		0.68	0.32	0.28
y7	0.57	0.72				0.86	0.14	0.39
y8	0.53	0.68				0.74	0.26	0.38
y9	0.54	0.67				0.74	0.26	0.40
y10	0.42		0.80			0.82	0.18	0.21
y11	0.40		0.78			0.76	0.24	0.21
y12	0.40		0.72			0.68	0.32	0.23

With eigenvalues of:

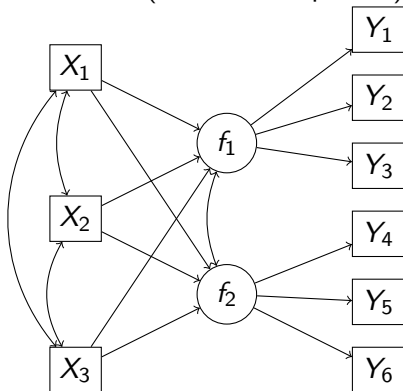
g	F1*	F2*	F3*	F4*
2.8	1.4	1.8	1.6	1.5

Latent variables:

f1 =~				
y1	0.704	0.045	15.768	0.000
y2	0.709	0.045	15.887	0.000
y3	0.694	0.045	15.314	0.000
f2 =~				
y4	0.752	0.042	18.112	0.000
y5	0.742	0.042	17.667	0.000
y6	0.705	0.043	16.453	0.000
f3 =~				
y7	0.713	0.047	15.248	0.000
y8	0.679	0.048	14.207	0.000
y9	0.646	0.049	13.211	0.000
f4 =~				
y10	0.798	0.039	20.266	0.000
y11	0.781	0.040	19.546	0.000
y12	0.718	0.041	17.473	0.000
g =~				
y1	0.543	0.057	9.490	0.000
y2	0.508	0.058	8.798	0.000
y3	0.431	0.059	7.356	0.000
y4	0.476	0.056	8.448	0.000
y5	0.435	0.057	7.644	0.000
y6	0.433	0.057	7.618	0.000
y7	0.588	0.059	10.039	0.000
y8	0.528	0.059	8.898	0.000
y9	0.563	0.059	9.565	0.000
y10	0.414	0.056	7.374	0.000
y11	0.392	0.056	6.944	0.000
y12	0.405	0.056	7.201	0.000

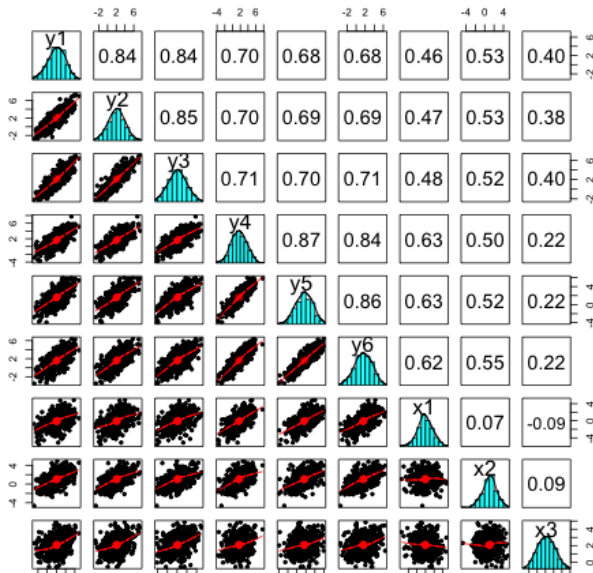
A MIMIC model

Consider the case of 6 variables with two latent factors that have 3 covariates (MPlus Example 5.8)



```
Data <- read.table("ex5.8.dat")
names(Data) <- c(paste("y", 1:6, sep=""),
                 paste("x", 1:3, sep=""))
model.ex5.8 <- ' f1 =~ y1 + y2 + y3
f2 =~ y4 + y5 + y6
f1 + f2 ~ x1 + x2 + x3 '
fit <- cfa(model.ex5.8, data=Data,
            estimator="ML",std.lv=TRUE)
summary(fit, standardized=TRUE,
        fit.measures=TRUE)
```

The MIMIC correlations 2 - factors



A MIMIC model

Lavaan (0.4-7) converged normally after 27 iterations

Number of observations	500				
Estimator	ML	Latent variables:			
Minimum Function Chi-square	20.023	f1 =~			
Degrees of freedom	20	y1	0.849	0.036	23.407
P-value	0.457	y2	0.876	0.037	23.627
		y3	0.867	0.037	23.733
		f2 =~			
Chi-square test baseline model:		y4	0.802	0.035	23.067
		y5	0.780	0.033	23.504
Minimum Function Chi-square	4176.601	y6	0.783	0.034	23.046
Degrees of freedom	33				
P-value	0.000				
		Regressions:			
Full model versus baseline model:		f1 ~			
		x1	0.585	0.037	15.786
		x2	0.667	0.043	15.414
Comparative Fit Index (CFI)	1.000	x3	0.814	0.058	13.950
Tucker-Lewis Index (TLI)	1.000	f2 ~			
		x1	0.846	0.045	18.783
Loglikelihood and Information Criteria:		x2	0.750	0.046	16.237
		x3	0.539	0.054	9.975
Loglikelihood user model (H0)	-6585.177				
Loglikelihood unrestricted model (H1)	-6575.166				
		Covariances:			
		f1 ~~			
Number of free parameters	19	f2	0.336	0.051	6.621
Akaike (AIC)	13208.355				
Bayesian (BIC)	13288.432				
Sample-size adjusted Bayesian (BIC)	13228.125				
		Variances:			
		y1	0.529	0.046	11.505

Consider the two factor measurement model

```
f2 <- fa(Data[1:6],2)
```

Factor Analysis using method = minres

Call: fa(r = Data[1:6], nfactors = 2)

Standardized loadings based

upon correlation matrix

	MR1	MR2	h2	u2
y1	-0.01	0.93	0.84	0.16
y2	0.00	0.93	0.85	0.15
y3	0.03	0.90	0.85	0.15
y4	0.88	0.05	0.85	0.15
y5	0.99	-0.05	0.90	0.10
y6	0.89	0.03	0.84	0.16

	MR1	MR2
SS loadings	2.56	2.56
Proportion Var	0.43	0.43
Cumulative Var	0.43	0.85

With factor correlations of

	MR1	MR2
MR1	1.00	0.81
MR2	0.81	1.00

Test of the hypothesis that 2 factors are sufficient.

The degrees of freedom for the null model are 15 and the objective function was 6.59 with Chi Square of 3270

The degrees of freedom for the model are 4 and the objective function was 0.01

The root mean square of the residuals is 0

The df corrected root mean square of the residuals is 0.01

The number of observations was 500 with

Chi Square = 3.33 with prob <

Tucker Lewis Index of factoring reliability = 1.001

RMSEA index = 0 and the 90 %

confidence intervals are 0 0.024

BIC = -21.53

Fit based upon off diagonal values = 1

Measures of factor score adequacy

	MR1	MR2
Correlation of scores with factors	0.98	0.97
Multiple R square of scores with factors	0.95	0.95
Minimum correlation of possible factor scores	0.91	0.90

Note the drastic difference in correlations from the two factor model versus the MIMIC model.

Consider the following 12 variables

```
> Data <- read.table("ex5.11.dat")
> names(Data) <- paste("y", 1:12, sep="")

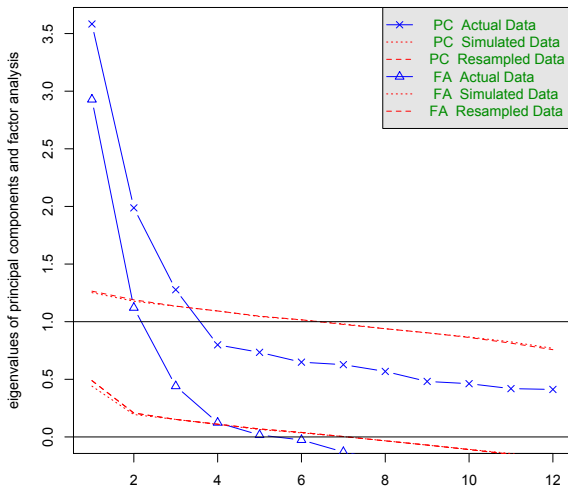
> round(cor(Data),2)
```

	y1	y2	y3	y4	y5	y6	y7	y8	y9	y10	y11	y12
y1	1.00	0.52	0.41	-0.02	0.00	-0.02	0.27	0.24	0.26	0.11	0.14	0.13
y2	0.52	1.00	0.52	0.01	-0.03	0.00	0.23	0.18	0.28	0.14	0.15	0.18
y3	0.41	0.52	1.00	0.02	-0.11	-0.01	0.16	0.17	0.20	0.09	0.08	0.12
y4	-0.02	0.01	0.02	1.00	0.44	0.39	0.34	0.34	0.30	0.16	0.12	0.08
y5	0.00	-0.03	-0.11	0.44	1.00	0.42	0.28	0.27	0.25	0.15	0.09	0.08
y6	-0.02	0.00	-0.01	0.39	0.42	1.00	0.30	0.31	0.34	0.11	0.13	0.12
y7	0.27	0.23	0.16	0.34	0.28	0.30	1.00	0.56	0.55	0.32	0.27	0.27
y8	0.24	0.18	0.17	0.34	0.27	0.31	0.56	1.00	0.53	0.28	0.25	0.23
y9	0.26	0.28	0.20	0.30	0.25	0.34	0.55	0.53	1.00	0.29	0.24	0.29
y10	0.11	0.14	0.09	0.16	0.15	0.11	0.32	0.28	0.29	1.00	0.42	0.31
y11	0.14	0.15	0.08	0.12	0.09	0.13	0.27	0.25	0.24	0.42	1.00	0.33
y12	0.13	0.18	0.12	0.08	0.08	0.12	0.27	0.23	0.29	0.31	0.33	1.00

12 variables, how many factors?

fa.parallel(Data)

Parallel Analysis Scree Plots



Try 3 factors

```
f3 <- fa(Data,3)
```

```
> f3 <- fa(Data,3)
```

```
> f3
```

```
Factor Analysis using method = minres
```

```
Call: fa(r = Data, nfactors = 3)
```

```
Standardized loadings based upon correlation matrixTest of the hypothesis that 3 factors are sufficient.
```

	MR1	MR2	MR3	h2	u2
y1	0.03	0.68	-0.02	0.46	0.54
y2	-0.02	0.76	0.00	0.57	0.43
y3	-0.03	0.67	-0.06	0.42	0.58
y4	0.65	-0.09	-0.07	0.38	0.62
y5	0.62	-0.15	-0.08	0.35	0.65
y6	0.63	-0.10	-0.07	0.36	0.64
y7	0.54	0.18	0.21	0.53	0.47
y8	0.55	0.17	0.16	0.48	0.52
y9	0.53	0.24	0.16	0.51	0.49
y10	0.03	-0.06	0.63	0.39	0.61
y11	-0.04	-0.05	0.66	0.39	0.61
y12	0.02	0.04	0.49	0.26	0.74

	MR1	MR2	MR3
SS loadings	2.17	1.68	1.27
Proportion Var	0.18	0.14	0.11
Cumulative Var	0.18	0.32	0.43

```
With factor correlations of
```

	MR1	MR2	MR3
MR1	1.00	0.17	0.43
MR2	0.17	1.00	0.37
MR3	0.43	0.37	1.00

```
The degrees of freedom for the null model are 66 and the
objective function was 3.05 with Chi Square of 15
```

```
The degrees of freedom for the model are 33 and the
objective function was 0.17
```

```
The root mean square of the residuals is 0.02
```

```
The df corrected root mean square of the residuals is 0.04
```

```
The number of observations was 500 with
```

```
Chi Square = 81.42 with prob
```

```
Tucker Lewis Index of factoring reliability = 0.932
```

```
RMSEA index = 0.055 and the 90 % confidence intervals are
```

```
BIC = -123.66
```

```
Fit based upon off diagonal values = 0.99
```

```
Measures of factor score adequacy
```

	MR1	MR2	MR3
Correlation of scores with factors	0.89	0.88	0.83
Multiple R square of scores with factors	0.80	0.77	0.70
Minimum correlation of possible factor scores	0.59	0.55	0.39

Try 4 factor solution

```
> f4 <- fa(Data,4)
> f4
```

Factor Analysis using method = minres

Call: fa(r = Data, nfactors = 4)

Standardized loadings based upon correlation matrix

	MR1	MR2	MR4	MR3	h2	u2
y1	0.21	0.56	-0.11	-0.04	0.43	0.57
y2	-0.07	0.85	0.05	0.04	0.69	0.31
y3	0.07	0.61	-0.07	-0.04	0.40	0.60
y4	0.12	0.00	0.58	-0.02	0.42	0.58
y5	-0.05	0.01	0.71	0.01	0.47	0.53
y6	0.11	-0.01	0.55	-0.01	0.38	0.62
y7	0.72	0.00	0.03	0.06	0.59	0.41
y8	0.74	-0.04	0.02	0.00	0.55	0.45
y9	0.62	0.10	0.08	0.04	0.53	0.47
y10	0.06	-0.02	0.02	0.60	0.40	0.60
y11	-0.03	0.00	-0.01	0.68	0.44	0.56
y12	0.11	0.05	-0.04	0.44	0.26	0.74

	MR1	MR2	MR4	MR3
SS loadings	1.75	1.48	1.25	1.08
Proportion Var	0.15	0.12	0.10	0.09
Cumulative Var	0.15	0.27	0.37	0.46

With factor correlations of

	MR1	MR2	MR4	MR3
MR1	1.00	0.38	0.54	0.52
MR2	0.38	1.00	-0.07	0.26
MR4	0.54	-0.07	1.00	0.26
MR3	0.52	0.26	0.26	1.00

Test of the hypothesis that 4 factors are sufficient.

The degrees of freedom for the null model are 66 and the objective function was 3.05 with Chi Square

The degrees of freedom for the model are 24 and the objective function was 0.06

The root mean square of the residuals is 0.01

The df corrected root mean square of the residuals is 0.03

The number of observations was 500 with Chi Square = 27.21 with prob < 0.29

Tucker Lewis Index of factoring reliability = 0.994

RMSEA index = 0.017 and the 90 % confidence intervals are 0.017 0.021

BIC = -121.94

Fit based upon off diagonal values = 1

Measures of factor score adequacy

	MR1	MR2	MR4
Correlation of scores with factors	0.90	0.89	0.85

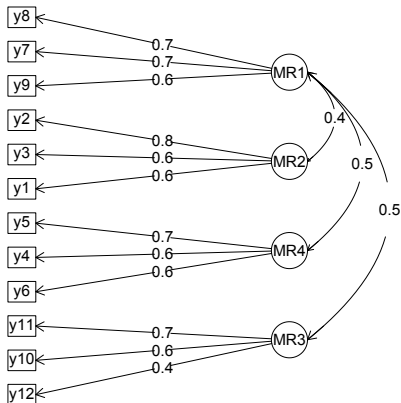
Multiple R square of scores with factors	0.81	0.79	0.72
--	------	------	------

Minimum correlation of possible factor scores	0.63	0.58	0.44
---	------	------	------

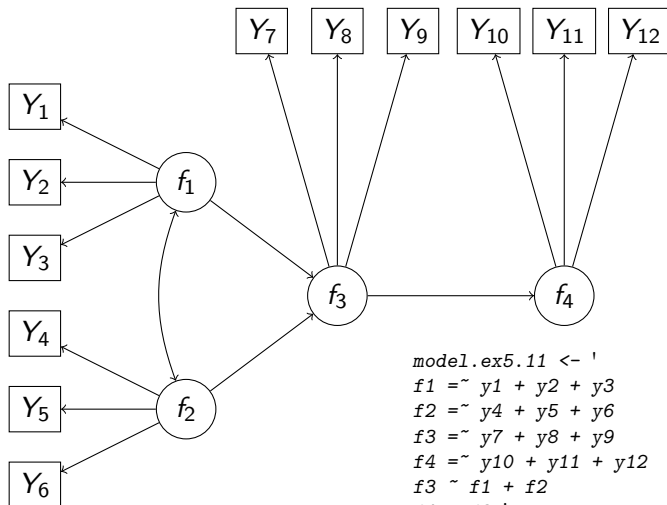
4 latent variables – somewhat correlated

```
diagram(f4,cut=.3)
```

Factor Analysis



Four latent variables with a particular structure



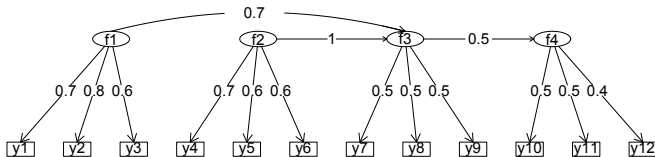
Fit the sem model

```
fit <- sem(model.ex5.11, data=Data, estimator="ML",
           std.ov=TRUE, std.lv=TRUE)
summary(fit, standardized=TRUE, fit.measures=TRUE)
```

			Estimate	Std.err	Z-value	P(> z)
Number of observations	500	Latent variables:				
		f1 =~				
Estimator	ML	y1	0.678	0.046	14.621	0.000
Minimum Function Chi-square	53.704	y2	0.779	0.046	16.826	0.000
Degrees of freedom	50	y3	0.637	0.047	13.680	0.000
P-value	0.334	f2 =~				
		y4	0.659	0.048	13.706	0.000
Chi-square test baseline model:		y5	0.643	0.048	13.349	0.000
		y6	0.632	0.048	13.107	0.000
Minimum Function Chi-square	1524.403	f3 =~				
Degrees of freedom	66	y7	0.487	0.040	12.109	0.000
P-value	0.000	y8	0.459	0.039	11.769	0.000
		y9	0.468	0.039	11.884	0.000
Full model versus baseline model:		f4 =~				
		y10	0.518	0.048	10.806	0.000
Comparative Fit Index (CFI)	0.997	y11	0.502	0.047	10.636	0.000
Tucker-Lewis Index (TLI)	0.997	y12	0.418	0.045	9.298	0.000
Loglikelihood and Information Criteria:		Regressions:				
		f3 ~				
Loglikelihood user model (H0)	-7772.275	f1	0.713	0.102	6.962	0.000
Loglikelihood unrestricted model (H1)	-7745.424	f2	1.004	0.127	7.906	0.000
		f4 ~				
Number of free parameters	28	f3	0.471	0.066	7.128	0.000
Akaike (AIC)	15600.551	Covariances:				
Bayesian (BIC)	15718.560	f1 ~~				
Sample-size adjusted Bayesian (BIC)	15629.686	f2	-0.034	0.062	-0.546	0.585

A SEM diagram using lavaan.diagram (from psych)

Example 5.11



Multiple group models can improve power

- Can improve power by “borrowing strength” from other samples
- Consider the case of figural invariance across groups
 - Items have same loadings on latent variables
 - Latent variable correlations differ across groups
 - Borrow strength in fixing figural invariance, and then study the difference in latent correlations

MIMIC with multiple groups

Consider the MIMIC model of Example 5.8, but with a grouping variable

```
Data <- read.table("ex5.14.dat")
names(Data) <- c("y1", "y2", "y3", "y4", "y5", "y6",
                 "x1", "x2", "x3", "g")
model.ex5.14 <- ' f1 =~ y1 + y2 + y3
f2 =~ y4 + y5 + y6
f1 + f2 ~ x1 + x2 + x3 '
fit <- cfa(model.ex5.14, data=Data, group="g",
           meanstructure=FALSE, group.equal=c("loadings"),
           group.partial=c("f1=~y3"), std.ov=TRUE)
summary(fit, standardized=TRUE, fit.measures=TRUE)

Lavaan (0.4-7) converged normally after 47 iterations
```

Group 1 [1]:

Latent variables:

f1 =~

y1

y2

y3

f2 =~

y4

y5

y6

Estimate	Std.err	Z-value	P(> z)
0.480	0.014	33.915	0.000
0.481	0.014	34.007	0.000
0.483	0.016	29.612	0.000
0.423	0.012	34.369	0.000
0.431	0.012	35.073	0.000
0.429	0.012	34.882	0.000

Number of observations per group

1

2

Estimator

Minimum Function Chi-square

Degrees of freedom

P-value

Chi-square for each group:

1

2

Chi-square test baseline model:

Minimum Function Chi-square

Degrees of freedom

P-value

Full model versus baseline model:

Regressions:

500 f1 ~

x1

x2

x3

45 f2 ~

x1

x2

x3

Covariances:

8731.537 f1 ~~

f2

Variances:

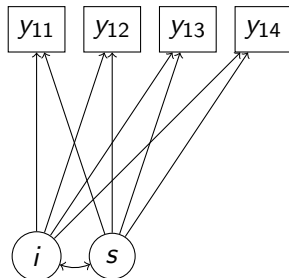
0.988	0.057	17.388	0.000
0.951	0.056	16.886	0.000
0.821	0.055	14.979	0.000
1.438	0.064	22.624	0.000
1.076	0.058	18.504	0.000
0.547	0.053	10.398	0.000

0.339	0.050	6.787	0.000
-------	-------	-------	-------

Growth models

- If traits change over time, one can try to model the growth in the means structure
 - Intercepts are set to be equal
 - Slope parameter models changes in the means
 - Variables can be observed or latent
- Can also model more complicated growth functions
 - Quadratic growth
 - Piecewise growth

Consider the following growth model



```
Data <- read.table("ex6.1.dat")
names(Data) <- c("y11", "y12", "y13", "y14")
```

```
> round(cor(Data), 2)
```

```
      y11 y12 y13 y14
y11 1.00 0.67 0.59 0.56
y12 0.67 1.00 0.78 0.74
y13 0.59 0.78 1.00 0.85
y14 0.56 0.74 0.85 1.00
```

```
> describe(Data)
```

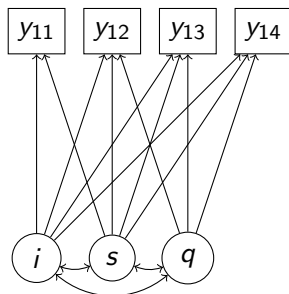
	var	n	mean	sd	median	trimmed	mad	min	max	range
y11	1	500	0.51	1.20	0.56	0.54	1.27	-2.69	3.60	6.29
y12	2	500	1.57	1.41	1.56	1.58	1.28	-3.06	5.96	9.02
y13	3	500	2.57	1.71	2.52	2.58	1.82	-2.75	8.43	11.18
y14	4	500	3.60	2.08	3.61	3.62	2.10	-2.36	9.18	11.54

Growth model for example 6.1

```
model.ex6.1 <- ' i =~ 1*y11 + 1*y12 + 1*y13 + 1*y14
s =~ 0*y11 + 1*y12 + 2*y13 + 3*y14 '
fit <- growth(model.ex6.1, data=Data)
summary(fit, standardized=TRUE, fit.measures=TRUE)
```

			Estimate	Std.err	Z-value	P(> z)
Lavaan (0.4-7) converged normally after 27 iterations						
Latent variables:						
	i =~					
Number of observations	500	y11	1.000			
		y12	1.000			
Estimator	ML	y13	1.000			
Minimum Function Chi-square	4.593	y14	1.000			
Degrees of freedom	5	s =~				
P-value	0.468	y11	0.000			
		y12	1.000			
Chi-square test baseline model:		y13	2.000			
		y14	3.000			
Minimum Function Chi-square	1439.722					
Degrees of freedom		6Covariances:				
P-value	0.000	i ~~				
		s	0.133	0.033	4.057	0.000
Full model versus baseline model:						
		Intercepts:				
Comparative Fit Index (CFI)	1.000	y11	0.000			
Tucker-Lewis Index (TLI)	1.000	y12	0.000			
		y13	0.000			
		y14	0.000			
Loglikelihood and Information Criteria:		i	0.523	0.051	10.153	0.000
Loglikelihood user model (H0)	-3016.386	s	1.026	0.025	40.268	0.000
Loglikelihood unrestricted model (H1)	-3014.089					
		Variances:				
Number of free parameters	9	y11	0.475	0.059	7.989	0.000
Akaike (AIC)	6050.772	y12	0.482	0.040	11.994	0.000

Consider the following growth model



```
Data <- read.table("ex6.9.dat")
names(Data) <- c("y11", "y12", "y13", "y14")
```

```
> round(cor(Data), 2)
```

```
      y11 y12 y13 y14
y11 1.00 0.50 0.33 0.24
y12 0.50 1.00 0.68 0.58
y13 0.33 0.68 1.00 0.89
y14 0.24 0.58 0.89 1.00
```

```
> describe(Data)
```

	var	n	mean	sd	median	trimmed	mad	min	max	ran
y11	1	500	0.52	1.39	0.57	0.55	1.46	-3.19	4.08	7.
y12	2	500	2.09	1.71	2.08	2.10	1.59	-4.25	7.61	11.
y13	3	500	4.62	2.82	4.38	4.64	3.04	-3.56	14.28	17.
y14	4	500	8.24	4.99	8.19	8.29	5.20	-6.18	22.45	28.

A quadratic growth model

Estimating quadratic and linear growth

```

model.ex6.9 <- '
i =~ 1*y11 + 1*y12 + 1*y13 + 1*y14
s =~ 0*y11 + 1*y12 + 2*y13 + 3*y14
q =~ 0*y11 + 1*y12 + 4*y13 + 9*y14
'

fit <- growth(model.ex6.9, data=Data)
summary(fit, standardized=TRUE, fit.measures=TRUE)

```

Lavaan (0.4-7) converged normally after 61 iterations

		Estimate	Std.err	Z-value	P(> z)
Number of observations	500				
Estimator	ML				
Minimum Function Chi-square	0.475				
Degrees of freedom	1				
P-value	0.490				

Chi-square test baseline model:

Minimum Function Chi-square	1231.275				
Degrees of freedom	6				
P-value	0.000				

Full model versus baseline model:

Comparative Fit Index (CFI)	1.000				
Tucker-Lewis Index (TLI)	1.003				

Latent variables:

i =~					
y11		1.000			
y12		1.000			
y13		1.000			
y14		1.000			
s =~					
y11		0.000			
y12		1.000			
y13		2.000			
y14		3.000			
q =~					
y11		0.000			
y12		1.000			
y13		4.000			
y14		9.000			

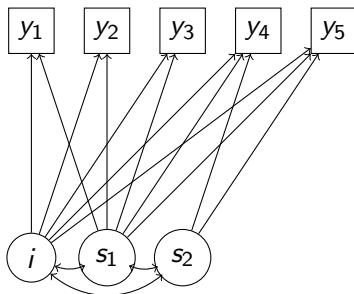
Covariances:

i ~~					
s		-0.173	0.300	-0.578	0.563
q		0.101	0.080	1.269	0.204
s ~~					
q		-0.179	0.117	-1.525	0.127

Intercepts:

y11		0.000			
y12		0.000			
v13		0.000			

Piecewise growth



```
> Data <- read.table("ex6.11.dat")
> names(Data) <- c("y1", "y2", "y3", "y4", "y5")
> round(cor(Data), 2)
> describe(Data)
```

	y1	y2	y3	y4	y5
y1	1.00	0.63	0.51	0.53	0.49
y2	0.63	1.00	0.70	0.68	0.62
y3	0.51	0.70	1.00	0.75	0.66
y4	0.53	0.68	0.75	1.00	0.79
y5	0.49	0.62	0.66	0.79	1.00

	var	n	mean	sd	median	trimmed	mad	min	max	r
y1	1	500	0.44	1.17	0.37	0.44	1.17	-2.64	3.61	0
y2	2	500	1.58	1.32	1.52	1.58	1.29	-2.28	5.76	0
y3	3	500	2.59	1.52	2.58	2.60	1.56	-1.46	7.15	0
y4	4	500	4.54	1.58	4.46	4.54	1.60	-0.25	9.11	0
y5	5	500	6.54	1.81	6.63	6.54	1.79	1.54	12.29	1

Modeling piecewise growth

```
model6.11 <- '

```

```
i =~ 1*y1 + 1*y2 + 1*y3 + 1*y4 + 1*y5

```

```
s1 =~ 0*y1 + 1*y2 + 2*y3 + 2*y4 + 2*y5

```

```
s2 =~ 0*y1 + 0*y2 + 0*y3 + 1*y4 + 2*y5
'

```

```
fit <- growth(model6.11, data=Data)

```

```
summary(fit, standardized=TRUE, fit.measures=TRUE)

```

			Estimate	Std.err	Z-value	P(> z)
Latent variables:						
i =~						
		y1	1.000			
Number of observations	500	y2	1.000			
		y3	1.000			
Estimator	ML	y4	1.000			
Minimum Function Chi-square	5.244	y5	1.000			
Degrees of freedom	6	s1 =~				
P-value	0.513	y1	0.000			
Chi-square test baseline model:						
		y2	1.000			
		y3	2.000			
		y4	2.000			
Minimum Function Chi-square	1587.697	y5	2.000			
Degrees of freedom	10	s2 =~				
P-value	0.000	y1	0.000			
Full model versus baseline model:						
		y2	0.000			
		y3	0.000			
		y4	1.000			
Comparative Fit Index (CFI)	1.000	y5	2.000			
Tucker-Lewis Index (TLI)	1.001					
Covariances:						
i ~~						
s1						
		s2	-0.029	0.049	-0.592	0.554
Loglikelihood user model (H0)	-3706.171	s1 ~~	0.059	0.036	1.644	0.100
Loglikelihood unrestricted model (H1)	-3703.549	s2	-0.031	0.026	-1.186	0.236