Psychology 454: Latent Variable Modeling
SEM over time
Changes in structure, changes in means

William Revelle

Department of Psychology
Northwestern University
Evanston, Illinois USA

NORTHWESTERN UNIVERSITY

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Outline

Measuring structure at two (or more) time points

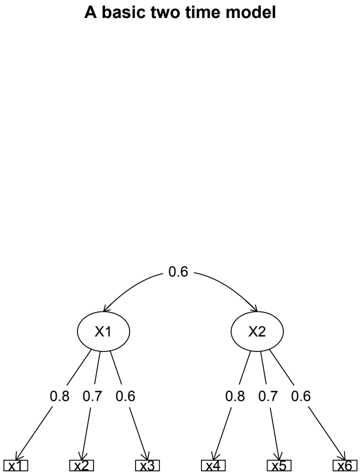
1. Is the structure the same
    - Structural Invariance (is the graph the same)
    - Measurement invariance (are the loadings the same)
    - Strong measurement invariance (are the item intercepts the same?)
    - Measuring change
2. Do the means change (is there growth)
    - This is the means of the latent trait, not the means of the items
3. Do the latent traits correlate across two or more occasions?
    - Just two occasions, can not separate trait from state effects
    - With > 2 occasions, can examine trait and state effects
4. Compare several different simulations

Create some basic data and add in some change

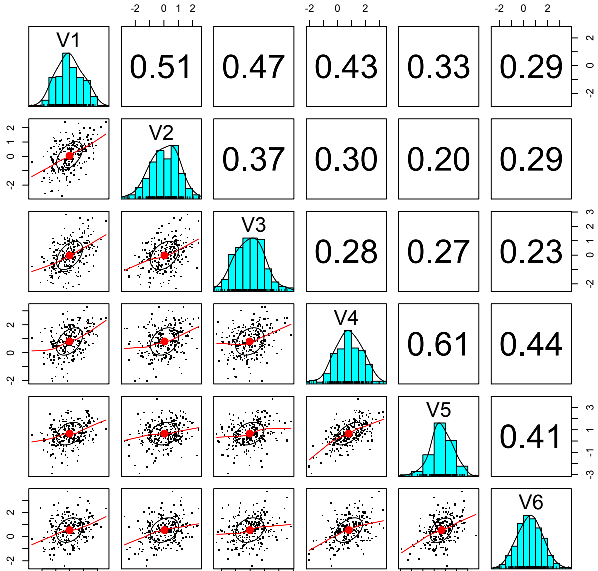
```
> set.seed(42)
 > fx <- matrix(c(.8,.7,.6,rep(0,6),.8,.7,.6),ncol=2)
> fx
     [,1] [,2]
[1,]  0.8  0.0
[2,]  0.7  0.0
[3,]  0.6  0.0
[4,]  0.0  0.8
[5,]  0.0  0.7
[6,]  0.0  0.6
> Phi <- matrix(c(1,.6,.6,1),ncol=2)
> Phi
     [,1] [,2]
[1,]  1.0  0.6
[2,]  0.6  1.0

> x.model <- sim(fx=fx,Phi=Phi,mu=c(0,1),n=250)
> x <- x.model$observed
> structure.diagram(fx,Phi,lr=FALSE,e.size=.3,main="A basic two time model")
> describe(x,skew=FALSE)

>  pairs.panels(x,pch=".")

   var   n  mean   sd median trimmed  mad   min  max range   se
V1   1 250 -0.01 0.94  -0.06   -0.02 1.00 -2.74 2.62  5.36 0.06
V2   2 250  0.04 0.98   0.11    0.06 0.99 -2.76 2.41  5.17 0.06
V3   3 250 -0.02 0.99  -0.03   -0.04 1.07 -2.32 2.84  5.17 0.06
V4   4 250  0.81 0.98   0.80    0.82 1.01 -2.03 3.29  5.32 0.06
V5   5 250  0.66 1.01   0.61    0.68 0.92 -2.89 3.72  6.61 0.06
V6   6 250  0.54 1.01   0.54    0.54 1.01 -2.42 3.71  6.13 0.06
```

1. Set the random seed
2. Create a one factor structure
3. Put in some change
4. Show the structural model
5. Describe it
6. Show the splom (with small pch)

A basic two occasion trait model



Splom of 6 basic variables



Multiple models

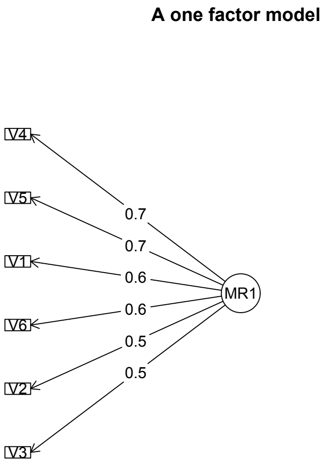
1. Ignore time, are the data congeneric?
    - Are they all measures of the same thing?
    - This is a one factor model
2. Include time, do we recover a correlation across time?
    - Try a two factor model
    - Plot the resulting structure

A one factor model

```
> fa(x)

Factor Analysis using method =  minres
Call: fa(r = x)
Standardized loadings (pattern matrix)
             based upon correlation matrix
    MR1   h2   u2
V1 0.64 0.41 0.59
V2 0.51 0.26 0.74
V3 0.50 0.25 0.75
V4 0.75 0.56 0.44
V5 0.66 0.43 0.57
V6 0.56 0.31 0.69

                MR1
SS loadings    2.22
Proportion Var 0.37

Test of the hypothesis that 1 factor is sufficient.
```

```
The degrees of freedom for the null model are  15
     and the objective function was  1.58
       with Chi Square of  388.97
The degrees of freedom for the model are 9
           and the objective function was  0.3

The root mean square of the residuals (RMSR) is  0.07
The df corrected root mean square of the
                  residuals is  0.12
The number of observations was  250
       with Chi Square =  73.6  with prob <  3e-12

Tucker Lewis Index of factoring reliability =  0.711
RMSEA index =  0.171  and the
            90 % confidence intervals are  0.135 0.206
BIC =  23.91
Fit based upon off diagonal values = 0.94
Measures of factor score adequacy
                                               MR1
Correlation of scores with factors            0.89
Multiple R square of scores with factors      0.79
Minimum correlation of possible factor scores 0.59
```

Fit a one factor model to the data



Try a two factor solution

```
> fa(x,2)

Loading required package: GPArotation
Factor Analysis using method =  minres
Call: fa(r = x, nfactors = 2)
Standardized loadings (pattern matrix)
         based upon correlation matrix
     MR1   MR2   h2   u2
V1  0.05  0.76 0.62 0.38
V2 -0.06  0.69 0.43 0.57
V3  0.04  0.55 0.33 0.67
V4  0.75  0.08 0.64 0.36
V5  0.81 -0.07 0.59 0.41
V6  0.47  0.12 0.30 0.70

                       MR1  MR2
SS loadings           1.49 1.43
Proportion Var        0.25 0.24
Cumulative Var        0.25 0.49
Proportion Explained  0.51 0.49
Cumulative Proportion 0.51 1.00
 With factor correlations of
     MR1  MR2
MR1 1.00 0.57
MR2 0.57 1.00
Measures of factor score adequacy
                                                 MR1  MR2
Correlation of scores with factors              0.89 0.87
Multiple R square of scores with factors        0.80 0.77
Minimum correlation of possible factor scores   0.59 0.53
```

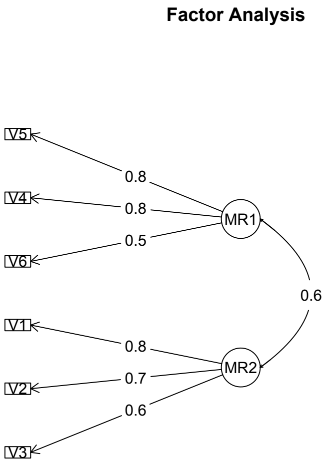
```
Test of the hypothesis that 2 factors are sufficient.

The degrees of freedom for the null model are  15
  and the objective function was  1.58
   with Chi Square of  388.97
The degrees of freedom for the model are 4
            and the objective function was  0.02

The root mean square of the residuals (RMSR) is  0.02
The df corrected root mean square of
                 the residuals is  0.04
The number of observations was  250
      with Chi Square =  5.87  with prob <  0.21

Tucker Lewis Index of factoring reliability =  0.981
RMSEA index =  0.045  and the
         90 % confidence intervals are  NA 0.112
BIC =  -16.21
Fit based upon off diagonal values = 1
```

Compare the two solutions

```
Factor Analysis using method =  minres
Call: fa(r = x)
Standardized loadings (pattern matrix)
    based upon correlation matrix
    MR1   h2   u2
V1 0.64 0.41 0.59
V2 0.51 0.26 0.74
V3 0.50 0.25 0.75
V4 0.75 0.56 0.44
V5 0.66 0.43 0.57
V6 0.56 0.31 0.69

                MR1
SS loadings    2.22
Proportion Var 0.37
```

```
Factor Analysis using method =  minres
Call: fa(r = x, nfactors = 2)
Standardized loadings (pattern matrix)
      based upon correlation matrix
     MR1   MR2   h2   u2
V1  0.05  0.76 0.62 0.38
V2 -0.06  0.69 0.43 0.57
V3  0.04  0.55 0.33 0.67
V4  0.75  0.08 0.64 0.36
V5  0.81 -0.07 0.59 0.41
V6  0.47  0.12 0.30 0.70

                       MR1  MR2
SS loadings           1.49 1.43
Proportion Var        0.25 0.24
Cumulative Var        0.25 0.49
Proportion Explained  0.51 0.49
Cumulative Proportion 0.51 1.00

 With factor correlations of
     MR1  MR2
MR1 1.00 0.57
MR2 0.57 1.00
```

EFA two factor solution



Now try three different sems (using lavaan)

1. Two correlated factors, free loadings
2. Two correlated factors, equal loadings across occasions
3. Two correlated factors, all loadings equal

A simple sem with a standardized solution

```
#factor model
mod2f <- F1 =~ V1 + V2 + V3
               F2 =~ V4 + V5 + V6
                  #correlation between factors
                 F1 ~~F2# now fit it and summarize it
> fit <- sem(mod2f, data=x.df,std.lv=TRUE)
> summary(fit, fit.measures=TRUE)
> standardizedSolution(fit)
```

```
   lhs op rhs est.std se  z pvalue
1   F1 =~  V1   0.819 NA NA     NA
2   F1 =~  V2   0.618 NA NA     NA
3   F1 =~  V3   0.579 NA NA     NA
4   F2 =~  V4   0.835 NA NA     NA
5   F2 =~  V5   0.720 NA NA     NA
6   F2 =~  V6   0.552 NA NA     NA
7   F1 ~~  F2   0.607 NA NA     NA
```

lavaan fit statistics

```
lavaan (0.4-14) converged normally after 16 iterations

  Number of observations                           250

  Estimator                                         ML
  Minimum Function Chi-square                    9.341
  Degrees of freedom                                 8
  P-value                                        0.314

Chi-square test baseline model:

  Minimum Function Chi-square                  395.026
  Degrees of freedom                                15
  P-value                                        0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    0.996
  Tucker-Lewis Index (TLI)                       0.993

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1909.112
  Loglikelihood unrestricted model (H1)      -1904.441

  Number of free parameters                         13
  Akaike (AIC)                                3844.224
  Bayesian (BIC)                              3890.003
  Sample-size adjusted Bayesian (BIC)         3848.791
```

```
Root Mean Square Error of Approximation:

  RMSEA                                          0.026
  90 Percent Confidence Interval          0.000  0.081
  P-value RMSEA <= 0.05                          0.699

Standardized Root Mean Square Residual:

  SRMR                                           0.029

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    V1                0.768    0.062   12.363    0.000
    V2                0.604    0.065    9.313    0.000
    V3                0.569    0.066    8.672    0.000
  F2 =~
    V4                0.818    0.062   13.296    0.000
    V5                0.727    0.064   11.346    0.000
    V6                0.558    0.066    8.472    0.000

Covariances:
  F1 ~~
    F2                0.607    0.062    9.861    0.000
```

Create two new models with some equality constraints

```
> mod2fa <- F1 =~ a*V1 + b*V2 + c*V3
                   F2 =~ a*V4 + b*V5 + c*V6
                   F1 ~~F2
 > fit2a <- sem(mod2fa,data=x.df,std.lv=TRUE)
> summary(fit2a,fit.measures=TRUE)

> mod2fe <- F1 =~ a*V1 + a*V2 + a*V3
                   F2 =~ a*V4 + a*V5 + a*V6
                   F1 ~~F2
 > fit2e <- sem(mod2fe,data=x.df,std.lv=TRUE)
> summary(fit2e,fit.measures=TRUE)
```

Equal across time

```
lavaan (0.4-14) converged normally after 15 iterations

  Number of observations                           250

  Estimator                                         ML
  Minimum Function Chi-square                   11.797
  Degrees of freedom                                11
  P-value                                        0.379

Chi-square test baseline model:

  Minimum Function Chi-square                  395.026
  Degrees of freedom                                15
  P-value                                        0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    0.998
  Tucker-Lewis Index (TLI)                       0.997

  Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1910.340
  Loglikelihood unrestricted model (H1)      -1904.441

  Number of free parameters                         10
  Akaike (AIC)                                3840.679
  Bayesian (BIC)                              3875.894
  Sample-size adjusted Bayesian (BIC)         3844.193
```

```
Root Mean Square Error of Approximation:

  RMSEA                                            0.017
  90 Percent Confidence Interval          0.000    0.070
  P-value RMSEA <= 0.05                            0.801

Standardized Root Mean Square Residual:

  SRMR                                             0.044

Parameter estimates:

  Information                                   Expected
  Standard Errors                               Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    V1        (a)     0.793    0.046   17.162    0.000
    V2        (b)     0.669    0.047   14.116    0.000
    V3        (c)     0.564    0.048   11.819    0.000
  F2 =~
    V4        (a)     0.793    0.046   17.162    0.000
    V5        (b)     0.669    0.047   14.116    0.000
    V6        (c)     0.564    0.048   11.819    0.000

Covariances:
  F1 ~~
    F2                0.606    0.061    9.869    0.000
```

All loadings equal

```
> summary(fit2e,fit.measures=TRUE)
lavaan (0.4-14) converged normally after 12 iterations

  Number of observations                           250

  Estimator                                         ML
  Minimum Function Chi-square                   25.457
  Degrees of freedom                                13
  P-value                                        0.020

Chi-square test baseline model:

  Minimum Function Chi-square                  395.026
  Degrees of freedom                                15
  P-value                                        0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    0.967
  Tucker-Lewis Index (TLI)                       0.962

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1917.169
  Loglikelihood unrestricted model (H1)      -1904.441

  Number of free parameters                          8
  Akaike (AIC)                                3850.339
  Bayesian (BIC)                              3878.510
  Sample-size adjusted Bayesian (BIC)         3853.150
```

```
Root Mean Square Error of Approximation:

  RMSEA                                          0.062
  90 Percent Confidence Interval          0.024  0.097
  P-value RMSEA <= 0.05                          0.260

Standardized Root Mean Square Residual:

  SRMR                                           0.071

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    V1        (a)     0.686    0.033   20.987    0.000
    V2        (a)     0.686    0.033   20.987    0.000
    V3        (a)     0.686    0.033   20.987    0.000
  F2 =~
    V4        (a)     0.686    0.033   20.987    0.000
    V5        (a)     0.686    0.033   20.987    0.000
    V6        (a)     0.686    0.033   20.987    0.000
Covariances:
  F1 ~~
    F2                0.624    0.063    9.909    0.000
```

Create a data set with non-invariant factor loadings

```
> set.seed(42)
 > fx <- matrix(c(.8,.7,.6,rep(0,6),.6,.7,.8),ncol=2)
> fx
     [,1] [,2]
[1,]  0.8  0.0
[2,]  0.7  0.0
[3,]  0.6  0.0
[4,]  0.0  0.6
[5,]  0.0  0.7
[6,]  0.0  0.8
> Phi <- matrix(c(1,.6,.6,1),ncol=2)
> Phi
> set.seed(42)
> x.model <- sim(fx=fx,Phi=Phi,mu=c(0,1),n=250)
>  x <- x.model$observed
>  structure.diagram(fx,Phi,lr=FALSE,e.size=.3,main="A basic two time model")
>  describe(x,skew=FALSE)

   var   n  mean   sd median trimmed  mad   min  max range   se
V1   1 250  0.02 0.99   0.01    0.03 1.02 -2.64 2.76  5.40 0.06
V2   2 250 -0.02 0.96  -0.01   -0.03 0.94 -2.66 3.12  5.78 0.06
V3   3 250  0.02 0.97  -0.06    0.01 0.95 -2.74 2.41  5.15 0.06
V4   4 250  0.61 0.99   0.59    0.65 0.99 -3.26 3.15  6.41 0.06
```

One factor model

```
> f1n <- fa(x)
> f1n

Factor Analysis using method =  minres
Call: fa(r = x)
Standardized loadings (pattern matrix) based upon correlation matrix
    MR1   h2   u2
V1 0.59 0.35 0.65
V2 0.56 0.31 0.69
V3 0.48 0.23 0.77
V4 0.54 0.29 0.71
V5 0.69 0.48 0.52
V6 0.72 0.52 0.48

                MR1
SS loadings    2.18
Proportion Var 0.36

Test of the hypothesis that 1 factor is sufficient.

The degrees of freedom for the null model are  15
         and the objective function was  1.58
          with Chi Square of  389.85
The degrees of freedom for the model are 9
           and the objective function was  0.33
```

```
The root mean square of the residuals (RMSR) is  0.07
The df corrected root mean square of the residuals is
The number of observations was  250  with Chi Square =

Tucker Lewis Index of factoring reliability =  0.684
RMSEA index =  0.179  and the 90 % confidence interval
BIC =  30.24
Fit based upon off diagonal values = 0.93
Measures of factor score adequacy
                                                 MR1
Correlation of scores with factors              0.89
Multiple R square of scores with factors        0.79
Minimum correlation of possible factor scores   0.57
```

Two factor model

```
> f2n <- fa(x,2)
> f2n


Factor Analysis using method =  minres
Call: fa(r = x, nfactors = 2)
Standardized loadings (pattern matrix) based upon correlation matrix
     MR1   MR2   h2   u2
V1 -0.01  0.79 0.62 0.38
V2 -0.01  0.73 0.53 0.47
V3  0.15  0.40 0.25 0.75
V4  0.51  0.07 0.30 0.70
V5  0.79 -0.03 0.60 0.40
V6  0.80  0.02 0.65 0.35

                       MR1  MR2
SS loadings           1.58 1.37
Proportion Var        0.26 0.23
Cumulative Var        0.26 0.49
Proportion Explained  0.53 0.47
Cumulative Proportion 0.53 1.00

 With factor correlations of
     MR1  MR2
MR1 1.00 0.54
MR2 0.54 1.00

Correlation of scores with factors            0.90 0.88
Multiple R square of scores with factors      0.80 0.77
Minimum correlation of possible factor scores 0.61 0.54
```

```
Test of the hypothesis that 2 factors are sufficient.

The degrees of freedom for the null model are  15
             and the objective function was  1.58
             with Chi Square of  389.85
The degrees of freedom for the model are 4
                     and the objective function was  0

The root mean square of the residuals (RMSR) is  0.01
The df corrected root mean square of the residuals is
The number of observations was  250  with
                 Chi Square =  4.7  with prob <  0.32

Tucker Lewis Index of factoring reliability =  0.993
RMSEA index =  0.028  and the 90 % confidence interval
BIC =  -17.39
Fit based upon off diagonal values = 1
Measures of factor score adequacy
                                                 MR1  M
```

Compare the two solutions

```
Factor Analysis using method =  minres
Call: fa(r = x)
Standardized loadings (pattern matrix)
  based upon correlation matrix
    MR1   h2   u2
V1 0.59 0.35 0.65
V2 0.56 0.31 0.69
V3 0.48 0.23 0.77
V4 0.54 0.29 0.71
V5 0.69 0.48 0.52
V6 0.72 0.52 0.48

                MR1
SS loadings    2.18
Proportion Var 0.36
```

```
Factor Analysis using method =  minres
Call: fa(r = x, nfactors = 2)
Standardized loadings (pattern matrix)
        based upon correlation matrix
     MR1   MR2   h2   u2
V1 -0.01  0.79 0.62 0.38
V2 -0.01  0.73 0.53 0.47
V3  0.15  0.40 0.25 0.75
V4  0.51  0.07 0.30 0.70
V5  0.79 -0.03 0.60 0.40
V6  0.80  0.02 0.65 0.35

                      MR1  MR2
SS loadings           1.58 1.37
Proportion Var        0.26 0.23
Cumulative Var        0.26 0.49
Proportion Explained  0.53 0.47
Cumulative Proportion 0.53 1.00

 With factor correlations of
     MR1  MR2
MR1 1.00 0.54
MR2 0.54 1.00
```

CFA 2 correlated factors

```
> y.df <- data.frame(y)
> fitn<- sem(mod2f,data=y.df,std.lv=TRUE)
> summary(fitn, fit.measures=TRUE)


lavaan (0.4-14) converged normally after 17 iterations
  Number of observations                           250
  Estimator                                         ML
  Minimum Function Chi-square                    2.552
  Degrees of freedom                                 8
  P-value                                        0.959
Chi-square test baseline model:

  Minimum Function Chi-square                  341.309
  Degrees of freedom                                15
  P-value                                        0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    1.000
  Tucker-Lewis Index (TLI)                       1.031

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1952.265
  Loglikelihood unrestricted model (H1)      -1950.989

  Number of free parameters                         13
  Akaike (AIC)                                3930.530
  Bayesian (BIC)                              3976.309
  Sample-size adjusted Bayesian (BIC)         3935.09
```

```
Root Mean Square Error of Approximation:

  RMSEA                                                  0.000
  90 Percent Confidence Interval              0.000     0.000
  P-value RMSEA <= 0.05                                  0.994

Standardized Root Mean Square Residual:

  SRMR                                                   0.014

Parameter estimates:

  Information                                         Expected
  Standard Errors                                     Standard

                     Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    V1                  0.750    0.063   11.880    0.000
    V2                  0.725    0.065   11.160    0.000
    V3                  0.621    0.066    9.410    0.000
  F2 =~
    V4                  0.585    0.074    7.896    0.000
    V5                  0.610    0.068    8.950    0.000
    V6                  0.700    0.066   10.545    0.000

Covariances:
  F1 ~~
    F2                  0.607    0.067    9.067    0.000

Variances:
```

Are the factors measurement invariant? Well, maybe.

```
> summary(fitn2a,fit.measures=TRUE)

lavaan (0.4-14) converged normally after 14 iterations

  Number of observations                           250

  Estimator                                         ML
  Minimum Function Chi-square                    8.093
  Degrees of freedom                                11
  P-value                                        0.705

Chi-square test baseline model:

  Minimum Function Chi-square                  341.309
  Degrees of freedom                                15
  P-value                                        0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    1.000
  Tucker-Lewis Index (TLI)                       1.012

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1955.036
  Loglikelihood unrestricted model (H1)      -1950.989

  Number of free parameters                         10
  Akaike (AIC)                                3930.072
  Bayesian (BIC)                              3965.286
```

```
Root Mean Square Error of Approximation:

  RMSEA                                          0.000
  90 Percent Confidence Interval          0.000  0.051
  P-value RMSEA <= 0.05                          0.946

Standardized Root Mean Square Residual:

  SRMR                                           0.049

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    V1        (a)     0.686    0.050   13.774    0.000
    V2        (b)     0.675    0.049   13.834    0.000
    V3        (c)     0.657    0.048   13.677    0.000
  F2 =~
    V4        (a)     0.686    0.050   13.774    0.000
    V5        (b)     0.675    0.049   13.834    0.000
    V6        (c)     0.657    0.048   13.677    0.000

Covariances:
  F1 ~~
    F2                0.610    0.067    9.089    0.000

Variances:
```

Create the original data set again

```
> fx <- matrix(c(.8,.7,.6,rep(0,6),.8,.7,.6),ncol=2)
> x.model <- sim(fx=fx,Phi=Phi,mu=c(0,1),n=250)
> x <- x.model$observed
> x.df <- data.frame(x)
```

```
> describe(x,skew=FALSE)
   var   n mean   sd median trimmed  mad   min  max range   se
V1   1 250 0.02 0.99  -0.01   -0.01 1.04 -2.38 3.26  5.64 0.06
V2   2 250 0.13 1.07   0.03    0.11 1.08 -2.53 3.65  6.19 0.07
V3   3 250 0.03 1.03   0.05    0.02 0.97 -2.68 3.12  5.80 0.07
V4   4 250 0.87 1.04   0.90    0.85 1.03 -1.63 3.61  5.24 0.07
V5   5 250 0.73 0.97   0.76    0.75 0.90 -2.36 3.29  5.65 0.06
V6   6 250 0.65 1.00   0.67    0.64 1.03 -2.58 3.17  5.75 0.06
```

Model change in the items

```
mod2fc <- F1 =~ a*V1 + b* V2 + c*V3
                F2 =~ a* V4 + b*V5 +c* V6
                  #correlation between factors
                 F2 ~  F1     #the regression
fit2c <- sem(mod2fc,data=x.df,meanstructure=TRUE)
summary(fit2c,fit.measures=TRUE)
```

frame

```
lavaan (0.4-14) converged normally after 23 iterations

  Number of observations                           250

  Estimator                                         ML
  Minimum Function Chi-square                   10.616
  Degrees of freedom                                10
  P-value                                        0.388

Chi-square test baseline model:

  Minimum Function Chi-square                  406.795
  Degrees of freedom                                15
  P-value                                        0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    0.998
  Tucker-Lewis Index (TLI)                       0.998

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1952.674
  Loglikelihood unrestricted model (H1)      -1947.365

  Number of free parameters                         17
  Akaike (AIC)                                3939.347
  Bayesian (BIC)                              3999.212
  Sample-size adjusted Bayesian (BIC)         3945.320
```

```
Root Mean Square Error of Approximation:

  RMSEA                                                   0.016
  90 Percent Confidence Interval                  0.000   0.071
  P-value RMSEA <= 0.05                                   0.793

Standardized Root Mean Square Residual:

  SRMR                                                    0.032

Parameter estimates:

  Information                                          Expected
  Standard Errors                                      Standard

                     Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    V1        (a)       1.000
    V2        (b)       0.925    0.076   12.243    0.000
    V3        (c)       0.816    0.072   11.386    0.000
  F2 =~
    V4        (a)       1.000
    V5        (b)       0.925    0.076   12.243    0.000
    V6        (c)       0.816    0.072   11.386    0.000

Regressions:
  F2 ~
    F1                  0.644    0.075    8.593    0.000

Intercepts:
```

With the intercepts

Intercepts:

V1	0.021	0.063	0.332	0.740
V2	0.135	0.066	2.033	0.042
V3	0.032	0.066	0.482	0.630
V4	0.865	0.065	13.294	0.000
V5	0.729	0.062	11.696	0.000
V6	0.649	0.063	10.369	0.000
F1	0.000			
F2	0.000			

Variances:

V1	0.383	0.060
V2	0.576	0.068
V3	0.670	0.071
V4	0.486	0.065
V5	0.481	0.061
V6	0.596	0.065
F1	0.613	0.088
F2	0.319	0.063

Try fitting a moments model

```
mod2fc <- F1 =~ a*V1 + b* V2 + c*V3
               F2 =~ a* V4 + b*V5 +c* V6
               means = ~ F1 + F2 + 1*V1 +1*V2+1*V3+1*V4+1*V5+1*V6
                 #correlation between factors

               F2 ~ F1     #the regression
fit2c <- sem(mod2fc,data=x.df,meanstructure=TRUE)
summary(fit2c,fit.measures=TRUE)
```

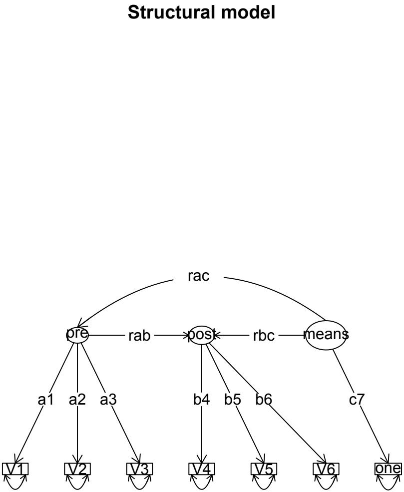
Modeling change using the moments matrix

1. McArdle (2009) Latent variable modeling of differences and changes with longitudinal data. Annual Review of Psychology, 60, 577-605
    - Use moments rather than covariances
2. Probably can be done in lavaan, but I don't know how. Can be done in sem package

Create the model to be fit in sem

```
 fxg
    pre  post means
V1  "a1" "0"  "0"
V2  "a2" "0"  "0"
V3  "a3" "0"  "0"
V4  "0"  "b4" "0"
V5  "0"  "b5" "0"
V6  "0"  "b6" "0"
one "0"  "0"  "c7"
> phi
   F1    F2  F3
F1 "1"   "0" "rac"
F2 "rab" "1" "rbc"
F3 "0"   "0" "1"

mod.mom1 <- structure.diagram(fog,phi,errors=TRUE)
```

Modeling the means in a moments matrix



the basic path model, with some editing

```
mod.mom1
      Path                Parameter Value
 [1,] "pre->V1"           "a1"      NA
 [2,] "pre->V2"           "a2"      NA
 [3,] "pre->V3"           "a3"      NA
 [4,] "post->V4"          "b4"      NA
 [5,] "post->V5"          "b5"      NA
 [6,] "post->V6"          "b6"      NA
 [7,] "means->one"        "c7"      NA
 [8,] "V1<->V1"           "x1e"     NA
 [9,] "V2<->V2"           "x2e"     NA
[10,] "V3<->V3"           "x3e"     NA
[11,] "V4<->V4"           "x4e"     NA
[12,] "V5<->V5"           "x5e"     NA
[13,] "V6<->V6"           "x6e"     NA
[14,] "one<->one"         NA    "1"      <-- edited
[15,] "pre ->post"        "rF1F2"   NA
[16,] "means->pre"        "rF3F1"   NA    <- edited
[17,] "means->post"       "rF3F2"   NA   <- edited
[18,] "pre<->pre"         NA        "1"
[19,] "post<->post"       NA        "1"
[20,] "means<->means"     NA        "1"
attr(,"class")
```

sem output

```
> sem.mom1 <- sem(mod.mom,MomMat,N=250,raw=TRUE)
> summary(sem.mom1)


Model fit to raw moment matrix.

 Model Chisquare =  12.077   Df =  12
                  Pr(>Chisq) = 0.43954
 AIC =  44.077
 AICc =  14.411
 BIC =  100.42
 CAIC =  -66.181

 Normalized Residuals
   Min.  1st Qu.   Median    Mean  3rd Qu.    Max.
-1.2500  -0.1150  0.0000  -0.0103  0.0972   0.97
```

```
 Parameter Estimates
      Estimate Std Error z value Pr(>|z|)
a1    0.830111 0.069823  11.8888  1.3536e-32 V1 <--- pre
a2    0.881776 0.076506  11.5256  9.7982e-31 V2 <--- pre
a3    0.698963 0.074824   9.3415  9.5005e-21 V3 <--- pre
b4    0.664838 0.062395  10.6553  1.6468e-26 V4 <--- post
b5    0.554498 0.053510  10.3626  3.6687e-25 V5 <--- post
b6    0.510270 0.051499   9.9084  3.8265e-23 V6 <--- post
c7    1.118034 0.050000  22.3607  9.5054e-111 one <--- means
x1e   0.525613 0.078258   6.7164  1.8623e-11 V1 <--> V1
x2e   0.673405 0.093381   7.2114  5.5387e-13 V2 <--> V2
x3e   0.836504 0.090362   9.2572  2.0983e-20 V3 <--> V3
x4e   0.567286 0.081353   6.9732  3.0990e-12 V4 <--> V4
x5e   0.628041 0.073463   8.5490  1.2412e-17 V5 <--> V5
x6e   0.750522 0.080300   9.3465  9.0592e-21 V6 <--> V6
rF1F2 0.893184 0.142998   6.2461  4.2076e-10 post <--- pre
rF3F1 0.086583 0.073172   1.1833  2.3669e-01 pre <--- means
rF3F2 1.375097 0.159247   8.6350  5.8723e-18 post <--- means

 Iterations =  21
```

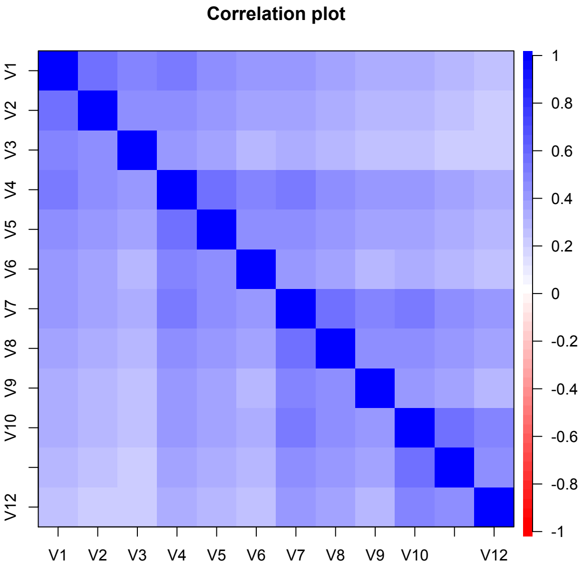
Traits and States and time

1. With just two time points, traits and states are confounded
    - Is the correlation a trait like stability
    - or does the state dissipate slowly?
2. With > 2 time points we can distinguish states and traits
    - States should have an autocorrelation component
    - Traits should be consistent across time
3. Consider the simplex structure of 4 time points
    - Clean within time factor structure
    - Simplex across time points

A factor simplex

```
simp <- sim()

 $model (Population correlation matrix)
      V1   V2   V3   V4   V5   V6   V7   V8   V9  V10  V11  V12
V1  1.00 0.56 0.48 0.51 0.45 0.38 0.41 0.36 0.31 0.33 0.29 0.25
V2  0.56 1.00 0.42 0.45 0.39 0.34 0.36 0.31 0.27 0.29 0.25 0.22
V3  0.48 0.42 1.00 0.38 0.34 0.29 0.31 0.27 0.23 0.25 0.22 0.18
V4  0.51 0.45 0.38 1.00 0.56 0.48 0.51 0.45 0.38 0.41 0.36 0.31
V5  0.45 0.39 0.34 0.56 1.00 0.42 0.45 0.39 0.34 0.36 0.31 0.27
V6  0.38 0.34 0.29 0.48 0.42 1.00 0.38 0.34 0.29 0.31 0.27 0.23
V7  0.41 0.36 0.31 0.51 0.45 0.38 1.00 0.56 0.48 0.51 0.45 0.38
V8  0.36 0.31 0.27 0.45 0.39 0.34 0.56 1.00 0.42 0.45 0.39 0.34
V9  0.31 0.27 0.23 0.38 0.34 0.29 0.48 0.42 1.00 0.38 0.34 0.29
V10 0.33 0.29 0.25 0.41 0.36 0.31 0.51 0.45 0.38 1.00 0.56 0.48
V11 0.29 0.25 0.22 0.36 0.31 0.27 0.45 0.39 0.34 0.56 1.00 0.42
V12 0.25 0.22 0.18 0.31 0.27 0.23 0.38 0.34 0.29 0.48 0.42 1.00
```

A simplex


Correlation plot
V1
V2
V3
V4
V5
V6
V7
V8
V9
V10
V12
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

Factor structure of a simplex

```
> fsimp <- fa(simp$model)
> fsimp

Factor Analysis using method =  minres
Call: fa(r = simp$model)
Standardized loadings (pattern matrix) based upon correlation matrix
     MR1   h2   u2
V1  0.64 0.41 0.59
V2  0.57 0.33 0.67
V3  0.49 0.24 0.76
V4  0.73 0.54 0.46
V5  0.65 0.42 0.58
V6  0.56 0.31 0.69
V7  0.73 0.54 0.46
V8  0.65 0.42 0.58
V9  0.56 0.31 0.69
V10 0.64 0.41 0.59
V11 0.57 0.33 0.67
V12 0.49 0.24 0.76

                MR1
SS loadings    4.50
Proportion Var 0.38
```

Factors over time

```
> fsimp4 <- fa(simp$model,4)
> fsimp4
```

```
Factor Analysis using method =  minres
Call: fa(r = simp$model, nfactors = 4)
Standardized loadings (pattern matrix)
            based upon correlation matrix
    MR3 MR1 MR2 MR4   h2   u2
V1  0.8 0.0 0.0 0.0 0.64 0.36
V2  0.7 0.0 0.0 0.0 0.49 0.51
V3  0.6 0.0 0.0 0.0 0.36 0.64
V4  0.0 0.0 0.0 0.8 0.64 0.36
V5  0.0 0.0 0.0 0.7 0.49 0.51
V6  0.0 0.0 0.0 0.6 0.36 0.64
V7  0.0 0.8 0.0 0.0 0.64 0.36
V8  0.0 0.7 0.0 0.0 0.49 0.51
V9  0.0 0.6 0.0 0.0 0.36 0.64
V10 0.0 0.0 0.8 0.0 0.64 0.36
V11 0.0 0.0 0.7 0.0 0.49 0.51
V12 0.0 0.0 0.6 0.0 0.36 0.64
```

```
                       MR3  MR1  MR2  MR4
SS loadings           1.49 1.49 1.49 1.49
Proportion Var        0.12 0.12 0.12 0.12
Cumulative Var        0.12 0.25 0.37 0.50
Proportion Explained  0.25 0.25 0.25 0.25
Cumulative Proportion 0.25 0.50 0.75 1.00

 With factor correlations of
     MR3  MR1  MR2  MR4
MR3 1.00 0.64 0.51 0.80
MR1 0.64 1.00 0.80 0.80
MR2 0.51 0.80 1.00 0.64
MR4 0.80 0.80 0.64 1.00
```