Psychology 454: Latent Variable Modeling
Week 4: Latent models of real data

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Outline

1. Example data sets
2. Thurstone 9 variable problem
   - Factor extraction: how many and what algorithm
   - Various Rotations or Transformations
3. Confirmatory fits
   - Using sem
   - Bifactor models
   - Confirmatory Bifactor model
4. Holzinger 14 cognitive variables
5. Using lavaan
   - Traditional analyses of the Holzinger Swineford data set
   - SEM analysis of Holzinger Swineford data set

Real data rather than simulated

- Prior examples were all artificial data sets.
  - These have the advantage that we know truth.
  - They have the disadvantage that they don't give us experience with real data.
- Some classic data sets are available in *psych*, *lavaan* and *sem*.
- Using the `data` function to see what is available.
  - data(package="psych")

Some of the psych data sets

```
data(package="psych")

Bechtoldt                     Seven data sets showing a bifactor solution.
Bechtoldt.1                   Seven data sets showing a bifactor solution.
Bechtoldt.2                   Seven data sets showing a bifactor solution.
Chen (Schmid)                 12 variables created by Schmid and Leiman to show the
                              Schmid-Leiman Transformation
Dwyer                         8 cognitive variables used by Dwyer for an example.
Gorsuch                       Example data set from Gorsuch (1997) for an example factor
                              extension.
Harman.5                      5 socio-economic variables from Harman (1967)
Harman.8                      Correlations of eight physical variables (from Harman, 196
Harman.Burt (Harman)          Two data sets from Harman (1967). 9 cognitive variables fro
                              Holzinger and 8 emotional variables from Burt
Harman.Holzinger (Harman)     Two data sets from Harman (1967). 9 cognitive variables fro
                              Holzinger and 8 emotional variables from Burt
Holzinger                     Seven data sets showing a bifactor solution.
Holzinger.9                   Seven data sets showing a bifactor solution.
Reise                         Seven data sets showing a bifactor solution.
Schmid                        12 variables created by Schmid and Leiman to show the
                              Schmid-Leiman Transformation
Thurstone                     Seven data sets showing a bifactor solution.
Thurstone.33                  Seven data sets showing a bifactor solution.
Tucker                        9 Cognitive variables discussed by Tucker and Lewis (1973)
West (Schmid)                 12 variables created by Schmid and Leiman to show the
                              Schmid-Leiman Transformation
all.income (income)           US family income from US census 2008
bfi                           25 Personality items representing 5 factors
blot                          Bonds Logical Operations Test - BLOT
bock.table (bock)             Bock and Liberman (1970) data set of 1000 observations of
                              the LSAT
```

More of the psych data sets

```
burt                      11 emotional variables from Burt (1915)
cities                    Distances between 11 US cities
city.location (cities)    Distances between 11 US cities
cubits                    Galtons example of the relationship between height and
                          cubit or forearm length
cushny                    A data set from Cushny and Peebles (1905) on the effect of
                          three drugs on hours of sleep, used by Student (1908)
epi.bfi                   13 personality scales from the Eysenck Personality Invento
                          and Big 5 inventory
flat (affect)             Two data sets of affect and arousal scores as a function c
                          personality and movie conditions
galton                    Galton Mid parent child height data
heights                   A data.frame of the Galton (1888) height and cubit data se
income                    US family income from US census 2008
iqitems                   16 multiple choice IQ items
lsat6 (bock)              Bock and Liberman (1970) data set of 1000 observations of
                          the LSAT
lsat7 (bock)              Bock and Liberman (1970) data set of 1000 observations of
                          the LSAT
maps (affect)             Two data sets of affect and arousal scores as a function c
                          personality and movie conditions
msq                       75 mood items from the Motivational State Questionnaire fo
                          3896 participants
neo                       NEO correlation matrix from the NEO_PI_R manual
peas                      Galtons Peas
sat.act                   3 Measures of ability: SATV, SATQ, ACT
schmid.leiman (Schmid)    12 variables created by Schmid and Leiman to show the
                          Schmid-Leiman Transformation
veg (vegetables)          Paired comparison of preferences for 9 vegetables
withinBetween             An example of the distinction between within group and
                          between group correlations
```

Data sets in sem and lavaan

```
data(package=ÔsemÕ)
data(package =ÔlavaanÕ)

Data sets in package ÔsemÕ:

Bollen       Bollen''s Data on Industrialization and Political Democracy
CNES         Variables from the 1997 Canadian National Election Study
Klein            Klein''s Data on the U. S. Economy'
Kmenta␣␣␣␣␣␣␣Partly␣Artificial␣Data␣on␣the␣U.␣S.␣Economy

Data␣sets␣in␣package␣ÔlavaanÕ:

Demo.growth␣␣␣␣␣␣Demo␣dataset␣for␣a␣illustrating␣a␣linear␣growth␣model.
HolzingerSwineford1939␣␣␣␣␣␣Holzinger␣and␣Swineford␣Dataset␣(9␣Variables)
PoliticalDemocracy␣␣␣␣␣␣␣␣␣␣Industrialization␣And␣Political␣Democracy␣Dataset
```

Also can read in from clipboard, web, from hard drive and import

- To read from clipboard: `read.clipboard()`
  - read.clipboard(header = TRUE, ...) #assumes headers and tab or space delimited
  - read.clipboard.csv(header=TRUE,sep=',',...) #assumes headers and comma delimited
  - read.clipboard.tab(header=TRUE,sep='\t',...) #assumes headers and tab delimited
  - read.clipboard.lower(diag=TRUE,names=FALSE,...) #read in a matrix given the lower off diagonal
  - read.clipboard.upper(diag=TRUE,names=FALSE,...)
  - read.clipboard.fwf(header=FALSE,widths=rep(1,10),...) #read in data using a fixed format width (see read.fwf for instructions)
- To read from a file
  - fn <- file.choose() #this opens the system directory – navigate it to your file
  - my.data <- read.table(fn)

More on getting your data

```
#specify the name and address of the remote file
datafilename <- "http://personality-project.org/r/datasets/maps.mixx.epi.bfi.data"
#datafilename <- Desktop/epi.big5.txt read from local directory or
# datafilename <- file.choose()  use the OS to find the file
#in all cases
person.data  <- read.table(datafilename,header=TRUE)  #read the data file

#Alternatively, to read in a comma delimited file:
#person.data <- read.table(datafilename,header=TRUE,sep=",")


names(person.data)  #list the names of the variables

library(foreign) #allows you to read SPSS files e.g.,
datafilename <- "/Users/bill/Library/Favorites/R.tutorial/datasets/finkel.sav"  #local
file
eli.data <- read.spss(datafilename, use.value.labels=TRUE, to.data.frame=TRUE)
```

Use the bifactor examples

The study of latent variable models in general and sem in particular is the combination of measurement models with structural models. The bifactor examples are useful to understand issues in reliability and estimating measurement models.

```
data(Thurstone)
data(Thurstone.33)
data(Holzinger)
data(Holzinger.9)
data(Bechtoldt)
data(Bechtoldt.1)
data(Bechtoldt.2)
data(Reise)
```

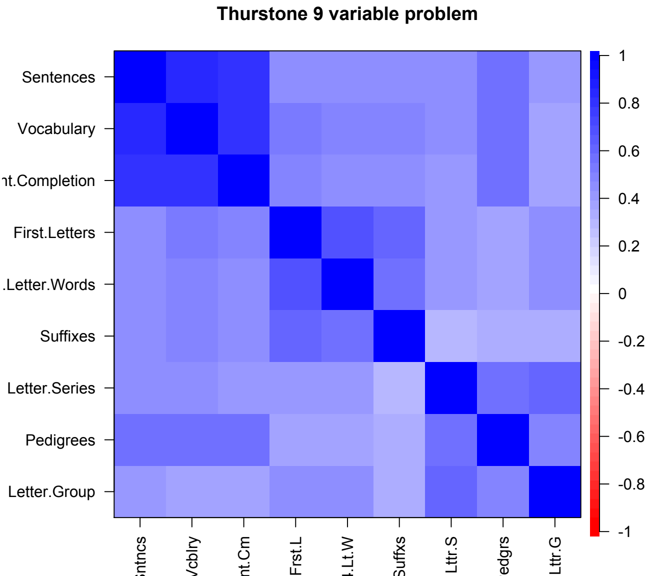
```
Two more data sets with similar structures are found in the Harman data set.

Bechtoldt.1: 17 x 17 correlation matrix of ability tests, N = 212.
Bechtoldt.2: 17 x 17 correlation matrix of ability tests, N = 213.
Holzinger: 14 x 14 correlation matrix of ability tests, N = 355
Holzinger.9: 9 x 9 correlation matrix of ability tests, N = 145
Reise: 16 x 16 correlation matrix of health satisfaction items. N = 35,000
Thurstone: 9 x 9 correlation matrix of ability tests, N = 213
Thurstone.33: Another 9 x 9 correlation matrix of ability items, N=4175
```

Thurstone correlation matrix

```
> colnames(Thurstone) <- abbreviate(rownames(Thurstone),6)
> Thurstone
```

```
                Sntncs Vcblry Snt.Cm Frst.L 4.Lt.W Suffxs Lttr.S Pedgrs Lttr.G
Sentences        1.000  0.828  0.776  0.439  0.432  0.447  0.447  0.541  0.380
Vocabulary       0.828  1.000  0.779  0.493  0.464  0.489  0.432  0.537  0.358
Sent.Completion  0.776  0.779  1.000  0.460  0.425  0.443  0.401  0.534  0.359
First.Letters    0.439  0.493  0.460  1.000  0.674  0.590  0.381  0.350  0.424
4.Letter.Words   0.432  0.464  0.425  0.674  1.000  0.541  0.402  0.367  0.446
Suffixes         0.447  0.489  0.443  0.590  0.541  1.000  0.288  0.320  0.325
Letter.Series    0.447  0.432  0.401  0.381  0.402  0.288  1.000  0.555  0.598
Pedigrees        0.541  0.537  0.534  0.350  0.367  0.320  0.555  1.000  0.452
Letter.Group     0.380  0.358  0.359  0.424  0.446  0.325  0.598  0.452  1.000
```

The Thurstone 9 variable problem from Bechtoldt



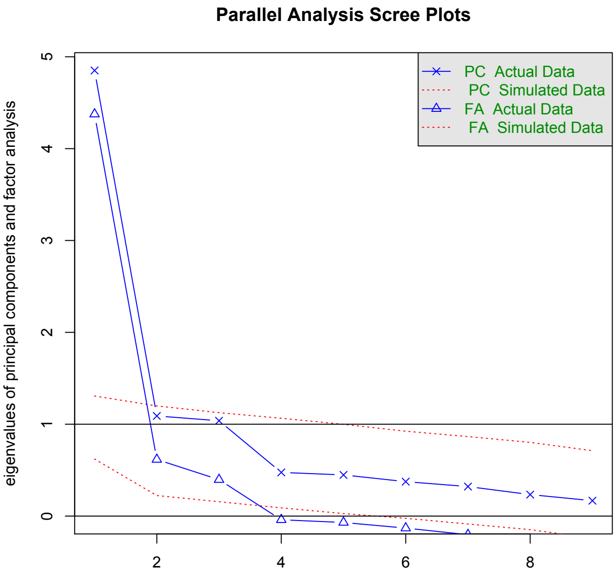
The number of factors problem

- "The number of factors problem is easy, I solve it everyday before breakfast. The problem is the right number" Kaiser.
  - Extract factors until χ^2 is not significant.
  - Extract factors until change in χ^2 is not significant.
  - Parallel analysis.
  - scree test
  - Very Simple Structure (VSS)
  - Minimum Absolute Partial (Velicer's MAP test)

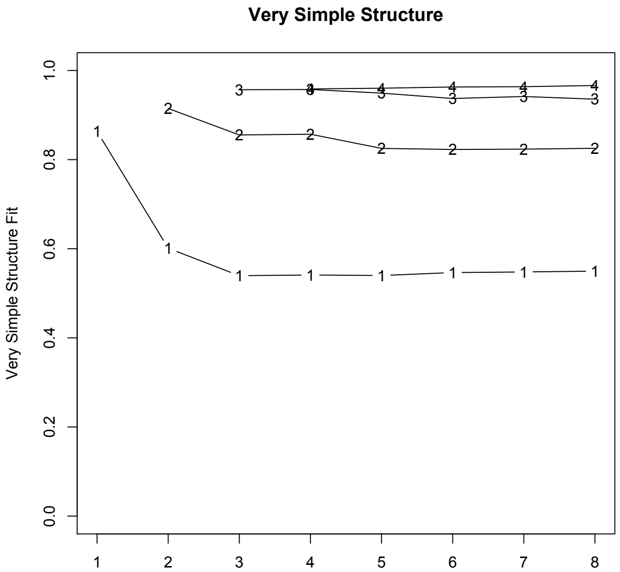
Number of factors?

```
> fa.parallel(Thurstone,n.obs=213)
> vss <- VSS(Thurstone,n.obs=213,SMC=FALSE)
> vss

Parallel analysis suggests that the number of factors =  3  and
                                  the number of components =  1

Very Simple Structure
Call: VSS(x = Thurstone, n.obs = 213, SMC = FALSE)
VSS complexity 1 achieves a maximimum of 0.86  with  1  factors
VSS complexity 2 achieves a maximimum of 0.91  with  2  factors

The Velicer MAP criterion achieves a minimum of 0.07  with  3  factors

Velicer MAP
[1] 0.07 0.07 0.07 0.11 0.20 0.31 0.59 1.00

Very Simple Structure Complexity 1
[1] 0.86 0.60 0.54 0.54 0.54 0.55 0.55 0.55

Very Simple Structure Complexity 2
[1] 0.00 0.91 0.86 0.86 0.83 0.82 0.82 0.83
```

Parallel analysis of the Thurstone data set



VSS fit function

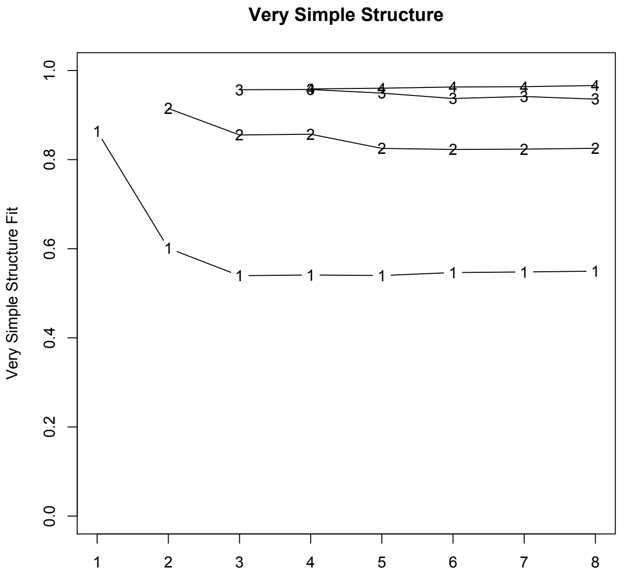


χ^2

```
> print(vss$vss.stats[,1:3],digits=3)
```

```
  dof    chisq     prob
1  27 2.32e+02 2.14e-34
2  19 8.30e+01 5.56e-10
3  12 2.82e+00 9.97e-01
4   6 1.49e+00 9.60e-01
5   1 2.44e-01 6.21e-01
6  -3 4.80e-07       NA
7  -6 4.16e-09       NA
8  -8 0.00e+00       NA
```

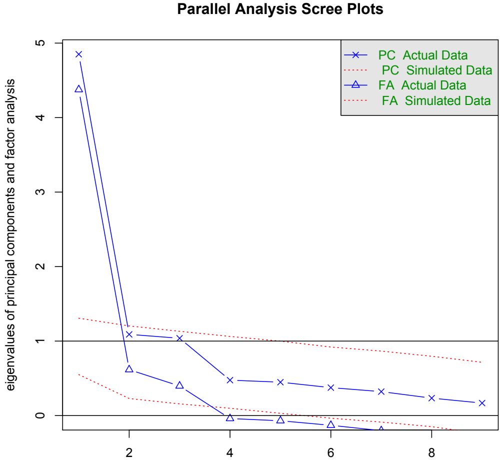
A Very Simple Structure Plot



Parallel analysis

> fa.parallel(Thurstone,n.obs=213)

Parallel analysis suggests that the number of factors = 3 and the number of components = 1



Minimal Residual FA

```
fa(Thurstone,n.obs=213)
```

```
> fa(Thurstone,n.obs=213)
Factor Analysis using method =  minres
Call: fa(r = Thurstone, n.obs = 213)
Standardized loadings based upon correlation matrix
    MR1   h2   u2
V1 0.87 0.75 0.25
V2 0.88 0.77 0.23
V3 0.83 0.70 0.30
V4 0.62 0.39 0.61
V5 0.61 0.37 0.63
V6 0.59 0.34 0.66
V7 0.57 0.32 0.68
V8 0.64 0.41 0.59
V9 0.52 0.27 0.73
                MR1
SS loadings    4.32
Proportion Var 0.48

Test of the hypothesis that 1 factor is sufficient.
The degrees of freedom for the null model are  36  and the objective function was
5.2 with Chi Square of  1081.97
The degrees of freedom for the model are 27  and the objective function was  1.12

The root mean square of the residuals is  0.08
The df corrected root mean square of the residuals is  0.13
The number of observations was  213  with Chi Square =  231.55  with prob <  2.1e-34

Tucker Lewis Index of factoring reliability =  0.738
RMSEA index =  0.191  and the 90 % confidence intervals are  0.19 0.194
BIC =  86.70
```

Maximum likelihood

```
> fa(Thurstone,n.obs=213,fm="mle")
Factor Analysis using method =  ml
Call: fa(r = Thurstone, n.obs = 213, fm = "mle")
Standardized loadings based upon correlation matrix
    ML1   h2   u2
V1 0.88 0.78 0.22
V2 0.90 0.80 0.20
V3 0.85 0.72 0.28
V4 0.59 0.35 0.65
V5 0.57 0.33 0.67
V6 0.57 0.32 0.68
V7 0.54 0.29 0.71
V8 0.63 0.40 0.60
V9 0.49 0.24 0.76
                ML1
SS loadings    4.22
Proportion Var 0.47

Test of the hypothesis that 1 factor is sufficient.
The degrees of freedom for the null model are  36  and the objective function was
5.2 with Chi Square of  1081.97
The degrees of freedom for the model are 27  and the objective function was  1.1

The root mean square of the residuals is  0.08
The df corrected root mean square of the residuals is  0.14
The number of observations was  213  with Chi Square =  228.59  with prob <  8e-34
Tucker Lewis Index of factoring reliability =  0.742
RMSEA index =  0.19  and the 90 % confidence intervals are  0.189 0.192
BIC =  83.83
Fit based upon off diagonal values = 0.94
Measures of factor score adequacy
                                                  ML1
```

Comparing min res and mle

```
Standardized loadings based upon correlation matrix
    MR1   h2   u2
V1 0.87 0.75 0.25
V2 0.88 0.77 0.23
V3 0.83 0.70 0.30
V4 0.62 0.39 0.61
V5 0.61 0.37 0.63
V6 0.59 0.34 0.66
V7 0.57 0.32 0.68
V8 0.64 0.41 0.59
V9 0.52 0.27 0.73

                MR1
SS loadings    4.32
Proportion Var 0.48
Chi Square =  231.55
     with prob <  2.1e-34
```

```
    ML1   h2   u2
V1 0.88 0.78 0.22
V2 0.90 0.80 0.20
V3 0.85 0.72 0.28
V4 0.59 0.35 0.65
V5 0.57 0.33 0.67
V6 0.57 0.32 0.68
V7 0.54 0.29 0.71
V8 0.63 0.40 0.60
V9 0.49 0.24 0.76

                ML1
SS loadings    4.22
Proportion Var 0.47
Chi Square =  228.59
     with prob <  8e-34
```

2 maximum likelihood factors

```
> f2 <- fa(Thurstone,2,n.obs=213,fm="mle")
> f2

Factor Analysis using method =  ml
Call: fa(r = Thurstone, nfactors = 2, n.obs = 213, fm = "mle")
Standardized loadings based upon correlation matrix
      ML1   ML2   h2   u2
V1   0.94 -0.05 0.83 0.17
V2   0.89  0.03 0.82 0.18
V3   0.85  0.02 0.73 0.27
V4  -0.02  0.84 0.68 0.32
V5  -0.04  0.84 0.66 0.34
V6   0.13  0.59 0.46 0.54
V7   0.28  0.35 0.32 0.68
V8   0.50  0.17 0.39 0.61
V9   0.12  0.49 0.32 0.68
                ML1  ML2
SS loadings     2.92 2.29
Proportion Var 0.32 0.25
Cumulative Var 0.32 0.58
 With factor correlations of
      ML1  ML2
ML1  1.00  0.64
```

2 factor goodness of fits

```
Test of the hypothesis that 2 factors are sufficient.

The degrees of freedom for the null model are  36  and the objecti
          was  5.2 with Chi Square of  1081.97
The degrees of freedom for the model are 19  and the
           objective function was  0.4

The root mean square of the residuals is  0.05
The df corrected root mean square of the residuals is  0.1
The number of observations was  213  with Chi Square =  82.84
                    with prob <  6e-10

Tucker Lewis Index of factoring reliability =  0.884
RMSEA index =  0.128  and the 90 % confidence intervals are
0.127 0.131
BIC =  -19.02
Fit based upon off diagonal values = 0.98
Measures of factor score adequacy
                                                  ML1  ML2
Correlation of scores with factors               0.97 0.93
Multiple R square of scores with factors         0.93 0.86
Minimum correlation of possible factor scores    0.86 0.72
```

3 MLE factors of Thurstone data

```
> f3 <- fa(Thurstone,3,n.obs=213,fm="mle")
> f3

Call: fa(r = Thurstone, nfactors = 3, n.obs = 213, fm = "mle")
Standardized loadings based upon correlation matrix
     ML1   ML2   ML3   h2   u2
V1  0.91 -0.04  0.04 0.83 0.17
V2  0.89  0.06 -0.03 0.84 0.16
V3  0.83  0.04  0.00 0.73 0.27
V4  0.00  0.86  0.01 0.73 0.27
V5 -0.01  0.74  0.10 0.63 0.37
V6  0.18  0.63 -0.08 0.50 0.50
V7  0.03 -0.01  0.84 0.72 0.28
V8  0.37 -0.05  0.47 0.50 0.50
V9 -0.06  0.21  0.64 0.53 0.47
                ML1  ML2  ML3
SS loadings     2.64 1.86 1.49
Proportion Var  0.29 0.21 0.17
Cumulative Var  0.29 0.50 0.67
 With factor correlations of
     ML1  ML2  ML3
ML1 1.00 0.59 0.54
ML2 0.59 1.00 0.52
ML3 0.54 0.52 1.00
```

with fit statistics

```
Test of the hypothesis that 3 factors are sufficient.

The degrees of freedom for the null model are  36  and the
        objective function was  5.2 with Chi Square of  1081.97
The degrees of freedom for the model are 12  and the
        objective function was  0.01

The root mean square of the residuals is  0
The df corrected root mean square of the residuals is  0.01
The number of observations was  213  with
          Chi Square =  2.82  with prob <  1

Tucker Lewis Index of factoring reliability =  1.027
RMSEA index =  0  and the 90 % confidence intervals are  0 0.023
BIC =  -61.51
Fit based upon off diagonal values = 1
Measures of factor score adequacy
                                                  ML1  ML2  ML3
Correlation of scores with factors               0.96 0.92 0.90
Multiple R square of scores with factors         0.93 0.85 0.81
Minimum correlation of possible factor scores    0.86 0.71 0.63
```

Orthogonal Rotations and oblique Transformations

- Should the latent variables be allowed to correlate?
  - Factors as extracted are orthogonal.
  - Factors as interpreted typically are allowed to be oblique
- Consider five cases
  - unrotated: factors as extracted (rotate="none")
  - rotated using VARIMAX (or Quartimax)
  - transformed using PROCRUSTES
  - transformed using oblimin
  - transformed using geominQ

Unrotated versus Varimax Rotation

```
> fa(Thurstone,3,n.obs=213,rotate="none")

Factor Analysis using method =
minres
Call: fa(r = Thurstone, nfactors = 3, n.obs = 213, rotate = "none")
Standardized loadings based upon correlation matrix
    MR1   MR2   MR3   h2   u2
V1 0.87 -0.27  0.02 0.82 0.18
V2 0.88 -0.24 -0.06 0.84 0.16
V3 0.83 -0.22 -0.03 0.73 0.27
V4 0.66  0.45 -0.32 0.73 0.27
V5 0.63  0.43 -0.21 0.63 0.37
V6 0.60  0.24 -0.29 0.50 0.50
V7 0.60  0.32  0.50 0.72 0.28
V8 0.65  0.05  0.29 0.50 0.50
V9 0.54  0.38  0.30 0.53 0.47

                MR1  MR2  MR3
SS loadings     4.47 0.86 0.67
Proportion Var  0.50 0.10 0.07
Cumulative Var  0.50 0.59 0.67
```

```
> fa(Thurstone,3,n.obs=213,rotate="Var
    MR1  MR2  MR3  h2   u2
V1 0.86 0.20 0.22 0.82 0.18
V2 0.85 0.27 0.18 0.84 0.16
V3 0.80 0.24 0.19 0.73 0.27
V4 0.29 0.78 0.20 0.73 0.27
V5 0.27 0.70 0.26 0.63 0.37
V6 0.36 0.60 0.10 0.50 0.50
V7 0.28 0.18 0.78 0.72 0.28
V8 0.48 0.15 0.50 0.50 0.50
V9 0.20 0.32 0.62 0.53 0.47

                MR1  MR2  MR3
SS loadings     2.73 1.78 1.48
Proportion Var  0.30 0.20 0.16
Cumulative Var  0.30 0.50 0.67
```

Varimax versus Quartimax

```
> fa(Thurstone,3,n.obs=213,rotate="varimax")
```

	MR1	MR2	MR3	h2	u2
V1	0.86	0.20	0.22	0.82	0.18
V2	0.85	0.27	0.18	0.84	0.16
V3	0.80	0.24	0.19	0.73	0.27
V4	0.29	0.78	0.20	0.73	0.27
V5	0.27	0.70	0.26	0.63	0.37
V6	0.36	0.60	0.10	0.50	0.50
V7	0.28	0.18	0.78	0.72	0.28
V8	0.48	0.15	0.50	0.50	0.50
V9	0.20	0.32	0.62	0.53	0.47

	MR1	MR2	MR3
SS loadings	2.73	1.78	1.48
Proportion Var	0.30	0.20	0.16
Cumulative Var	0.30	0.50	0.67

```
> fa(Thurstone,3,n.obs=213,rotate="qua
```

	MR1	MR2	MR3	h2	u2
V1	0.91	0.02	0.05	0.82	0.18
V2	0.91	0.09	0.01	0.84	0.16
V3	0.85	0.07	0.03	0.73	0.27
V4	0.47	0.71	0.11	0.73	0.27
V5	0.45	0.63	0.18	0.63	0.37
V6	0.48	0.51	0.01	0.50	0.50
V7	0.45	0.12	0.71	0.72	0.28
V8	0.58	0.05	0.40	0.50	0.50
V9	0.37	0.27	0.56	0.53	0.47

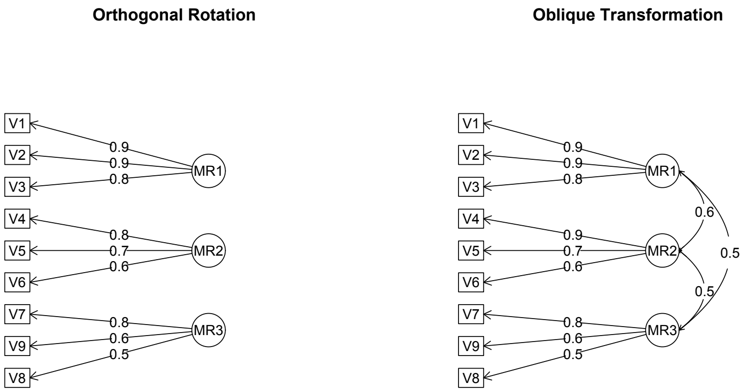
	MR1	MR2	MR3
SS loadings	3.70	1.27	1.03
Proportion Var	0.41	0.14	0.11
Cumulative Var	0.41	0.55	0.67

Varimax versus Quartimin

```
> fa(Thurstone,3,n.obs=213,
        rotate="Varimax")

    MR1  MR2  MR3   h2   u2
V1 0.86 0.20 0.22 0.82 0.18
V2 0.85 0.27 0.18 0.84 0.16
V3 0.80 0.24 0.19 0.73 0.27
V4 0.29 0.78 0.20 0.73 0.27
V5 0.27 0.70 0.26 0.63 0.37
V6 0.36 0.60 0.10 0.50 0.50
V7 0.28 0.18 0.78 0.72 0.28
V8 0.48 0.15 0.50 0.50 0.50
V9 0.20 0.32 0.62 0.53 0.47

                MR1  MR2  MR3
SS loadings    2.73 1.78 1.48
Proportion Var 0.30 0.20 0.16
Cumulative Var 0.30 0.50 0.67
```

```
> fa(Thurstone,3,n.obs=213,rotate="qua

     MR1   MR2   MR3   h2   u2
V1  0.91 -0.04  0.04 0.82 0.18
V2  0.89  0.06 -0.03 0.84 0.16
V3  0.83  0.04  0.00 0.73 0.27
V4  0.00  0.86  0.00 0.73 0.27
V5 -0.01  0.74  0.10 0.63 0.37
V6  0.18  0.63 -0.08 0.50 0.50
V7  0.03 -0.01  0.84 0.72 0.28
V8  0.37 -0.05  0.47 0.50 0.50
V9 -0.06  0.21  0.64 0.53 0.47

                MR1  MR2  MR3
SS loadings    2.64 1.86 1.50
Proportion Var 0.29 0.21 0.17
Cumulative Var 0.29 0.50 0.67
 With factor correlations of
     MR1  MR2  MR3
MR1 1.00 0.59 0.54
MR2 0.59 1.00 0.52
MR3 0.54 0.52 1.00
```

Visualizing the difference: Varimax versus quartimin



Specifying the parameters in sem using the RAM notation

- Need to specify each path
- Need to specify the error variance paths as well
    - A short cut can be done by using *psych*
    - See the vignette ‘`psych for sem`‘” for more details
- Apply this to the Thurstone problem

create the sem commands by using psych

```
f3 <- fa(Thurstone,3,fm='mle')
mod3 <- structure.diagram(f3,cut=.45,errors=TRUE)
mod3

      Path           Parameter Value
 [1,] "ML1->V1"      "F1V1"    NA
 [2,] "ML1->V2"      "F1V2"    NA
 [3,] "ML1->V3"      "F1V3"    NA
 [4,] "ML2->V4"      "F2V4"    NA
 [5,] "ML2->V5"      "F2V5"    NA
 [6,] "ML2->V6"      "F2V6"    NA
 [7,] "ML3->V7"      "F3V7"    NA
 [8,] "ML3->V8"      "F3V8"    NA
 [9,] "ML3->V9"      "F3V9"    NA
[10,] "V1<->V1"      "x1e"     NA
[11,] "V2<->V2"      "x2e"     NA
...
[18,] "V9<->V9"      "x9e"     NA
[19,] "ML2<->ML1"    "rF2F1"   NA
[20,] "ML3<->ML1"    "rF3F1"   NA
[21,] "ML3<->ML2"    "rF3F2"   NA
[22,] "ML1<->ML1"    NA        "1"
[23,] "ML2<->ML2"    NA        "1"
```

Running sem

```
> rownames(Thurstone) <- colnames(Thurstone) #to get the names to
match the modle
> sem3 <- sem(mod3, Thurstone,N=213)
> summary(sem3, digits=2)
```

```
Model Chisquare =  38   Df =  24 Pr(>Chisq) = 0.033
 Chisquare (null model) =  1102   Df =  36
 Goodness-of-fit index =  0.96
 Adjusted goodness-of-fit index =  0.92
RMSEA index =  0.053   90% CI: (0.015, 0.083)
 Bentler-Bonnett NFI =  0.97
 Tucker-Lewis NNFI =  0.98
 Bentler CFI =  0.99
SRMR =  0.044
BIC =  -90

 Normalized Residuals
   Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
 -0.97    -0.42     0.00     0.04     0.09     1.63
```

With parameter estimates

```
 Parameter Estimates
      Estimate Std Error z value Pr(>|z|)
F1V1  0.90     0.054     16.7    0.0e+00  V1 <--- ML1
F1V2  0.91     0.054     17.0    0.0e+00  V2 <--- ML1
F1V3  0.86     0.056     15.3    0.0e+00  V3 <--- ML1
F2V4  0.84     0.061     13.8    0.0e+00  V4 <--- ML2
F2V5  0.80     0.062     12.9    0.0e+00  V5 <--- ML2
F2V6  0.70     0.064     10.9    0.0e+00  V6 <--- ML2
F3V7  0.78     0.065     12.0    0.0e+00  V7 <--- ML3
F3V8  0.72     0.067     10.7    0.0e+00  V8 <--- ML3
F3V9  0.70     0.067     10.5    0.0e+00  V9 <--- ML3
x1e   0.18     0.028      6.4    1.7e-10  V1 <--> V1
x2e   0.16     0.028      5.9    3.0e-09  V2 <--> V2
x3e   0.27     0.033      8.0    1.6e-15  V3 <--> V3
x4e   0.30     0.051      5.9    2.7e-09  V4 <--> V4
x5e   0.36     0.052      7.0    3.4e-12  V5 <--> V5
x6e   0.51     0.060      8.4    0.0e+00  V6 <--> V6
x7e   0.39     0.062      6.3    2.3e-10  V7 <--> V7
x8e   0.48     0.065      7.4    1.8e-13  V8 <--> V8
x9e   0.51     0.065      7.7    9.5e-15  V9 <--> V9
rF2F1 0.64     0.051     12.6    0.0e+00  ML1 <--> ML2
rF3F1 0.67     0.054     12.5    0.0e+00  ML1 <--> ML3
rF3F2 0.64     0.059     10.7    0.0e+00  ML2 <--> ML3
```

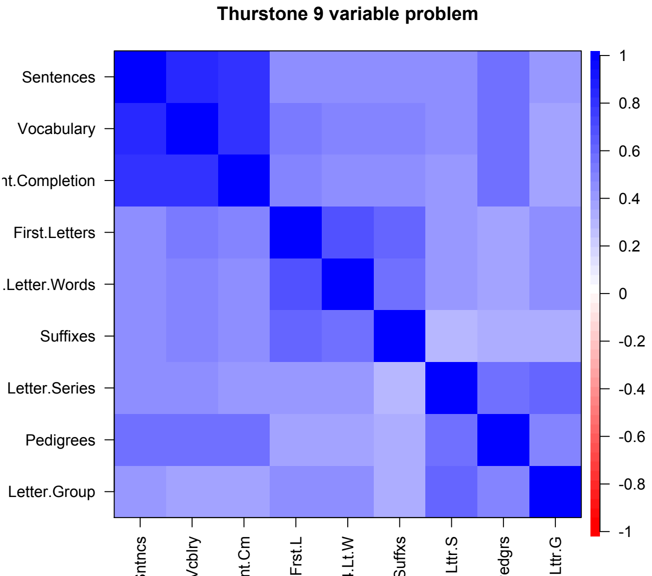
Exploratory bifactor model

```
omt <- omega(Thurstone)
omega.diagram(omt, sl=FALSE)

Omega
Call: omega(m = Thurstone)
Alpha:                  0.89
G.6:                    0.91
Omega Hierarchical:     0.74
Omega H asymptotic:     0.79
Omega Total             0.93

Schmid Leiman Factor loadings greater than  0.2
      g   F1*   F2*   F3*   h2   u2   p2
V1 0.71  0.57              0.82 0.18 0.61
V2 0.73  0.55              0.84 0.16 0.63
V3 0.68  0.52              0.73 0.27 0.63
V4 0.65        0.56        0.73 0.27 0.57
V5 0.62        0.49        0.63 0.37 0.61
V6 0.56        0.41        0.50 0.50 0.63
V7 0.59              0.61  0.72 0.28 0.48
V8 0.58  0.23        0.34  0.50 0.50 0.66
V9 0.54              0.46  0.53 0.47 0.56
```

With fit statistics

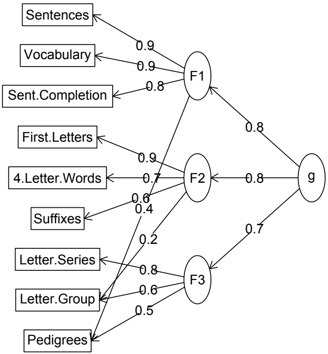
```
With eigenvalues of:
   g  F1*  F2*  F3*
3.58 0.96 0.74 0.71
general/max  3.71   max/min =   1.35
mean percent general =  0.6    with sd =  0.05 and cv of  0.09
The degrees of freedom are 12  and the fit is  0.01
The number of observations was  213  with Chi Square =  2.82
with prob <  1
The root mean square of the residuals is  0
The df corrected root mean square of the residuals is  0.01
RMSEA index =  0  and the 90 % confidence intervals are  0 0.023
BIC =  -61.51
Compare this with the adequacy of just a general factor and no gro
The degrees of freedom for just the general factor are 27  and the
1.48
The number of observations was  213  with Chi Square =  307.1
with prob <  2.8e-49
The root mean square of the residuals is  0.1
The df corrected root mean square of the residuals is  0.16
RMSEA index =  0.224  and the 90 % confidence intervals are
0.223 0.226
BIC =  162.35
Measures of factor score adequacy
```

A correlation plot to show a general factor

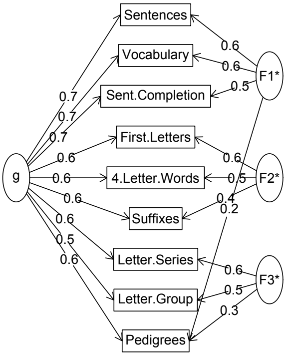


Two ways of showing a hierarchical structure

Hierarchical (multilevel) Structure



Omega



Exploratory versus confirmatory bifactor model

- `omega` will do exploratory bifactoring
- `omegaSem` will do exploratory and then confirmatory based upon that solution.
  - This is not true "confirmatory" in that the solution was decided on in an exploratory fashion.
  - True confirmatory tests a prior hypothesis
- `omegaSem` reports both exploratory and confirmatory results.
- The sem model is hidden as part of the structure but may be found from the `str` command

omegaSem

```
Alpha:                  0.89
G.6:                    0.91
Omega Hierarchical:     0.74
Omega H asymptotic:     0.79
Omega Total             0.93

Schmid Leiman Factor loadings greater than
0.2
                        g     F1*    F2*
F3*    h2    u2    p2
Sentences          0.71  0.57
0.82  0.18  0.61
Vocabulary         0.73  0.55
0.84  0.16  0.63
Sent.Completion    0.68  0.52
0.73  0.27  0.63
First.Letters      0.65         0.56
0.73  0.27  0.57
4.Letter.Words     0.62         0.49
0.63  0.37  0.61
Suffixes           0.56         0.41
0.50  0.50  0.63
Letter.Series      0.59
0.61  0.72  0.28  0.48
Pedigrees          0.58  0.23
0.34  0.50  0.50  0.66
Letter.Group       0.54
0.46  0.53  0.47  0.56
```

```
 Omega Hierarchical from a confirmatory model
      using sem =  0.79
 Omega Total  from a confirmatory model
      using sem =  0.93
With loadings of
                  g   F1*   F2*   F3*
h2    u2
Sentences        0.77  0.49
0.83  0.17
Vocabulary       0.79  0.45
0.83  0.17
Sent.Completion  0.75  0.40
0.73  0.27
First.Letters    0.61        0.61
0.75  0.25
4.Letter.Words   0.60        0.51
0.61  0.39
Suffixes         0.57        0.39
0.48  0.52
Letter.Series    0.57
0.73  0.85  0.15
Pedigrees        0.66
0.25  0.50  0.50
Letter.Group     0.53
0.41  0.45  0.55

With eigenvalues of:
   g   F1*   F2*   F3*
```

The sem model is an object in omegaSem output

```
omts <- omegaSem(Thurstone,n.obs=213)
omts$omegaSem$model
```

```
     Path                              Parameter              Initial Value
 [1,] "g->Sentences"                    "Sentences"            NA
 [2,] "g->Vocabulary"                   "Vocabulary"           NA
 [...
 [9,] "g->Letter.Group"                 "Letter.Group"         NA
[10,] "F1*->Sentences"                  "F1*Sentences"         NA
[11,] "F1*->Vocabulary"                 "F1*Vocabulary"        NA
[12,] "F1*->Sent.Completion"            "F1*Sent.Completion"   NA
[13,] "F2*->First.Letters"              "F2*First.Letters"     NA
[14,] "F2*->4.Letter.Words"             "F2*4.Letter.Words"    NA
[15,] "F2*->Suffixes"                   "F2*Suffixes"          NA
[16,] "F3*->Letter.Series"              "F3*Letter.Series"     NA
[17,] "F3*->Pedigrees"                  "F3*Pedigrees"         NA
[18,] "F3*->Letter.Group"               "F3*Letter.Group"      NA
[19,] "Sentences<->Sentences"           "e1"                   NA
[20,] "Vocabulary<->Vocabulary"         "e2"                   NA
...
[27,] "Letter.Group<->Letter.Group"     "e9"                   NA
[28,] "F1*<->F1*"                       NA                     "1"
[29,] "F2*<->F2*"                       NA                     "1"
[30,] "F3*<->F3*"                       NA                     "1"
[31,] "g <->g"                          NA                     "1"
```

Using the sem model from omeaSem

```
> sem.t.om <- sem(omts$omegaSem$model, Thurstone, N=213)
> summary(sem.t.om)
```

```
 Model Chisquare =  24.216   Df =  18 Pr(>Chisq) = 0.14807
 Chisquare (null model) =  1101.9   Df =  36
 Goodness-of-fit index =  0.97578
 Adjusted goodness-of-fit index =  0.93944
 RMSEA index =  0.040361   90% CI: (NA, 0.077994)
 Bentler-Bonnett NFI =  0.97802
 Tucker-Lewis NNFI =  0.98834
 Bentler CFI =  0.99417
 SRMR =  0.034895
 BIC =  -72.287

 Normalized Residuals
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.8210  -0.3340   0.0000   0.0282   0.1560   1.8000
```

sem parameter estimates

```
Parameter Estimates
                     Estimate  Std Error  z value   Pr(>|z|)
Sentences            0.76787   0.072626  10.57291  0.0000e+00 Sentences <--- g
Vocabulary           0.79092   0.072418  10.92170  0.0000e+00 Vocabulary <--- g
Sent.Completion      0.75362   0.073402  10.26709  0.0000e+00 Sent.Completion <--- g
First.Letters        0.60838   0.072201   8.42617  0.0000e+00 First.Letters <--- g
4.Letter.Words       0.59733   0.073851   8.08843  6.6613e-16 4.Letter.Words <--- g
Suffixes             0.57179   0.071492   7.99792  1.3323e-15 Suffixes <--- g
Letter.Series        0.56689   0.074271   7.63282  2.2871e-14 Letter.Series <--- g
Pedigrees            0.66233   0.069321   9.55455  0.0000e+00 Pedigrees <--- g
Letter.Group         0.52995   0.078985   6.70955  1.9522e-11 Letter.Group <--- g
F1*Sentences         0.48787   0.085457   5.70898  1.1366e-08 Sentences <--- F1*
F1*Vocabulary        0.45232   0.090422   5.00233  5.6640e-07 Vocabulary <--- F1*
F1*Sent.Completion   0.40445   0.093402   4.33024  1.4895e-05 Sent.Completion <--- F1*
F2*First.Letters     0.61405   0.085794   7.15733  8.2268e-13 First.Letters <--- F2*
F2*4.Letter.Words    0.50581   0.084848   5.96130  2.5024e-09 4.Letter.Words <--- F2*
F2*Suffixes          0.39432   0.078289   5.03671  4.7359e-07 Suffixes <--- F2*
F3*Letter.Series     0.72730   0.159499   4.55988  5.1184e-06 Letter.Series <--- F3*
F3*Pedigrees         0.24684   0.089011   2.77317  5.5513e-03 Pedigrees <--- F3*
F3*Letter.Group      0.40915   0.122180   3.34875  8.1177e-04 Letter.Group <--- F3*
e1                   0.17236   0.034113   5.05265  4.3571e-07 Sentences <--> Sentences
e2                   0.16984   0.030037   5.65438  1.5641e-08 Vocabulary <--> Vocabulary
e3                   0.26847   0.033188   8.08958  6.6613e-16 Sent.Completion <--> Sent.Com
e4                   0.25281   0.079472   3.18115  1.4669e-03 First.Letters <--> First.Lette
e5                   0.38735   0.063194   6.12960  8.8103e-10 4.Letter.Words <--> 4.Letter.V
e6                   0.51757   0.059639   8.67838  0.0000e+00 Suffixes <--> Suffixes
e7                   0.14967   0.223242   0.67044  5.0257e-01 Letter.Series <--> Letter.Seri
e8                   0.50039   0.059655   8.38800  0.0000e+00 Pedigrees <--> Pedigrees
e9                   0.55175   0.084725   6.51223  7.4044e-11 Letter.Group <--> Letter.Group
```

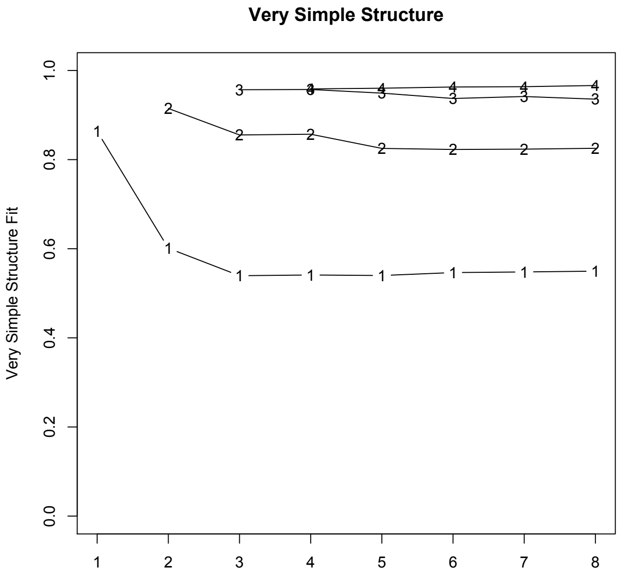
14 cognitive variables from Holzinger

```
> lowerCor(Holzinger)

     T1    T2    T3.4  T6    T28   T29   T32   T34   T35   T36a  T13   T18   T25b
T77
T1    1.00
T2    0.52  1.00
T3.4  0.49  0.79  1.00
T6    0.42  0.42  0.28  1.00
T28   0.47  0.28  0.17  0.48  1.00
T29   0.33  0.41  0.29  0.62  0.50  1.00
T32  -0.44 -0.42 -0.31 -0.57 -0.52 -0.59  1.00
T34  -0.11 -0.31 -0.40  0.11 -0.07 -0.13  0.13  1.00
T35  -0.23 -0.56 -0.42 -0.15 -0.16 -0.39  0.14  0.41  1.00
T36a -0.03  0.02 -0.22  0.20 -0.01  0.13 -0.15  0.40  0.01  1.00
T13   0.04  0.59  0.47  0.07 -0.09  0.19 -0.29 -0.45 -0.71  0.02  1.00
T18  -0.02  0.54  0.34  0.08 -0.15  0.22 -0.33 -0.46 -0.70  0.08  0.87  1.00
T25b -0.03  0.47  0.42  0.01 -0.19  0.07 -0.26 -0.59 -0.68 -0.14  0.81  0.81  1.00
T77  -0.01  0.44  0.35 -0.03 -0.15  0.10 -0.26 -0.61 -0.70 -0.13  0.79  0.82  0.79
1.00
>
```

How many factors

```
> vss.h <- vss(Holzinger, n.obs=355)
> vss.h

Very Simple Structure
Call: VSS(x = x, n = n, rotate = rotate, diagonal = diagonal, fm = fm,
    n.obs = n.obs, plot = plot, title = title)
VSS complexity 1 achieves a maximimum of 0.75  with  1  factors
VSS complexity 2 achieves a maximimum of 0.83  with  2  factors

The Velicer MAP criterion achieves a minimum of 0.03  with  2  factors

Velicer MAP
[1] 0.03 0.03 0.03 0.04 0.06 0.07 0.10 0.14

Very Simple Structure Complexity 1
[1] 0.75 0.59 0.56 0.51 0.49 0.48 0.44 0.44

Very Simple Structure Complexity 2
[1] 0.00 0.83 0.82 0.77 0.73 0.70 0.62 0.61
```

Holzinger 14 variable problem



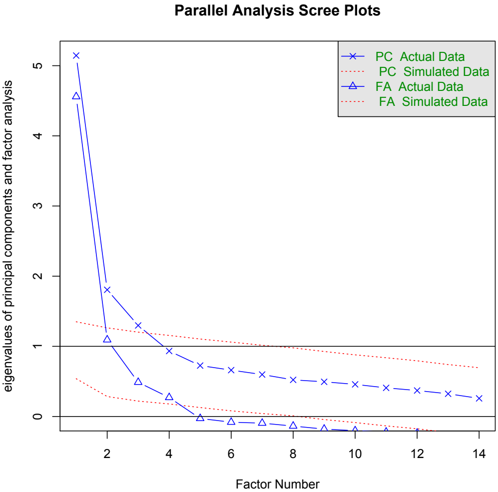
Chi square test

```
> print(vss.h$vss.stats[,1:4],digits=3)
```

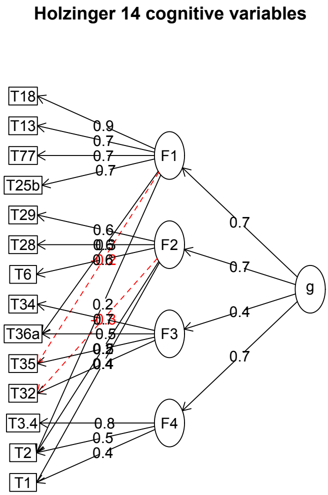
```
  dof  chisq     prob sqresid
1  77 543.38 2.64e-71    8.84
2  64 231.86 7.26e-21    5.85
3  52 118.10 4.74e-07    4.51
4  41  41.66 4.42e-01    3.65
5  31  27.16 6.64e-01    3.45
6  22  13.58 9.16e-01    3.01
7  14   5.79 9.71e-01    2.87
8   7   2.65 9.15e-01    2.77
```

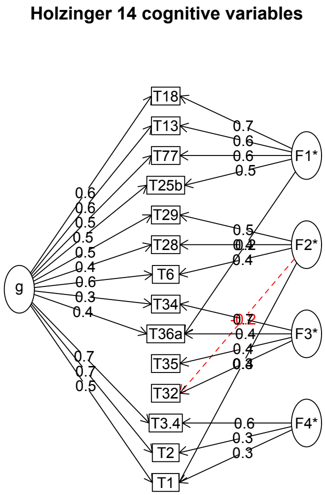
Parallel analysis of Holzinger 14 variables

> fa.parallel(Holzinger,n.obs=355) Parallel analysis suggests that the number of factors = 4 and the number of components = 3



Exploratory Omega solution for Holzinger 14 variable problem





Omega for Holzinger – exploratory and confirmatory

```
Call: omegaSem(m = Holzinger, nfactors = 4, n.obs = 355)
Omega

Alpha:                    0.85
G.6:                      0.88
Omega Hierarchical:       0.64
Omega H asymptotic:       0.71
Omega Total               0.9

Schmid Leiman Factor loadings greater than
0.2
         g    F1*   F2*   F3*   F4*
h2   u2   p2
T1    0.52        0.27
0.27 0.42 0.58 0.64
T2    0.69
0.34 0.66 0.34 0.72
T3.4  0.65
0.57 0.75 0.25 0.56
T6    0.57        0.42
0.54 0.46 0.59
T28   0.44        0.44
0.40 0.60 0.49
T29   0.51        0.47
0.50 0.50 0.54
T32              -0.25  0.38
0.26 0.74 0.13
T34   0.32              0.66
0.54 0.46 0.19
T35                     0.44
```

```
 Omega Hierarchical from a confirmatory model
0.75
 Omega Total
from a confirmatory model using sem =
0.9
With loadings of
        g   F1*  F2*  F3*  F4*   h2   u2
T1    0.59                      0.37 0.63
T2    0.80                 0.26 0.71 0.29
T3.4  0.64                 0.57 0.74 0.26
T6    0.65       0.36           0.55 0.45
T28   0.50       0.34           0.36 0.64
T29   0.57       0.45           0.53 0.47
T32                   0.32      0.13 0.87
T34   0.36            0.65      0.55 0.45
T35                   0.44      0.22 0.78
T36a  0.51            0.38      0.41 0.59
T13   0.64  0.51                0.67 0.33
T18   0.60  0.57                0.68 0.32
T25b  0.40  0.55                0.55 0.45
```

lavaan

- Mimics input of LISREL and MPlus
  - Easier to set up than sem
  - Clearly targets MPlus
  - Further help available at `http://lavaan.ugent.be/`
  - Example data sets from MPlus

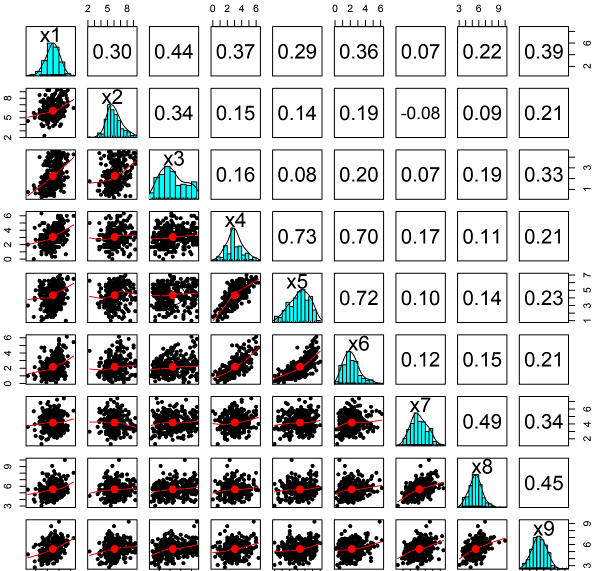
An example from lavaan

1. Holzinger Swineford data set from three schools
   - First describe the data
   - descriptive stats and graphics
2. Then consider the structure of the 9 variables
   - Using fa
   - Using sem/lavaan

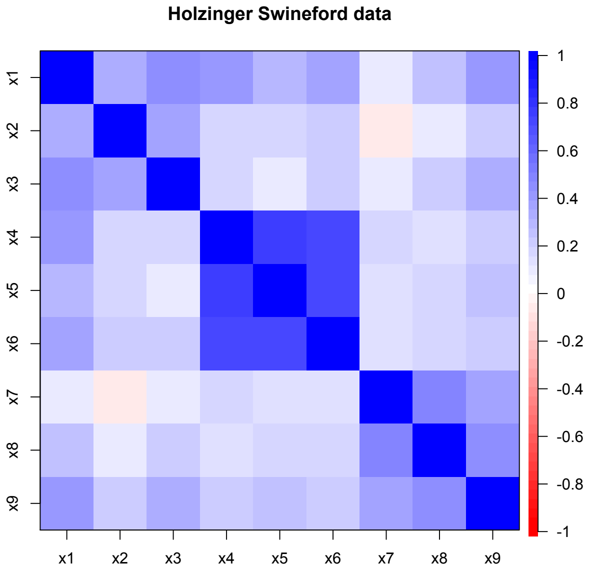
Holzinger descriptive statistics

```
> describe(HolzingerSwineford1939)

         var   n   mean     sd median trimmed    mad   min    max  range  skew kurtosis
se
id         1 301 176.55 105.94 163.00  176.78 140.85  1.00 351.00 350.00 -0.01
-1.36  6.11
sex        2 301   1.51   0.50   2.00    1.52   0.00  1.00   2.00   1.00 -0.06
-2.00  0.03
ageyr      3 301  13.00   1.05  13.00   12.89   1.48 11.00  16.00   5.00  0.69
0.20  0.06
agemo      4 301   5.38   3.45   5.00    5.32   4.45  0.00  11.00  11.00  0.09
-1.22  0.20
school*    5 301   1.52   0.50   2.00    1.52   0.00  1.00   2.00   1.00 -0.07
-2.00  0.03
grade      6 300   7.48   0.50   7.00    7.47   0.00  7.00   8.00   1.00  0.09
-2.00  0.03
x1         7 301   4.94   1.17   5.00    4.96   1.24  0.67   8.50   7.83 -0.25
0.31  0.07
x2         8 301   6.09   1.18   6.00    6.02   1.11  2.25   9.25   7.00  0.47
0.33  0.07
x3         9 301   2.25   1.13   2.12    2.20   1.30  0.25   4.50   4.25  0.38
-0.91  0.07
x4        10 301   3.06   1.16   3.00    3.02   0.99  0.00   6.33   6.33  0.27
0.08  0.07
x5        11 301   4.34   1.29   4.50    4.40   1.48  1.00   7.00   6.00 -0.35
-0.55  0.07
x6        12 301   2.19   1.10   2.00    2.09   1.06  0.14   6.14   6.00  0.86
0.82  0.06
x7        13 301   4.19   1.09   4.09    4.16   1.10  1.30   7.43   6.13  0.25
-0.31  0.06
x8        14 301   5.53   1.01   5.50    5.40   0.96  3.05  10.00   6.95  0.53
```

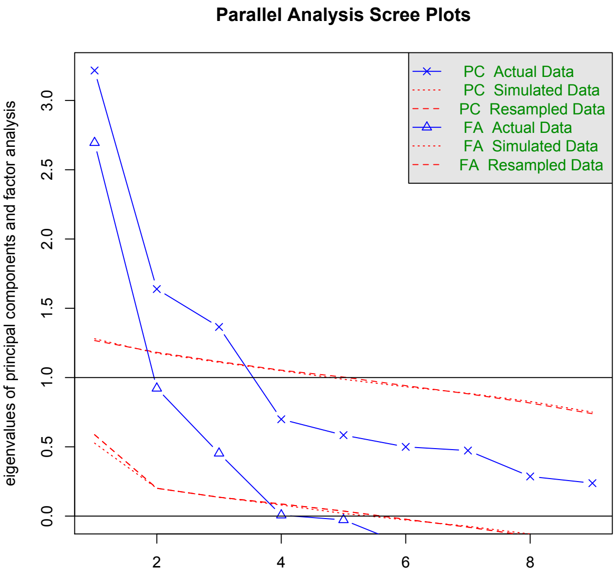
Pairs panels of HolzingerSwineford1939



Weaker correlations than the Thurstone data set



Parallel Analysis of the Holzlinger Swineford data set



Extract 3 factors from Holzinger Swineford data set

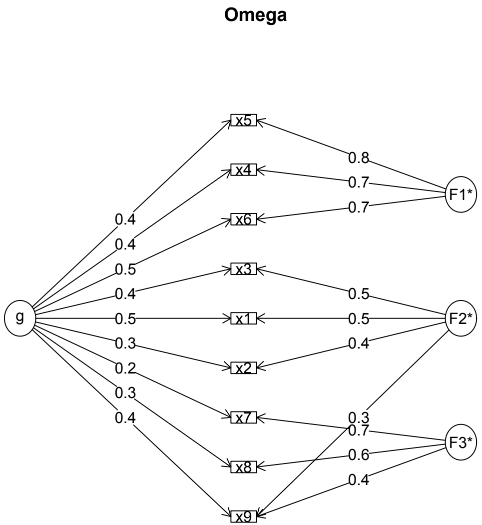
```
> hs.fa <- fa(HolzingerSwineford1939[,7:15],3)
> hs.fa
```

```
Factor Analysis using method =  minres
Call: fa(r = HolzingerSwineford1939[, 7:15], nfactors = 3)
Standardized loadings (pattern matrix) based upon correlation matrix
     MR1   MR3   MR2   h2   u2
x1  0.19  0.60  0.03 0.49 0.51
x2  0.04  0.51 -0.12 0.25 0.75
x3 -0.07  0.69  0.02 0.46 0.54
x4  0.84  0.02  0.01 0.72 0.28
x5  0.89 -0.07  0.01 0.76 0.24
x6  0.81  0.08 -0.01 0.69 0.31
x7  0.04 -0.15  0.72 0.50 0.50
x8 -0.03  0.10  0.70 0.53 0.47
x9  0.03  0.37  0.46 0.46 0.54

                       MR1  MR3  MR2
SS loadings           2.24 1.34 1.28
Proportion Var        0.25 0.15 0.14
Cumulative Var        0.25 0.40 0.54
Proportion Explained  0.46 0.28 0.26
Cumulative Proportion 0.46 0.74 1.00

 With factor correlations of
     MR1  MR3  MR2
MR1 1.00 0.33 0.22
MR3 0.33 1.00 0.27
MR2 0.22 0.27 1.00
```

With goodness of fit stats

```
Test of the hypothesis that 3 factors are sufficient.

The degrees of freedom for the null model are  36  and the objective function was
3.05 with Chi Square of  904.1
The degrees of freedom for the model are 12  and the objective function was  0.08

The root mean square of the residuals (RMSR) is  0.01
The df corrected root mean square of the residuals is  0.03
The number of observations was  301  with Chi Square =  22.38  with prob <  0.034

Tucker Lewis Index of factoring reliability =  0.964
RMSEA index =  0.055  and the 90 % confidence intervals are  0.015 0.088
BIC =  -46.11
Fit based upon off diagonal values = 1
Measures of factor score adequacy
                                                  MR1  MR3  MR2
Correlation of scores with factors               0.94 0.84 0.85
Multiple R square of scores with factors         0.89 0.71 0.72
Minimum correlation of possible factor scores    0.78 0.42 0.45
```

Weak evidence for hierarchical structure



Omega analysis of Holzinger Swineford data

```
> om<- omega(HolzingerSwineford1939[,7:15])
> om
Omega
Call: omega(m = HolzingerSwineford1939[, 7:15])
Alpha:                 0.76
G.6:                   0.81
Omega Hierarchical:    0.45
Omega H asymptotic:    0.53
Omega Total            0.85

Schmid Leiman Factor loadings greater than  0.2
      g   F1*   F2*   F3*   h2   u2   p2
x1 0.49       0.46        0.49 0.51 0.50
x2 0.30       0.39        0.25 0.75 0.35
x3 0.41       0.53        0.46 0.54 0.38
x4 0.45 0.72              0.72 0.28 0.28
x5 0.41 0.76              0.76 0.24 0.23
x6 0.46 0.69              0.69 0.31 0.30
x7 0.23             0.66 0.50 0.50 0.11
x8 0.35             0.64 0.53 0.47 0.23
x9 0.45       0.28  0.42 0.46 0.54 0.44

With eigenvalues of:
   g  F1*  F2*  F3*
1.46 1.62 0.75 1.02
```

Omega fit statistics for Holzinger Swineford

```
general/max  0.9   max/min =   2.15
mean percent general =  0.31    with sd =  0.12 and cv of  0.38

The degrees of freedom are 12  and the fit is  0.08
The number of observations was  301  with Chi Square =  22.38  with prob <  0.034
The root mean square of the residuals is  0.01
The df corrected root mean square of the residuals is  0.03
RMSEA index =  0.055  and the 90 % confidence intervals are  0.015 0.088
BIC =  -46.11

Compare this with the adequacy of just a general factor and no group factors
The degrees of freedom for just the general factor are 27  and the fit is  1.75
The number of observations was  301  with Chi Square =  517.18  with prob <  4.4e-92
The root mean square of the residuals is  0.14
The df corrected root mean square of the residuals is  0.23

RMSEA index =  0.248  and the 90 % confidence intervals are  0.227 0.264
BIC =  363.09

Measures of factor score adequacy
                                                 g  F1*   F2*  F3*
Correlation of scores with factors            0.68 0.84  0.67 0.78
Multiple R square of scores with factors      0.46 0.71  0.44 0.61
Minimum correlation of factor score estimates -0.07 0.43 -0.11 0.22
```

The lavaan commands

```
# The Holzinger and Swineford (1939) example
HS.model <- ' visual  =~ x1 + x2 + x3
              textual =~ x4 + x5 + x6
              speed   =~ x7 + x8 + x9 '

fit <- lavaan(HS.model, data=HolzingerSwineford1939,
              auto.var=TRUE, auto.fix.first=TRUE,
              auto.cov.lv.x=TRUE)
summary(fit, fit.measures=TRUE)
```

lavaan output

```
lavaan (0.4-14) converged normally after 41 iterations

  Number of observations                          301

  Estimator                                        ML
  Minimum Function Chi-square                  85.306
  Degrees of freedom                               24
  P-value                                       0.000
Chi-square test baseline model:
  Minimum Function Chi-square                 918.852
  Degrees of freedom                               36
  P-value                                       0.000
Full model versus baseline model:
  Comparative Fit Index (CFI)                   0.931
  Tucker-Lewis Index (TLI)                      0.896
Loglikelihood and Information Criteria:
  Loglikelihood user model (H0)             -3737.745
  Loglikelihood unrestricted model (H1)     -3695.092
  Number of free parameters                        21
  Akaike (AIC)                               7517.490
  Bayesian (BIC)                             7595.339
  Sample-size adjusted Bayesian (BIC)        7528.739
Root Mean Square Error of Approximation:
  RMSEA                                         0.092
  90 Percent Confidence Interval       0.071    0.114
  P-value RMSEA <= 0.05                         0.001
```

and the parameter estimates

```
Parameter estimates:
  Information                                     Expected
  Standard Errors                                 Standard
                     Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  visual =~
    x1                 1.000
    x2                 0.553    0.100    5.554    0.000
    x3                 0.729    0.109    6.685    0.000
  textual =~
    x4                 1.000
    x5                 1.113    0.065   17.014    0.000
    x6                 0.926    0.055   16.703    0.000
  speed =~
    x7                 1.000
    x8                 1.180    0.165    7.152    0.000
    x9                 1.082    0.151    7.155    0.000
Covariances:
  visual ~~
    textual            0.408    0.074    5.552    0.000
    speed              0.262    0.056    4.660    0.000
  textual ~~
    speed              0.173    0.049    3.518    0.000
Variances:
    x1                 0.549    0.114
    x2                 1.134    0.102
    x3                 0.844    0.091
    x4                 0.371    0.048
    x5                 0.446    0.058
    x6                 0.356    0.043
    x7                 0.799    0.081
    x8                 0.488    0.074
```

Do this again, but fix the latent variable variance to 1

```
                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  visual =~
    x1                0.900    0.081   11.127    0.000
    x2                0.498    0.077    6.429    0.000
    x3                0.656    0.074    8.817    0.000
  textual =~
    x4                0.990    0.057   17.474    0.000
    x5                1.102    0.063   17.576    0.000
    x6                0.917    0.054   17.082    0.000
  speed =~
    x7                0.619    0.070    8.903    0.000
    x8                0.731    0.066   11.090    0.000
    x9                0.670    0.065   10.305    0.000
Covariances:
  visual ~~
    textual           0.459    0.064    7.189    0.000
    speed             0.471    0.073    6.461    0.000
  textual ~~
    speed             0.283    0.069    4.117    0.000
Variances:
    x1                0.549    0.114
    x2                1.134    0.102
    x3                0.844    0.091
    x4                0.371    0.048
    x5                0.446    0.058
    x6                0.356    0.043
    x7                0.799    0.081
    x8                0.488    0.074
    x9                0.566    0.071
    visual            1.000
```