Latent models of real data

Exploring the structure of three data sets

Prior examples were with artificial data sets now lets explore real data

- 9 cognitive variables from Thurstone
- 14 cognitive variables from Holzinger
- 16 medical service items from Reise

9 Cognitive Variables

- Thurstone, Primary Mental Abilities
- Example data set available as part of sem package, also part of PROC CALIS example, also used by R. McDonald
- Part of bifactor data set (psych 1.0-64)

Thurstone correlation matrix

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]
Sentences	1.00	0.83	0.78	0.44	0.43	0.45	0.45	0.54	0.38
Vocabulary	0.83	1.00	0.78	0.49	0.46	0.49	0.43	0.54	0.36
Sent.Completion	0.78	0.78	1.00	0.46	0.42	0.44	0.40	0.53	0.36
First.Letters	0.44	0.49	0.46	1.00	0.67	0.59	0.38	0.35	0.42
4.Letter.Words	0.43	0.46	0.42	0.67	1.00	0.54	0.40	0.37	0.45
Suffixes	0.45	0.49	0.44	0.59	0.54	1.00	0.29	0.32	0.32
Letter.Series	0.45	0.43	0.40	0.38	0.40	0.29	1.00	0.56	0.60
Pedigrees	0.54	0.54	0.53	0.35	0.37	0.32	0.56	1.00	0.45
Letter.Group	0.38	0.36	0.36	0.42	0.45	0.32	0.60	0.45	1.00

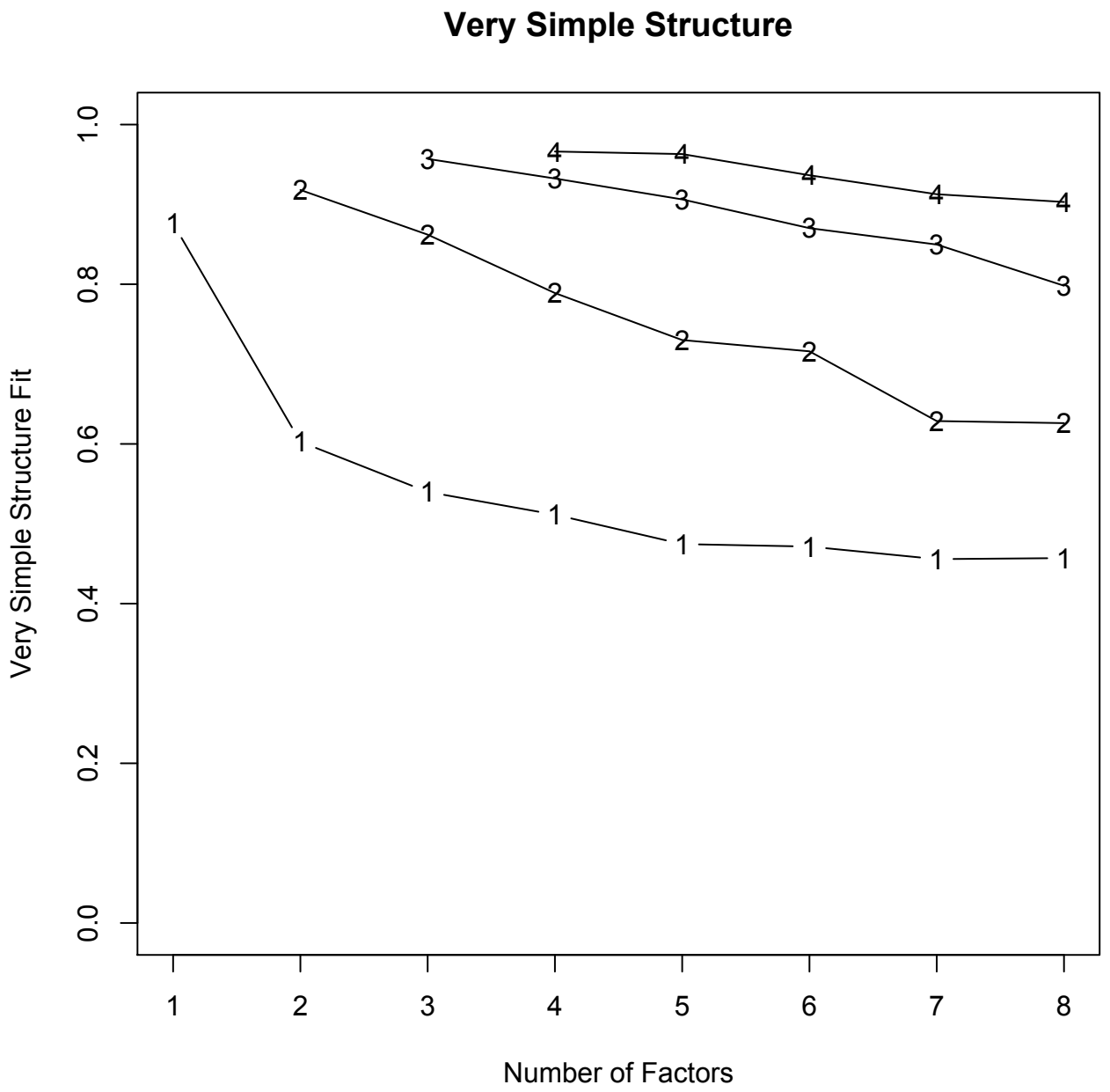
Number of factors?

- Extract factors until chi square is not significant
- Extract factors until change in chi square n.s.
- Parallel analysis
- scree
- Very Simple Structure (VSS)
- Minimum Absolute Partial (MAP)

VSS function

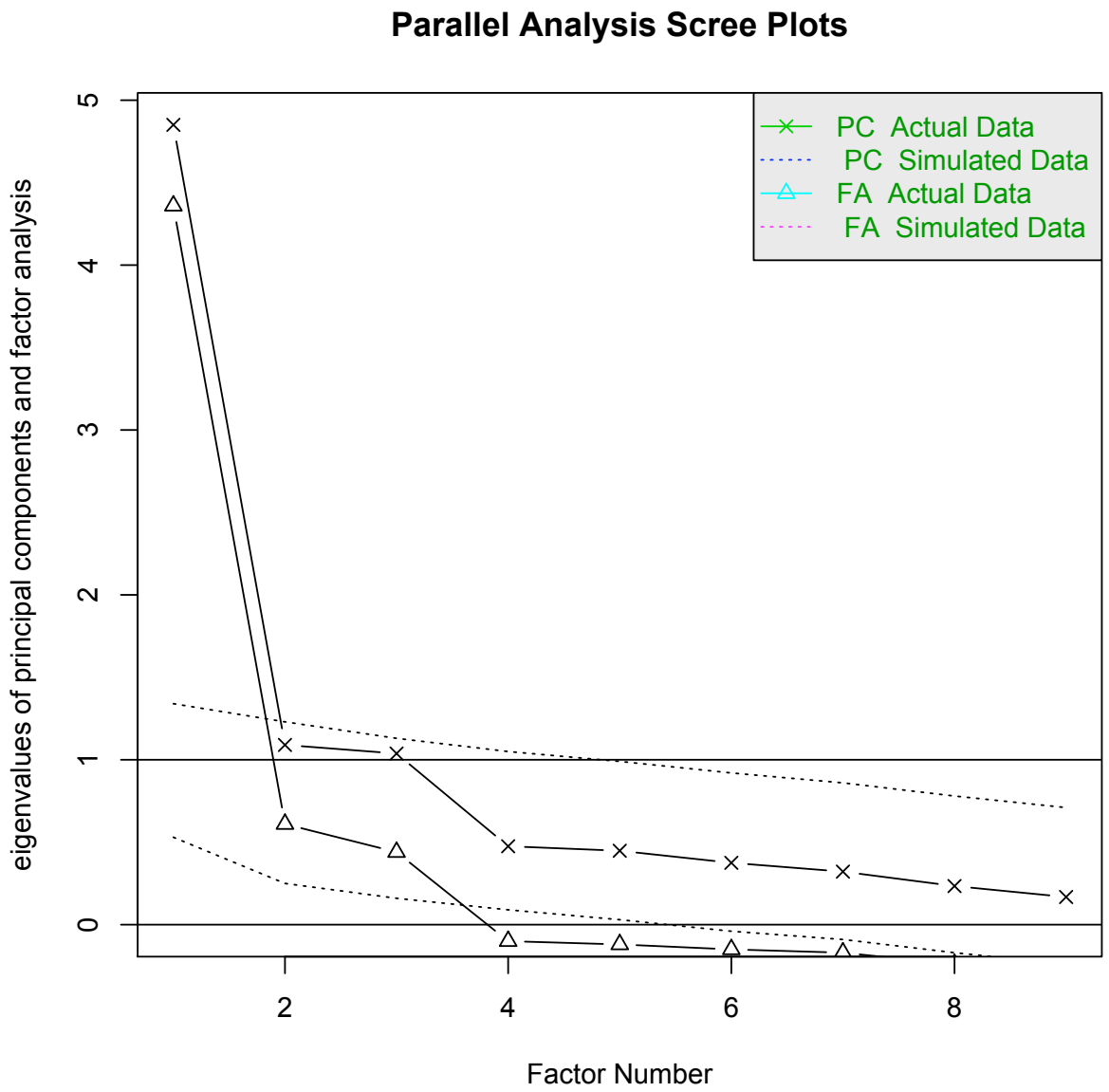
vss <- VSS(Thurstone,n.obs=213,SMC=FALSE)

```
Very Simple Structure
VSS complexity 1 achieves a maximimum of 0.88  with  1
factors
VSS complexity 2 achieves a maximimum of 0.92  with  2
factors

The Velicer MAP criterion achieves a minimum of 1  with  3
factors
 Velicer MAP
[1] 0.07 0.07 0.07 0.11 0.20 0.31 0.59 1.00

Very Simple Structure Complexity 1
[1] 0.88 0.60 0.54 0.51 0.47 0.47 0.46 0.46

Very Simple Structure Complexity 2
[1] 0.00 0.92 0.86 0.79 0.73 0.72 0.63 0.63
```


Very Simple Structure
Very Simple Structure Fit
0.0
0.2
0.4
0.6
0.8
1.0
1
2
3
4
5
6
7
8
Number of Factors

Chi square => 3 factors

```
> print(vss$vss.stats[,1:3],digits=3)
  dof    chisq     prob
1  27 2.55e+02 5.05e-39
2  19 8.67e+01 1.29e-10
3  12 3.10e+00 9.95e-01
4   6 1.58e+00 9.54e-01
5   1 1.99e+00 1.58e-01
6  -3 1.15e+00       NA
7  -6 1.10e-02       NA
8  -8 4.48e-04       NA
```



Parallel analysis => 1 PC, 3 PA

```
factanal(covmat=Thurstone,factors=1,n.obs=213)

factanal(factors = 1, covmat = Thurstone, n.obs = 213)

Loadings:
                 Factor1
Sentences        0.883
Vocabulary       0.896
Sent.Completion  0.848
First.Letters    0.590
4.Letter.Words   0.570
Suffixes         0.565
Letter.Series    0.543
Pedigrees        0.632
Letter.Group     0.487

                Factor1
SS loadings       4.225
Proportion Var    0.469
Test of the hypothesis that 1 factor is sufficient.
The chi square statistic is 228.59 on 27 degrees of freedom
The p-value is 8.04e-34
```

MLE factanal

```
 factanal(covmat=Thurstone,factors=2,n.obs=213)
                Factor1 Factor2
Sentences         0.866   0.287
Vocabulary        0.839   0.343
Sent.Completion   0.795   0.314
First.Letters     0.255   0.783
4.Letter.Words    0.235   0.775
Suffixes          0.317   0.602
Letter.Series     0.372   0.426
Pedigrees         0.524   0.336
Letter.Group      0.269   0.501
                Factor1 Factor2
SS loadings       2.793   2.420
Proportion Var    0.310   0.269
Cumulative Var    0.310   0.579
Test of the hypothesis that 2 factors are sufficient.
The chi square statistic is 82.84 on 19 degrees of freedom.
The p-value is 5.99e-10
```

MLE factanal 2

```
Call:
factanal(factors = 3, covmat = Thurstone, n.obs = 213)
Loadings:
                 Factor1 Factor2 Factor3
Sentences          0.834   0.244   0.264
Vocabulary         0.827   0.318   0.223
Sent.Completion    0.775   0.284   0.227
First.Letters      0.228   0.792   0.230
4.Letter.Words     0.213   0.706   0.291
Suffixes           0.314   0.616   0.134
Letter.Series      0.232   0.179   0.795
Pedigrees          0.446   0.166   0.527
Letter.Group       0.154   0.311   0.638

                Factor1 Factor2 Factor3
SS loadings       2.454   1.902   1.642
Proportion Var    0.273   0.211   0.182
Cumulative Var    0.273   0.484   0.666

Test of the hypothesis that 3 factors are sufficient.
The chi square statistic is 2.82 on 12 degrees of freedom.
The p-value is 0.997
```

MLE factanal 3

Oblique rotation (Promax)

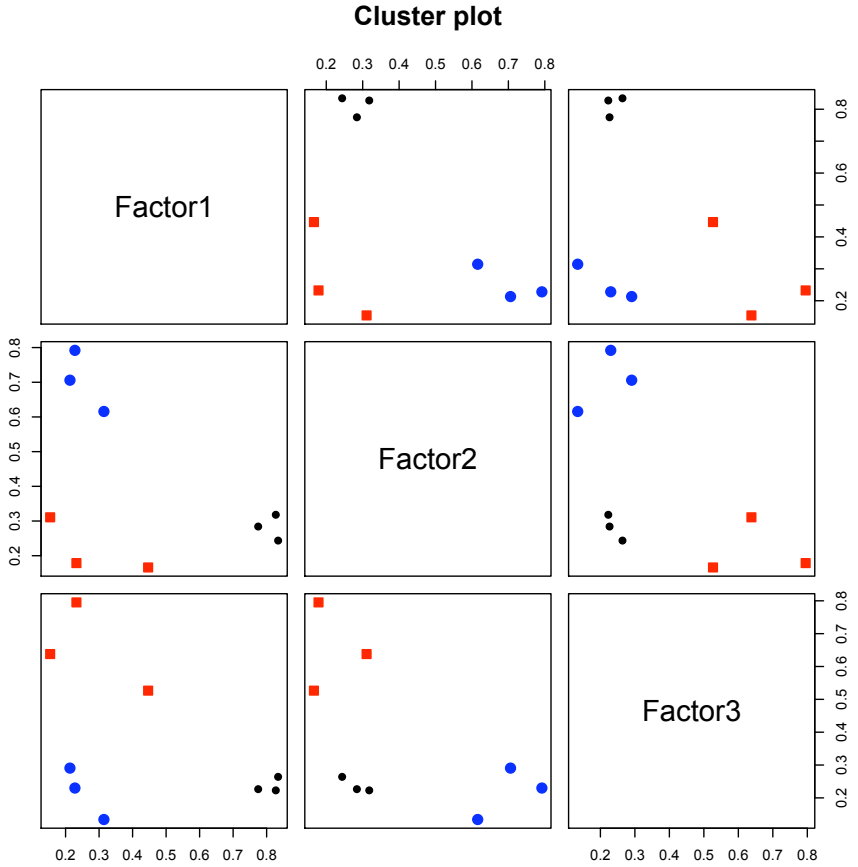
```
> t3p <- Promax(t3)
> t3p
                 V Factor1 Factor2 Factor3
Sentences        1   0.919                
Vocabulary       2   0.901                
Sent.Completion  3   0.842                
First.Letters    4           0.876        
4.Letter.Words   5           0.756        
Suffixes         6           0.645        
Letter.Series    7                   0.893
Pedigrees        8   0.347           0.494
Letter.Group     9                   0.675
                Factor1 Factor2 Factor3
SS loadings        2.53    1.81    1.52
Proportion Var     0.28    0.20    0.17
Cumulative Var     0.28    0.48    0.65
 With factor correlations of
         Factor1 Factor2 Factor3
Factor1    1.00    0.62    0.61
Factor2    0.62    1.00    0.58
Factor3    0.61    0.58    1.00
```

Oblique rotation (oblimin)

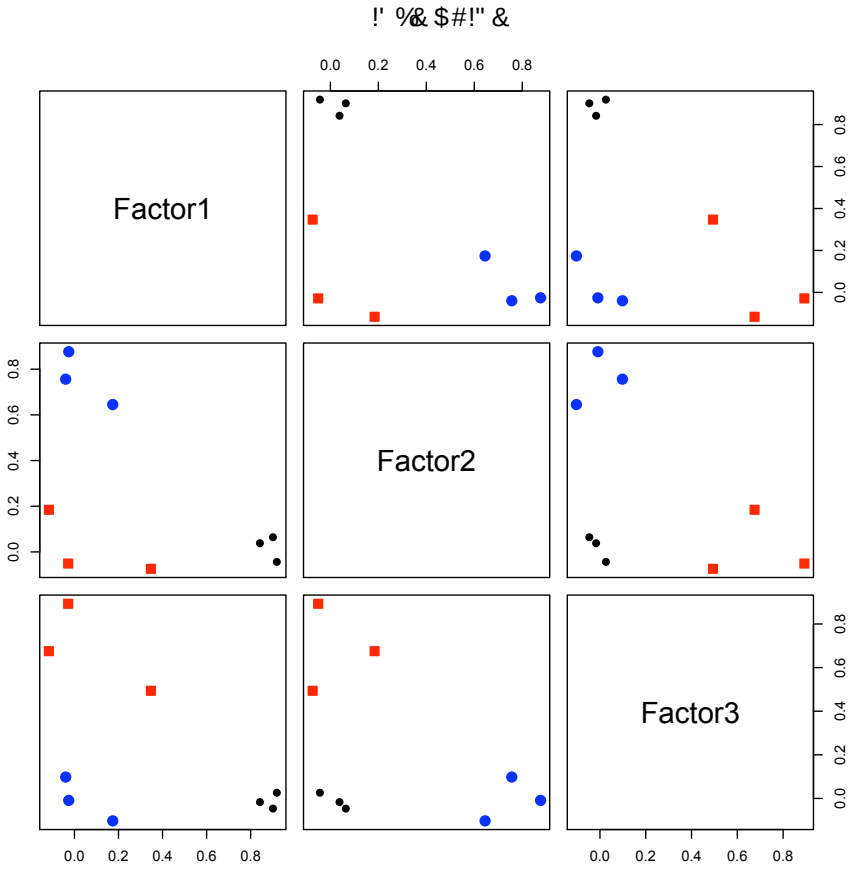
```
> t3o
Oblique rotation method Oblimin Quartimin converged.
Loadings:
                 Factor1 Factor2   Factor3
Sentences         0.90761 -0.0375  0.041132
Vocabulary        0.88807  0.0648 -0.025735
Sent.Completion   0.83096  0.0403  0.000702
First.Letters    -0.00397  0.8552  0.005231
4.Letter.Words   -0.01342  0.7420  0.103793
Suffixes          0.18084  0.6265 -0.082924
Letter.Series     0.03201 -0.0149  0.837698
Pedigrees         0.37463 -0.0514  0.469343
Letter.Group     -0.06352  0.2059  0.635489

Rotating matrix:
       [,1]   [,2]   [,3]
[1,]  1.255 -0.344 -0.248
[2,] -0.290  1.236 -0.266
[3,] -0.261 -0.196  1.186

Phi:
      [,1]  [,2]  [,3]
[1,] 1.000 0.591 0.536
[2,] 0.591 1.000 0.518
[3,] 0.536 0.518 1.000
```

Orthogonal vs. Oblique loadings

> cluster.plot(t3)



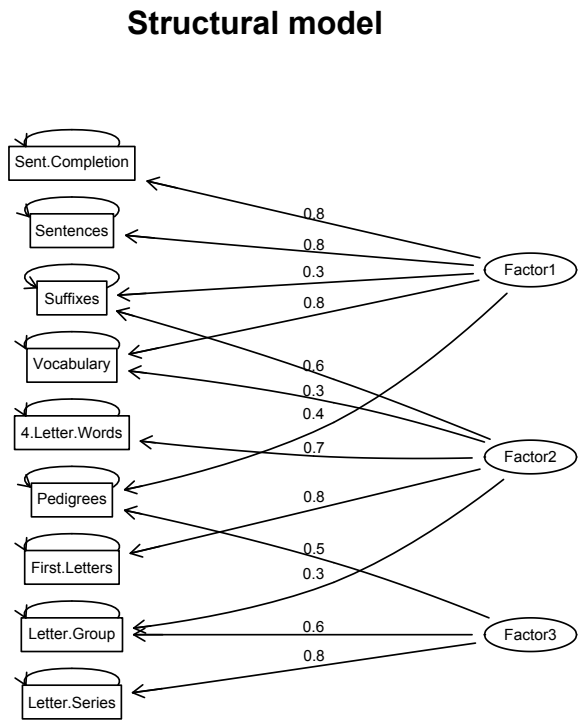
> cluster.plot(t3p)

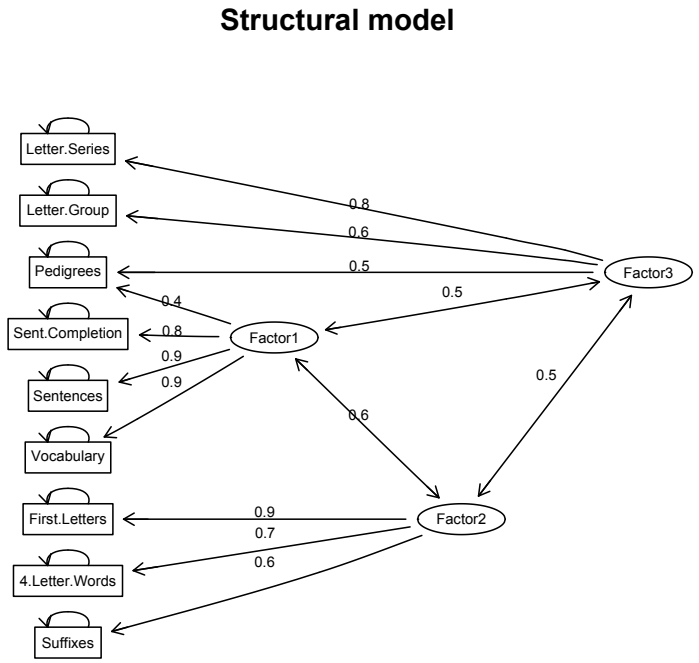


Orthogonal vs. Oblique

s3 <- structure.graph(t3)

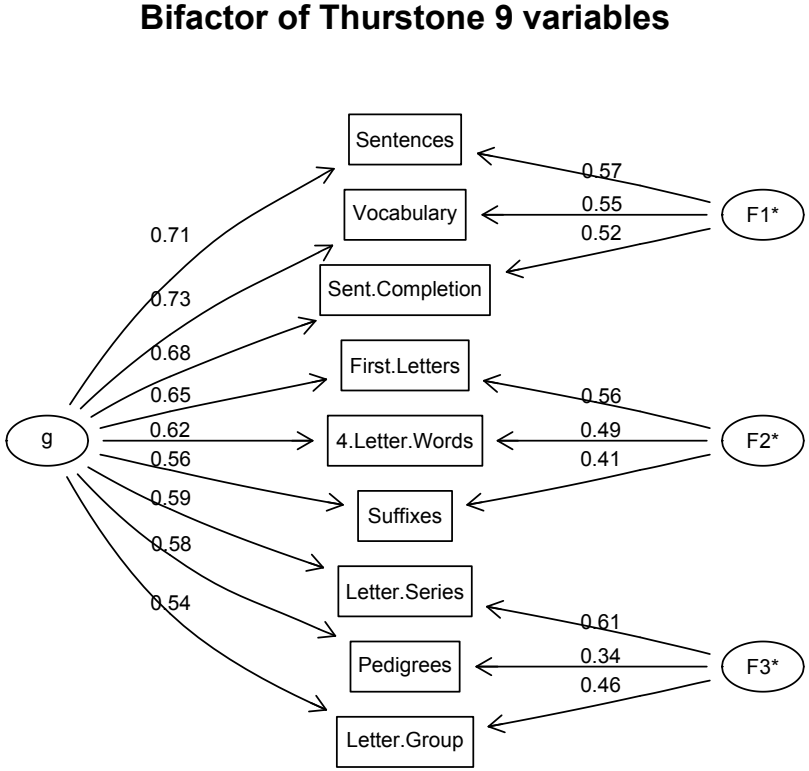
s3o <- structure.graph(t3o)



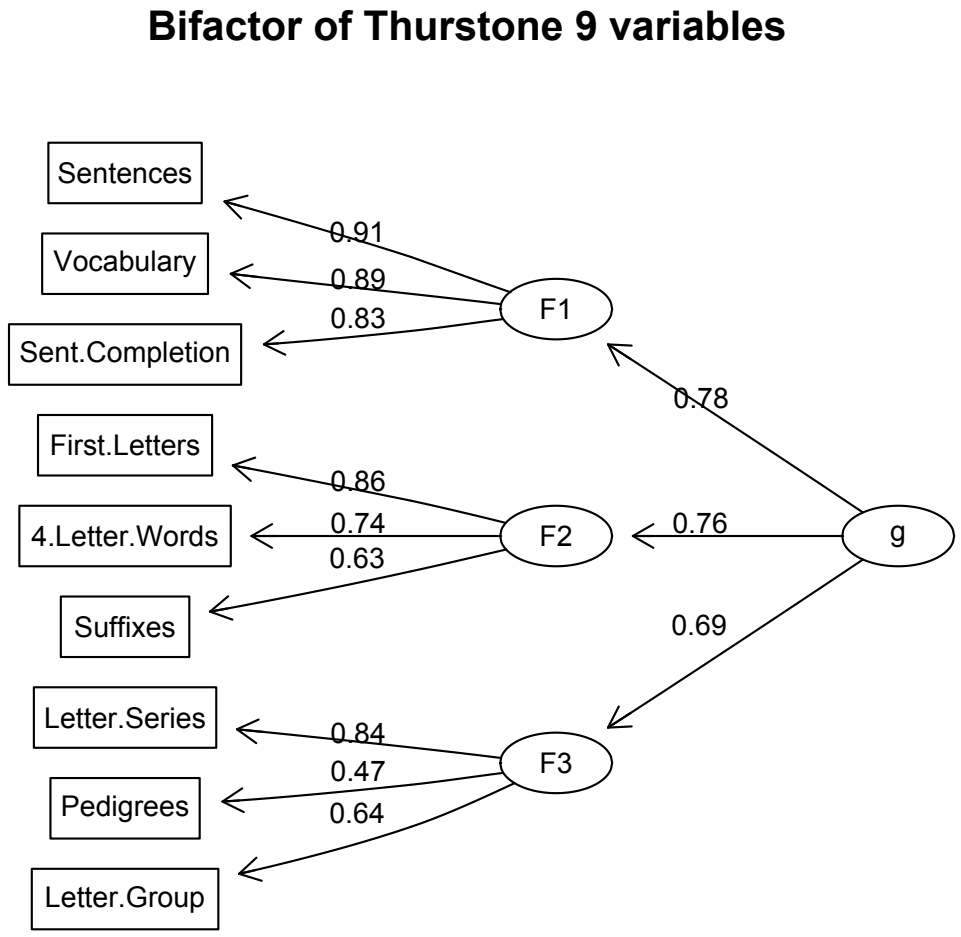


exploratory bifactor model

```
omt <- omega(Thurstone,digits=3,title="Bifactor of Thurstone
        9 variables",labels=rownames(Thurstone))
```



```
omt.hi <- omega(Thurstone,digits=3,sl=FALSE,title="Bifactor
   of Thurstone 9 variables",labels=rownames(Thurstone))
```



Omega of Thurstone

```
> omt
Bifactor of Thurstone 9 variables
Alpha:  0.8914575
Lambda.6:
Omega Hierarchical:   0.7363219
Omega Total   0.930733
Schmid Leiman Factor loadings greater than  0.2
   g     F1*   F2*   F3*   h2    u2
V1 0.710 0.566              0.825
V2 0.726 0.554              0.835
V3 0.681 0.518              0.732 0.268
V4 0.647       0.560        0.732 0.268
V5 0.621       0.486        0.628 0.372
V6 0.558       0.410        0.496 0.504
V7 0.589              0.609 0.718 0.282
V8 0.576 0.234        0.341 0.504 0.496
V9 0.542              0.462 0.527 0.473
With eigenvalues of:
   g  F1*  F2*  F3*
3.58 0.97 0.74 0.71
general/max  3.71   max/min =   1.36
The degrees of freedom for the model is 12  and the fit was
0.01
```

	Path	Parameter	Initial Value
[1,]	"g->Sentences"	"Sentences"	NA
[2,]	"g->Vocabulary"	"Vocabulary"	NA
[3,]	"g->Sent.Completion"	"Sent.Completion"	NA
[4,]	"g->First.Letters"	"First.Letters"	NA
[5,]	"g->4.Letter.Words"	"4.Letter.Words"	NA
[6,]	"g->Suffixes"	"Suffixes"	NA
[7,]	"g->Letter.Series"	"Letter.Series"	NA
[8,]	"g->Pedigrees"	"Pedigrees"	NA
[9,]	"g->Letter.Group"	"Letter.Group"	NA
[10,]	"F1*->Sentences"	"F1*Sentences"	NA
[11,]	"F1*->Vocabulary"	"F1*Vocabulary"	NA
[12,]	"F1*->Sent.Completion"	"F1*Sent.Completion"	NA
[13,]	"F2*->First.Letters"	"F2*First.Letters"	NA
[14,]	"F2*->4.Letter.Words"	"F2*4.Letter.Words"	NA
[15,]	"F2*->Suffixes"	"F2*Suffixes"	NA
[16,]	"F3*->Letter.Series"	"F3*Letter.Series"	NA
[17,]	"F3*->Pedigrees"	"F3*Pedigrees"	NA
[18,]	"F3*->Letter.Group"	"F3*Letter.Group"	NA
[19,]	"Sentences<->Sentences"	"e1"	NA
[20,]	"Vocabulary<->Vocabulary"	"e2"	NA
[21,]	"Sent.Completion<->Sent.Completion"	"e3"	NA
[22,]	"First.Letters<->First.Letters"	"e4"	NA
[23,]	"4.Letter.Words<->4.Letter.Words"	"e5"	NA
[24,]	"Suffixes<->Suffixes"	"e6"	NA
[25,]	"Letter.Series<->Letter.Series"	"e7"	NA
[26,]	"Pedigrees<->Pedigrees"	"e8"	NA
[27,]	"Letter.Group<->Letter.Group"	"e9"	NA
[28,]	"F1*<->F1*"	NA	"1"
[29,]	"F2*<->F2*"	NA	"1"
[30,]	"F3*<->F3*"	NA	"1"
[31,]	"g <->g"	NA	"1"

sem of Thurstone bifactor

sem output

```
> summary(sem.t.sl)

 Model Chisquare =  24.216   Df =  18 Pr(>Chisq) = 0.14807
 Chisquare (null model) =  1101.9   Df =  36
 Goodness-of-fit index =  0.97578
 Adjusted goodness-of-fit index =  0.93944
 RMSEA index =  0.040361   90% CI: (NA, 0.077994)
 Bentler-Bonnett NFI =  0.97802
 Tucker-Lewis NNFI =  0.98834
 Bentler CFI =  0.99417
 SRMR =  0.034895
 BIC =  -72.287

 Normalized Residuals
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-8.21e-01 -3.34e-01 -8.92e-07  2.82e-02  1.56e-01  1.80e+00
```

Parameter Estimates

	Estimate	Std Error	z value	Pr(>|z|)
Sentences	0.76787	0.072626	10.57292	0.0000e+00
Vocabulary	0.79092	0.072418	10.92170	0.0000e+00
Sent.Completion	0.75362	0.073402	10.26709	0.0000e+00
First.Letters	0.60838	0.072201	8.42617	0.0000e+00
4.Letter.Words	0.59733	0.073851	8.08843	6.6613e-16
Suffixes	0.57179	0.071492	7.99792	1.3323e-15
Letter.Series	0.56689	0.074271	7.63281	2.2871e-14
Pedigrees	0.66233	0.069321	9.55455	0.0000e+00
Letter.Group	0.52995	0.078985	6.70955	1.9522e-11
F1*Sentences	0.48787	0.085457	5.70898	1.1366e-08
F1*Vocabulary	0.45232	0.090422	5.00233	5.6640e-07
F1*Sent.Completion	0.40445	0.093402	4.33024	1.4895e-05
F2*First.Letters	0.61405	0.085794	7.15733	8.2268e-13
F2*4.Letter.Words	0.50581	0.084848	5.96130	2.5024e-09
F2*Suffixes	0.39432	0.078289	5.03671	4.7360e-07
F3*Letter.Series	0.72730	0.159499	4.55988	5.1183e-06
F3*Pedigrees	0.24684	0.089011	2.77317	5.5513e-03
F3*Letter.Group	0.40915	0.122180	3.34875	8.1177e-04
e1	0.17236	0.034113	5.05265	4.3571e-07
e2	0.16984	0.030037	5.65438	1.5641e-08
e3	0.26847	0.033188	8.08958	6.6613e-16

standardized coefficients

Sentences	Sentences	0.76787	Sentences <--- g
Vocabulary	Vocabulary	0.79092	Vocabulary <--- g
Sent.Completion	Sent.Completion	0.75362	Sent.Completion <--
First.Letters	First.Letters	0.60838	First.Letters <---
4.Letter.Words	4.Letter.Words	0.59733	4.Letter.Words <---
Suffixes	Suffixes	0.57179	Suffixes <--- g
Letter.Series	Letter.Series	0.56690	Letter.Series <---
Pedigrees	Pedigrees	0.66233	Pedigrees <--- g
Letter.Group	Letter.Group	0.52995	Letter.Group <--- g
F1*Sentences	F1*Sentences	0.48787	Sentences <--- F1*
F1*Vocabulary	F1*Vocabulary	0.45232	Vocabulary <--- F1*
F1*Sent.Completion	F1*Sent.Completion	0.40445	Sent.Completion <--
F2*First.Letters	F2*First.Letters	0.61405	First.Letters <---
F2*4.Letter.Words	F2*4.Letter.Words	0.50581	4.Letter.Words <---
F2*Suffixes	F2*Suffixes	0.39432	Suffixes <--- F2*
F3*Letter.Series	F3*Letter.Series	0.72730	Letter.Series <---
F3*Pedigrees	F3*Pedigrees	0.24684	Pedigrees <--- F3*
F3*Letter.Group	F3*Letter.Group	0.40915	Letter.Group <--- F

Alternative model

	Path	Parameter	Initial Value
[1,]	"g->F1"	"gF1"	NA
[2,]	"g->F2"	"gF2"	NA
[3,]	"g->F3"	"gF3"	NA
[4,]	"F1->Sentences"	"F1Sentences"	NA
[5,]	"F1->Vocabulary"	"F1Vocabulary"	NA
[6,]	"F1->Sent.Completion"	"F1Sent.Completion"	NA
[7,]	"F2->First.Letters"	"F2First.Letters"	NA
[8,]	"F2->4.Letter.Words"	"F24.Letter.Words"	NA
[9,]	"F2->Suffixes"	"F2Suffixes"	NA
[10,]	"F3->Letter.Series"	"F3Letter.Series"	NA
[11,]	"F3->Pedigrees"	"F3Pedigrees"	NA
[12,]	"F3->Letter.Group"	"F3Letter.Group"	NA
[13,]	"Sentences<->Sentences"	"e1"	NA
[14,]	"Vocabulary<->Vocabulary"	"e2"	NA
[15,]	"Sent.Completion<->Sent.Completion"	"e3"	NA
[16,]	"First.Letters<->First.Letters"	"e4"	NA
[17,]	"4.Letter.Words<->4.Letter.Words"	"e5"	NA
[18,]	"Suffixes<->Suffixes"	"e6"	NA
[19,]	"Letter.Series<->Letter.Series"	"e7"	NA
[20,]	"Pedigrees<->Pedigrees"	"e8"	NA
[21,]	"Letter.Group<->Letter.Group"	"e9"	NA
[22,]	"F1<->F1"	NA	"1"
[23,]	"F2<->F2"	NA	"1"
[24,]	"F3<->F3"	NA	"1"
[25,]	"g<->g"	NA	"1"

sem hierarchical model

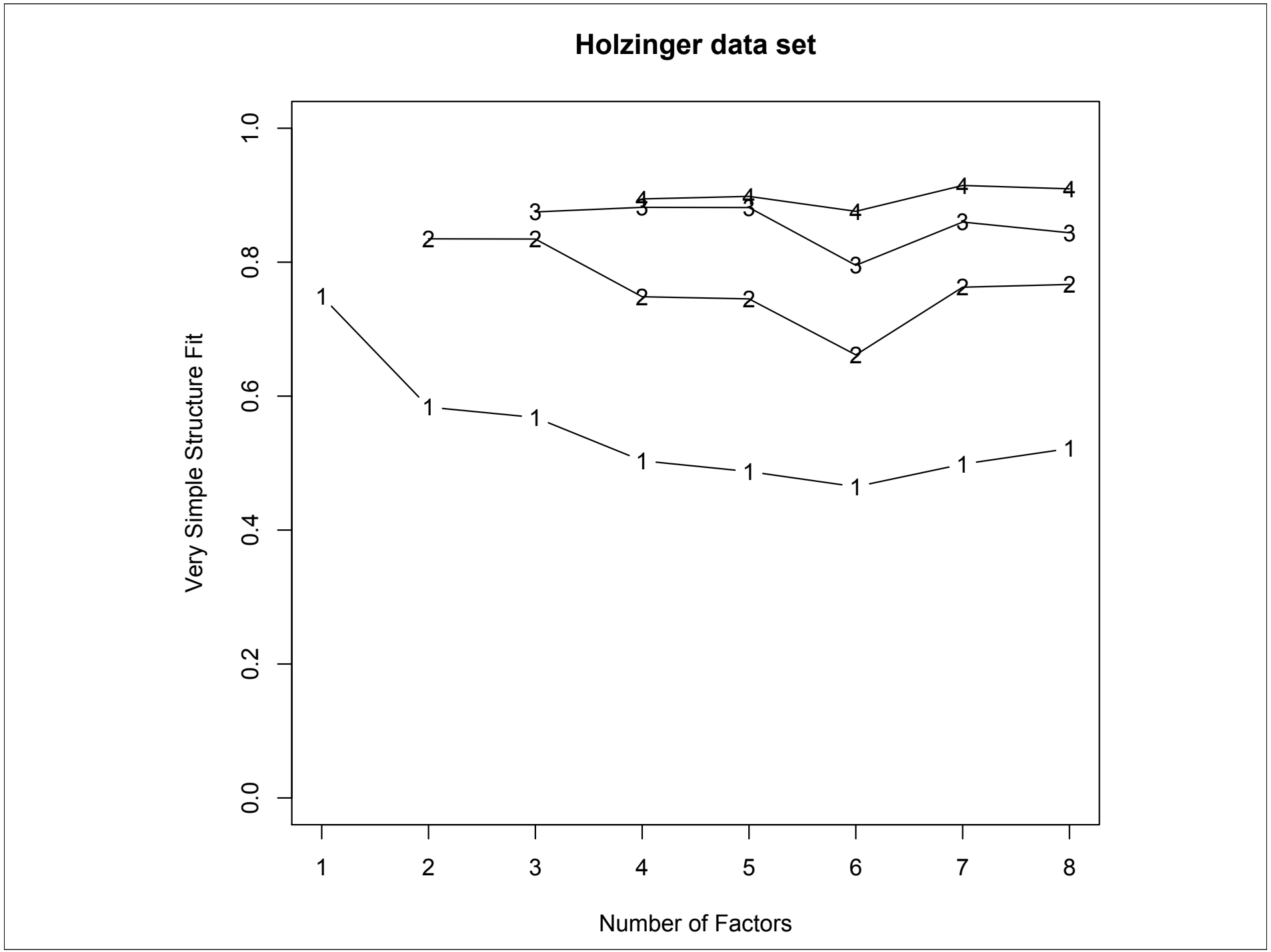
```
> sem.t.hi <- sem(omt.hi$model,Thurstone,213)
> summary(sem.t.hi)

 Model Chisquare =  38.196   Df =  24 Pr(>Chisq) = 0.033101
 Chisquare (null model) =  1101.9   Df =  36
 Goodness-of-fit index =  0.95957
 Adjusted goodness-of-fit index =  0.9242
 RMSEA index =  0.052822   90% CI: (0.015262, 0.083067)
 Bentler-Bonnett NFI =  0.96534
 Tucker-Lewis NNFI =  0.98002
 Bentler CFI =  0.98668
 SRMR =  0.043595
 BIC =  -90.475
```

Standardized coefficients

```
> std.coef(sem.t.hi)
                                    Std. Estimate
gF1                 gF1             0.82208       F1 <--- g
gF2                 gF2             0.78180       F2 <--- g
gF3                 gF3             0.81501       F3 <--- g
F1Sentences         F1Sentences     0.90471       Sentences <--- F1
F1Vocabulary        F1Vocabulary    0.91382       Vocabulary <--- F1
F1Sent.Completion   F1Sent.Completion 0.85608     Sent.Completion <---
F2First.Letters     F2First.Letters 0.83576       First.Letters <--- F
F24.Letter.Words    F24.Letter.Words 0.79718      4.Letter.Words <---
F2Suffixes          F2Suffixes      0.70256       Suffixes <--- F2
F3Letter.Series     F3Letter.Series 0.78081       Letter.Series <--- F
F3Pedigrees         F3Pedigrees     0.72016       Pedigrees <--- F3
F3Letter.Group      F3Letter.Group  0.70349       Letter.Group <--- F3
```

Holzinger 14 cognitive

	T1	T2	T3.4	T6	T28	T29	T32	T34	T35	T36a	T13	T18	T25b	T77
T1	1.000	0.514	0.477	0.433	0.424	0.350	0.083	0.239	0.140	0.286	0.305	0.260	0.231	0.250
T2	0.514	1.000	0.662	0.497	0.397	0.427	0.152	0.254	0.083	0.368	0.545	0.526	0.437	0.426
T3.4	0.477	0.662	1.000	0.415	0.319	0.376	0.173	0.172	0.137	0.229	0.482	0.373	0.424	0.368
T6	0.433	0.497	0.415	1.000	0.444	0.530	0.064	0.371	0.214	0.394	0.354	0.348	0.310	0.279
T28	0.424	0.397	0.319	0.444	1.000	0.437	0.027	0.211	0.139	0.267	0.262	0.193	0.160	0.189
T29	0.350	0.427	0.376	0.530	0.437	1.000	0.018	0.224	0.066	0.340	0.349	0.368	0.245	0.273
T32	0.083	0.152	0.173	0.064	0.027	0.018	1.000	0.264	0.203	0.191	0.166	0.115	0.129	0.133
T34	0.239	0.254	0.172	0.371	0.211	0.224	0.264	1.000	0.334	0.442	0.202	0.159	0.053	0.039
T35	0.140	0.083	0.137	0.214	0.139	0.066	0.203	0.334	1.000	0.234	0.007	-0.014	-0.030	-0.037
T36a	0.286	0.368	0.229	0.394	0.267	0.340	0.191	0.442	0.234	1.000	0.360	0.372	0.235	0.241
T13	0.305	0.545	0.482	0.354	0.262	0.349	0.166	0.202	0.007	0.360	1.000	0.677	0.603	0.586
T18	0.260	0.526	0.373	0.348	0.193	0.368	0.115	0.159	-0.014	0.372	0.677	1.000	0.596	0.613
T25b	0.231	0.437	0.424	0.310	0.160	0.245	0.129	0.053	-0.030	0.235	0.603	0.596	1.000	0.559
T77	0.250	0.426	0.368	0.279	0.189	0.273	0.133	0.039	-0.037	0.241	0.586	0.613	0.559	1.000

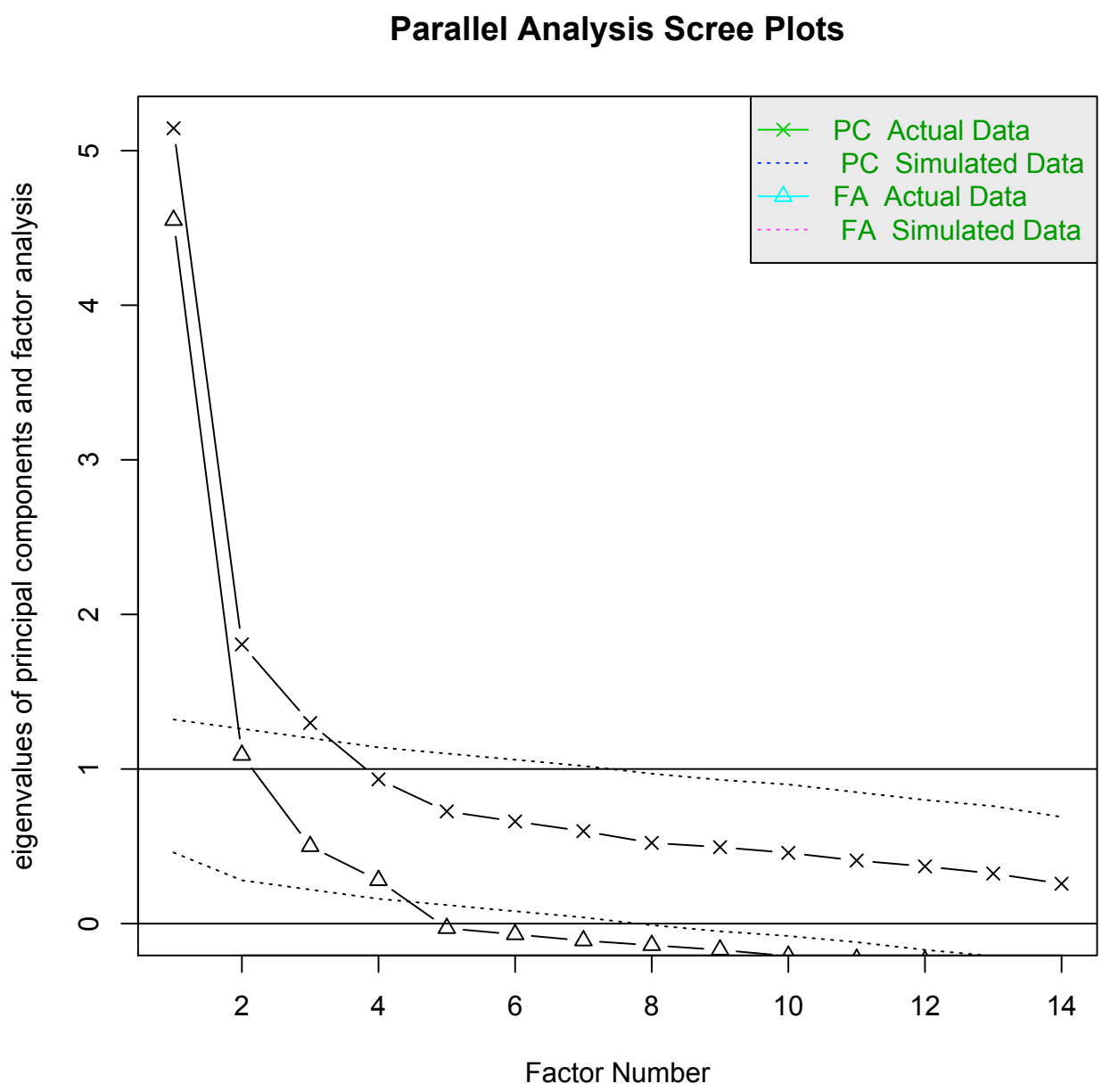

Holzinger data set
Very Simple Structure Fit
0.0
0.2
0.4
0.6
0.8
1.0
1
2
3
4
5
6
7
8
Number of Factors

```
> vss.h <- VSS(Holzinger,n.obs=355,title="Holzinger data
set")
> vss.h

Very Simple Structure of  Holzinger data set
VSS complexity 1 achieves a maximimum of 0.75  with  1
factors
VSS complexity 2 achieves a maximimum of 0.83  with  2
factors

The Velicer MAP criterion achieves a minimum of 0.14  with
2  factors
Velicer MAP
[1] 0.03 0.03 0.03 0.04 0.06 0.07 0.10 0.14

Very Simple Structure Complexity 1
[1] 0.75 0.58 0.57 0.50 0.49 0.46 0.50 0.52
Very Simple Structure Complexity 2
[1] 0.00 0.83 0.83 0.75 0.75 0.66 0.76 0.77
```

Chi square

```
> print(vss.h$vss.stats[,1:4],digits=3)
      dof chisq     prob sqresid
1  77 547.4 4.60e-72    8.72
2  64 238.2 6.87e-22    5.75
3  52 124.5 7.01e-08    4.35
4  41  42.8 3.96e-01    3.67
5  31  28.1 6.14e-01    3.43
6  22  16.0 8.17e-01    3.08
7  14  19.3 1.52e-01    1.92
8   7  17.7 1.33e-02    1.47
```

Parallel analysis

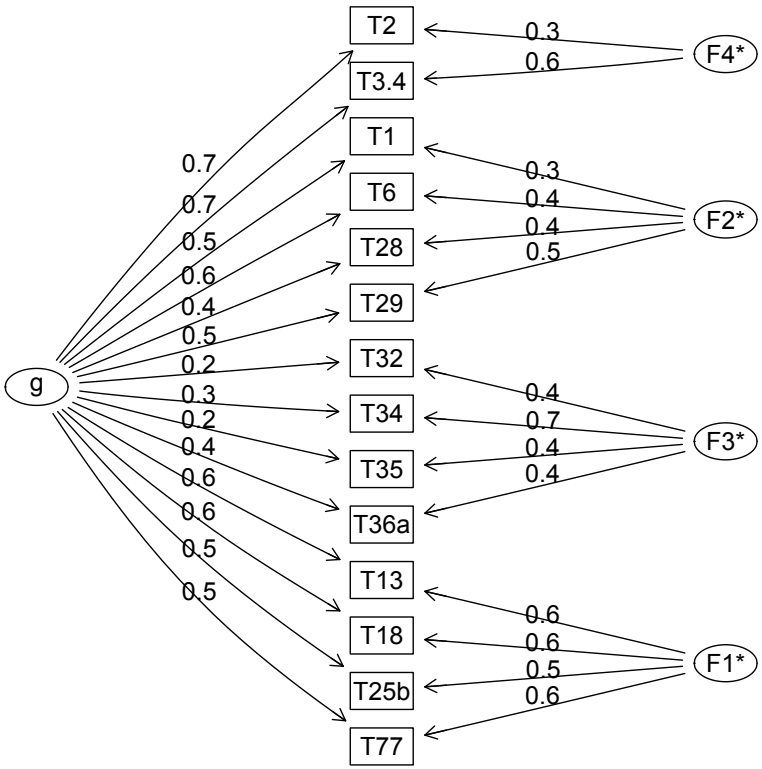


Omega analysis of Holzinger 14 variables

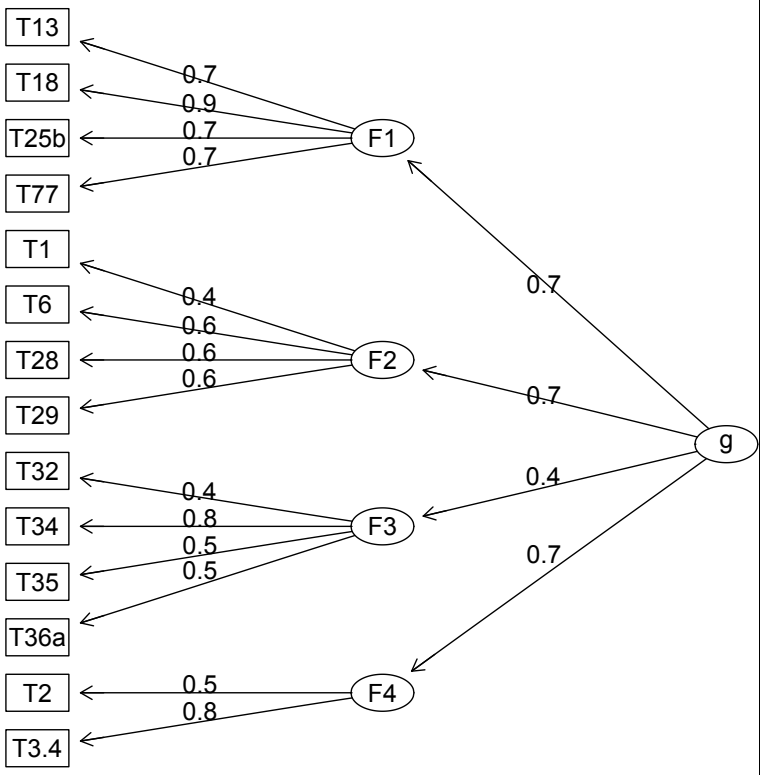
```
> om.holz.bi <- omega(Holzinger,4,title="Holzinger 14 cognitive variables")
```

```
om.holz.hi <- omega(Holzinger,4,sl=FALSE, title="Holzinger 14 cognitive variables")
```

Holzinger 14 cognitive variables



Holzinger 14 cognitive variables



Holzinger bifactor model

```
> om.holz.bi$model
      Path           Parameter Initial Value
 [1,] "g->T1"        "T1"      NA
 [2,] "g->T2"        "T2"      NA
 [3,] "g->T3.4"      "T3.4"    NA
 [4,] "g->T6"        "T6"      NA
 [5,] "g->T28"       "T28"     NA
 [6,] "g->T29"       "T29"     NA
 [7,] "g->T32"       "T32"     NA
 [8,] "g->T34"       "T34"     NA
 [9,] "g->T35"       "T35"     NA
[10,] "g->T36a"      "T36a"    NA
[11,] "g->T13"       "T13"     NA
[12,] "g->T18"       "T18"     NA
[13,] "g->T25b"      "T25b"    NA
[14,] "g->T77"       "T77"     NA
[15,] "F2*->T1"      "F2*T1"   NA
[16,] "F4*->T2"      "F4*T2"   NA
[17,] "F4*->T3.4"    "F4*T3.4" NA
[18,] "F2*->T6"      "F2*T6"   NA
[19,] "F2*->T28"     "F2*T28"  NA
[20,] "F2*->T29"     "F2*T29"  NA
[21,] "F3*->T32"     "F3*T32"  NA
[22,] "F3*->T34"     "F3*T34"  NA
[23,] "F3*->T35"     "F3*T35"  NA
[24,] "F3*->T36a"    "F3*T36a" NA
[25,] "F1*->T13"     "F1*T13"  NA
[26,] "F1*->T18"     "F1*T18"  NA
[27,] "F1*->T25b"    "F1*T25b" NA
[28,] "F1*->T77"     "F1*T77"  NA
[29,] "T1<->T1"      "e1"      NA
[30,] "T2<->T2"      "e2"      NA
[31,] "T3.4<->T3.4"  "e3"      NA
[32,] "T6<->T6"      "e4"      NA
[33,] "T28<->T28"    "e5"      NA
[34,] "T29<->T29"    "e6"      NA
[35,] "T32<->T32"    "e7"      NA
[36,] "T34<->T34"    "e8"      NA
[37,] "T35<->T35"    "e9"      NA
[38,] "T36a<->T36a"  "e10"     NA
[39,] "T13<->T13"    "e11"     NA
[40,] "T18<->T18"    "e12"     NA
[41,] "T25b<->T25b"  "e13"     NA
[42,] "T77<->T77"    "e14"     NA
[43,] "F1*<->F1*"    NA        "1"
[44,] "F2*<->F2*"    NA        "1"
[45,] "F3*<->F3*"    NA        "1"
[46,] "F4*<->F4*"    NA        "1"
[47,] "g <->g"       NA        "1"
```

sem analysis: bifactor

```
> summary(sem.holz.bi)

 Model Chisquare =  147.18   Df =  63 Pr(>Chisq) =
1.0880e-08
 Chisquare (null model) =  1889.1   Df =  91
 Goodness-of-fit index =  0.9435
 Adjusted goodness-of-fit index =  0.90582
 RMSEA index =  0.061437   90% CI: (0.04856, 0.07441)
 Bentler-Bonnett NFI =  0.9221
 Tucker-Lewis NNFI =  0.93238
 Bentler CFI =  0.95319
 SRMR =  0.05007
 BIC =  -222.76
```

Parameter Estimates

	Estimate	Std Error	z value	Pr(>|z|)	
T1	0.590094	0.053919	10.944016	0.0000e+00	T1 <--- g
T2	0.835497	0.050235	16.631777	0.0000e+00	T2 <--- g
T3.4	0.693958	0.055586	12.484463	0.0000e+00	T3.4 <--- g
T6	0.610566	0.054343	11.235376	0.0000e+00	T6 <--- g
T28	0.464242	0.056586	8.204160	2.2204e-16	T28 <--- g
T29	0.535398	0.055572	9.634298	0.0000e+00	T29 <--- g
T32	0.178126	0.057931	3.074797	2.1065e-03	T32 <--- g
T34	0.334306	0.058047	5.759248	8.4489e-09	T34 <--- g
T35	0.131645	0.058615	2.245944	2.4708e-02	T35 <--- g
T36a	0.482660	0.057024	8.464141	0.0000e+00	T36a <--- g
T13	0.642623	0.052203	12.309980	0.0000e+00	T13 <--- g
T18	0.595635	0.053720	11.087844	0.0000e+00	T18 <--- g
T25b	0.506603	0.054958	9.217943	0.0000e+00	T25b <--- g
T77	0.489791	0.055092	8.890400	0.0000e+00	T77 <--- g
F2*T1	0.190911	0.075060	2.543438	1.0977e-02	T1 <--- F2*
F4*T2	0.106426	0.243799	0.436530	6.6245e-01	T2 <--- F4*
F4*T3.4	0.772350	1.718960	0.449312	6.5321e-01	T3.4 <--- F4*
F2*T6	0.415974	0.073810	5.635740	1.7431e-08	T6 <--- F2*
F2*T28	0.433821	0.085226	5.090248	3.5759e-07	T28 <--- F2*
F2*T29	0.441482	0.077094	5.726550	1.0249e-08	T29 <--- F2*
F3*T32	0.312653	0.068937	4.535327	5.7514e-06	T32 <--- F3*
F3*T34	0.671950	0.082360	8.158736	4.4409e-16	T34 <--- F3*
F3*T35	0.440678	0.072623	6.067981	1.2953e-09	T35 <--- F3*
F3*T36a	0.405607	0.063429	6.394624	1.6094e-10	T36a <--- F3*
F1*T13	0.507630	0.049204	10.316873	0.0000e+00	T13 <--- F1*
F1*T18	0.568381	0.050748	11.200168	0.0000e+00	T18 <--- F1*
F1*T25b	0.537670	0.053866	9.981622	0.0000e+00	T25b <--- F1*
F1*T77	0.558644	0.053751	10.393121	0.0000e+00	T77 <--- F1*

Parameter estimates

Holzinger bifactor continued

```
e1        0.615336 0.051597  11.925721 0.0000e+00 T1 <--> T1
e2        0.290615 0.065375   4.445323 8.7760e-06 T2 <--> T2
e3       -0.078103 2.655986  -0.029406 9.7654e-01 T3.4 <--> T3.4
e4        0.454171 0.054651   8.310325 0.0000e+00 T6 <--> T6
e5        0.596277 0.069623   8.564393 0.0000e+00 T28 <--> T28
e6        0.518439 0.062206   8.334234 0.0000e+00 T29 <--> T29
e7        0.870518 0.070999  12.260925 0.0000e+00 T32 <--> T32
e8        0.436721 0.096993   4.502592 6.7130e-06 T34 <--> T34
e9        0.788469 0.074329  10.607775 0.0000e+00 T35 <--> T35
e10       0.602520 0.058251  10.343536 0.0000e+00 T36a <--> T36a
e11       0.329344 0.034051   9.672048 0.0000e+00 T13 <--> T13
e12       0.322158 0.036679   8.783066 0.0000e+00 T18 <--> T18
e13       0.454260 0.043656  10.405461 0.0000e+00 T25b <--> T25b
e14       0.448018 0.044288  10.115984 0.0000e+00 T77 <--> T77

 Iterations =  43
```

Holzinger hierarchical

```
> om.holz.hi$model
      Path          Parameter Initial Value
 [1,] "g->F1"       "gF1"     NA
 [2,] "g->F2"       "gF2"     NA
 [3,] "g->F3"       "gF3"     NA
 [4,] "g->F4"       "gF4"     NA
 [5,] "F2->T1"      "F2T1"    NA
 [6,] "F4->T2"      "F4T2"    NA
 [7,] "F4->T3.4"    "F4T3.4"  NA
 [8,] "F2->T6"      "F2T6"    NA
 [9,] "F2->T28"     "F2T28"   NA
[10,] "F2->T29"     "F2T29"   NA
[11,] "F3->T32"     "F3T32"   NA
[12,] "F3->T34"     "F3T34"   NA
[13,] "F3->T35"     "F3T35"   NA
[14,] "F3->T36a"    "F3T36a"  NA
[15,] "F1->T13"     "F1T13"   NA
[16,] "F1->T18"     "F1T18"   NA
[17,] "F1->T25b"    "F1T25b"  NA
[18,] "F1->T77"     "F1T77"   NA
[19,] "T1<->T1"     "e1"      NA
[20,] "T2<->T2"     "e2"      NA
[21,] "T3.4<->T3.4" "e3"      NA
[22,] "T6<->T6"     "e4"      NA
[23,] "T28<->T28"   "e5"      NA
[24,] "T29<->T29"   "e6"      NA
[25,] "T32<->T32"   "e7"      NA
[26,] "T34<->T34"   "e8"      NA
[27,] "T35<->T35"   "e9"      NA
[28,] "T36a<->T36a" "e10"     NA
[29,] "T13<->T13"   "e11"     NA
[30,] "T18<->T18"   "e12"     NA
[31,] "T25b<->T25b" "e13"     NA
[32,] "T77<->T77"   "e14"     NA
[33,] "F1<->F1"     NA        "1"
[34,] "F2<->F2"     NA        "1"
[35,] "F3<->F3"     NA        "1"
[36,] "F4<->F4"     NA        "1"
[37,] "g<->g"       NA        "1"
```

Holzinger hierarchical fit

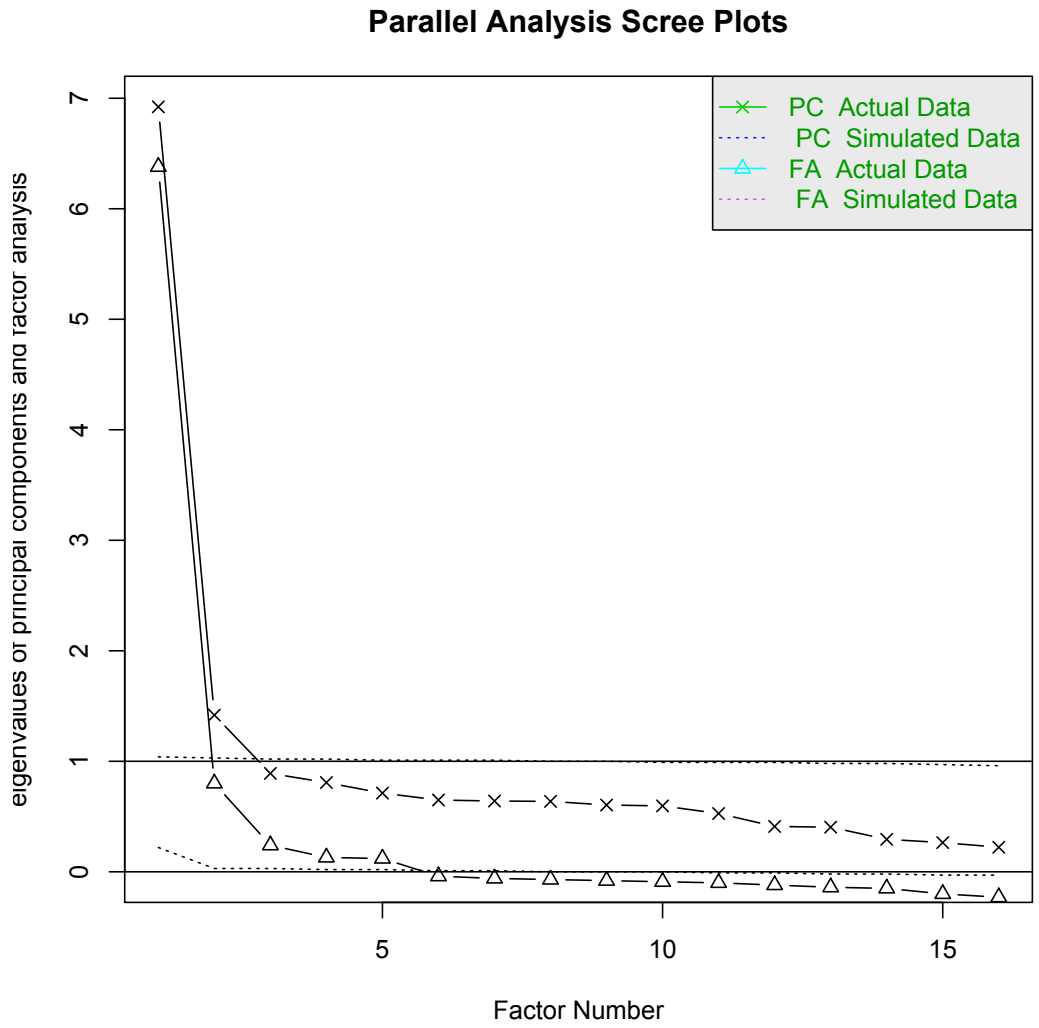
```
> sem.holz.hi <- sem(om.holz.hi$model,Holzinger,355)
> summary(sem.holz.hi)

 Model Chisquare =  183.05   Df =  73 Pr(>Chisq) =
2.0132e-11
 Chisquare (null model) =  1889.1   Df =  91
 Goodness-of-fit index =  0.92993
 Adjusted goodness-of-fit index =  0.89922
 RMSEA index =  0.065257   90% CI: (0.053516, 0.07714)
 Bentler-Bonnett NFI =  0.9031
 Tucker-Lewis NNFI =  0.9237
 Bentler CFI =  0.9388
 SRMR =  0.061793
 BIC =  -245.62

 Normalized Residuals
    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-2.9300 -0.7510 -0.0728 -0.1390  0.5620  2.6600
```

Hierarchical parameters

	Estimate	Std Error	z value	Pr(>|z|)	
gF1	1.04339	0.112796	9.2502	0.0000e+00	F1 <--- g
gF2	1.52786	0.235384	6.4909	8.5313e-11	F2 <--- g
gF3	0.70479	0.113768	6.1950	5.8272e-10	F3 <--- g
gF4	2.72694	0.899891	3.0303	2.4431e-03	F4 <--- g
F2T1	0.34709	0.044292	7.8364	4.6629e-15	T1 <--- F2
F4T2	0.30621	0.092606	3.3067	9.4418e-04	T2 <--- F4
F4T3.4	0.25626	0.074621	3.4342	5.9437e-04	T3.4 <--- F4
F2T6	0.40530	0.050219	8.0706	6.6613e-16	T6 <--- F2
F2T28	0.33208	0.046154	7.1950	6.2461e-13	T28 <--- F2
F2T29	0.36272	0.047445	7.6450	2.0872e-14	T29 <--- F2
F3T32	0.28041	0.054646	5.1314	2.8759e-07	T32 <--- F3
F3T34	0.53501	0.063477	8.4284	0.0000e+00	T34 <--- F3
F3T35	0.32056	0.056846	5.6391	1.7097e-08	T35 <--- F3
F3T36a	0.56915	0.053207	10.6971	0.0000e+00	T36a <--- F3
F1T13	0.57368	0.039200	14.6349	0.0000e+00	T13 <--- F1
F1T18	0.56886	0.040114	14.1811	0.0000e+00	T18 <--- F1
F1T25b	0.50546	0.039892	12.6707	0.0000e+00	T25b <--- F1
F1T77	0.50291	0.040077	12.5485	0.0000e+00	T77 <--- F1

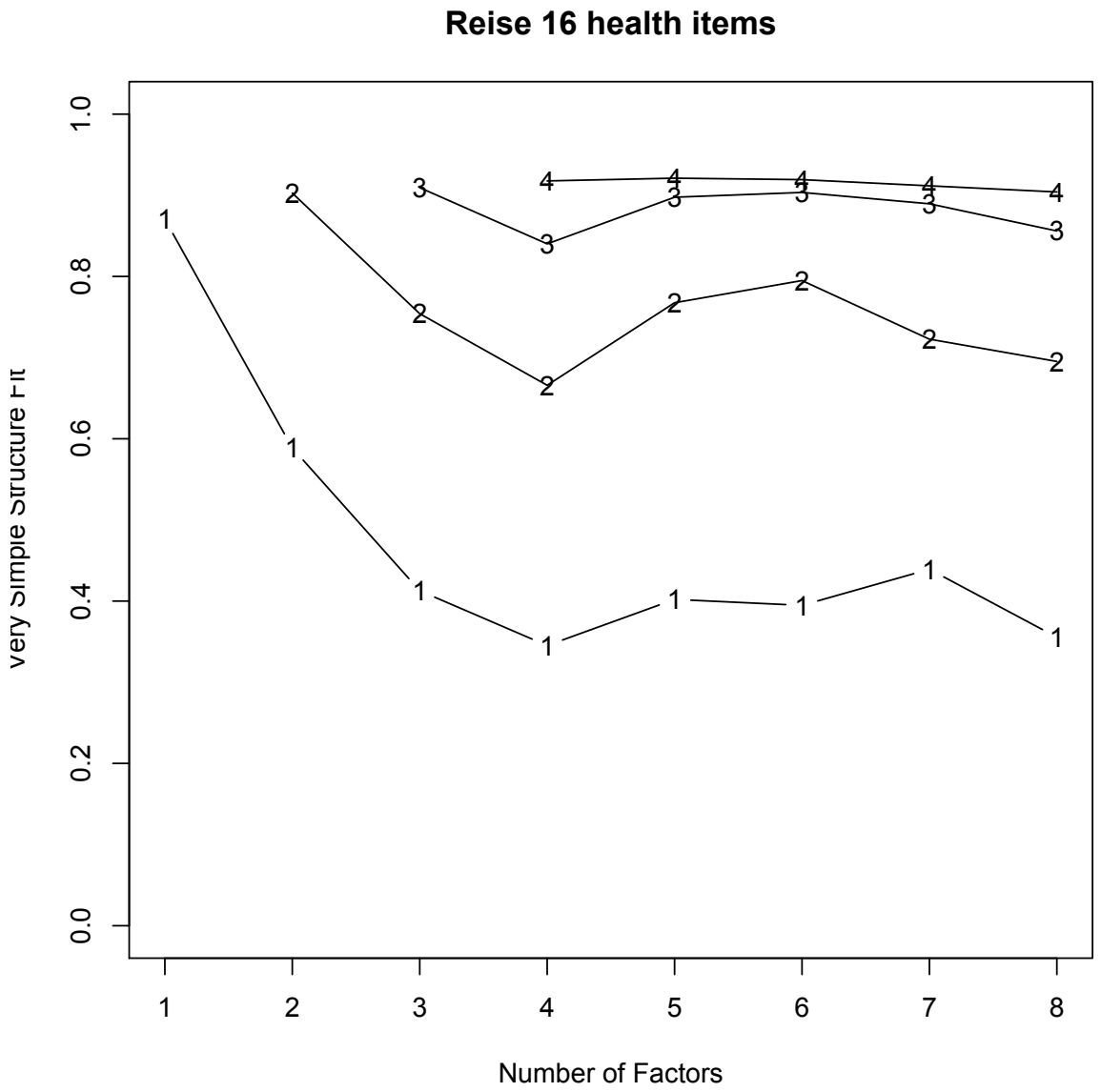
Reise 16 medical variables

	phone	routine	illness	listen	explain	respect	time	courtesy	helpful	happy	referral	necessary	delay	problem	help
phone	1.00	0.44	0.42	0.50	0.40	0.48	0.48	0.47	0.52	0.38	0.32	0.42	0.36	0.27	0.29
routine	0.44	1.00	0.39	0.43	0.36	0.41	0.42	0.41	0.43	0.30	0.25	0.37	0.28	0.23	0.23
illness	0.42	0.39	1.00	0.44	0.34	0.42	0.41	0.37	0.41	0.34	0.30	0.40	0.33	0.27	0.28
listen	0.50	0.43	0.44	1.00	0.62	0.77	0.71	0.60	0.65	0.41	0.32	0.48	0.38	0.32	0.31
explain	0.40	0.36	0.34	0.62	1.00	0.67	0.60	0.49	0.52	0.32	0.26	0.40	0.30	0.26	0.25
respect	0.48	0.41	0.42	0.77	0.67	1.00	0.72	0.61	0.63	0.41	0.33	0.46	0.36	0.32	0.31
time	0.48	0.42	0.41	0.71	0.60	0.72	1.00	0.56	0.61	0.39	0.30	0.43	0.35	0.30	0.31
courtesy	0.47	0.41	0.37	0.60	0.49	0.61	0.56	1.00	0.73	0.36	0.29	0.41	0.33	0.26	0.25
helpful	0.52	0.43	0.41	0.65	0.52	0.63	0.61	0.73	1.00	0.38	0.32	0.44	0.37	0.28	0.29
happy	0.38	0.30	0.34	0.41	0.32	0.41	0.39	0.36	0.38	1.00	0.34	0.42	0.38	0.31	0.34
referral	0.32	0.25	0.30	0.32	0.26	0.33	0.30	0.29	0.32	0.34	1.00	0.44	0.41	0.26	0.28
necessary	0.42	0.37	0.40	0.48	0.40	0.46	0.43	0.41	0.44	0.42	0.44	1.00	0.57	0.31	0.34
delay	0.36	0.28	0.33	0.38	0.30	0.36	0.35	0.33	0.37	0.38	0.41	0.57	1.00	0.33	0.38
problem	0.27	0.23	0.27	0.32	0.26	0.32	0.30	0.26	0.28	0.31	0.26	0.31	0.33	1.00	0.34
help	0.29	0.23	0.28	0.31	0.25	0.31	0.31	0.25	0.29	0.34	0.28	0.34	0.38	0.34	1.00
paperwork	0.28	0.22	0.27	0.32	0.24	0.31	0.30	0.27	0.29	0.31	0.25	0.29	0.32	0.29	0.33



Reise 16 variables parallel analysis

Reise health items

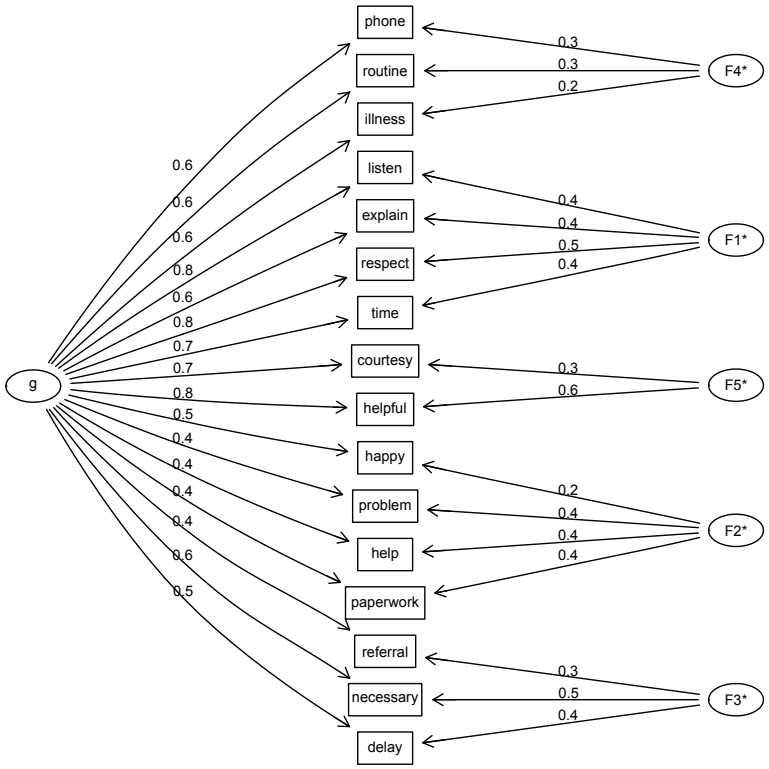


VSS Reise

```
> vss.reise

Very Simple Structure of  Reise 16 health items
VSS complexity 1 achieves a maximimum of 0.87  with  1
factors
VSS complexity 2 achieves a maximimum of 0.9  with  2
factors

The Velicer MAP criterion achieves a minimum of 0.1  with  2
factors
Velicer MAP
[1] 0.02 0.02 0.02 0.03 0.04 0.06 0.09 0.10

Very Simple Structure Complexity 1
[1] 0.87 0.59 0.41 0.35 0.40 0.39 0.44 0.36
Very Simple Structure Complexity 2
[1] 0.00 0.90 0.75 0.67 0.77 0.79 0.72 0.70
```

Omega Reise

```
> om.reise<- omega(Reise,5,n.obs=35000)
```




```
om.reise.hi<- omega(Reise,5,n.obs=35000,sl=FALSE)
```