Psychology 454: Latent Variable Modeling

Notes for a course in Latent Variable Modeling to accompany
Psychometric Theory with Applications in R

William Revelle

Department of Psychology
Northwestern University
Evanston, Illinois USA

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Outline

Texts and readings

- Loehlin, J. C. Latent Variable Models (4th ed). Lawrence Erlbaum Associates, Mahwah, N.J. 2004 (recommended)
- Revelle, W. (in prep) An introduction to Psychometric Theory with Applications in R. Springer. Chapters available at http://personality-project.org/r/book
- Various web based readings about SEM
  - e.g., Barrett (2007), Bollen (2002), McArdle (2009), Widaman & Thompson (2003), Preacher (2015)
- Syllabus and handouts are available at http:personality-project.org/revelle/syllabi/454/454.syllabus.pdf
  - Syllabus is subject to modification as we go through the course.
  - Lecture notes will appear no later than 3 hours before class.
- R tutorial is at http:personality-project.org/r

Requirements and Evaluation

1. Basic knowledge of psychometrics
   - Preferably have taken 405 or equivalent course.
   - Alternatively, willing to read some key chapters and catch up
     - Chapters available at http://personality-project.org/r/book/
     - Basic concepts of measurement and scaling (Chapters 1-3)
     - Correlation and Regression (Chapters 4 & 5)
     - Factor Analysis (Chapter 6)
     - Reliability (Chapter 7)
2. Familiarity with basic linear algebra (Appendix E) (or, at least, a willingness to learn)
3. Willingness to use computer programs, particularly R, comparing alternative solutions, playing with data.
4. Willingness to ask questions
5. Weekly problem sets/final brief paper

Outline of Course

1. Review of correlation/regression/reliability/matrix algebra (405 in a week)
2. Basic model fitting/path analysis
3. Simple models
4. Goodness of fit–what is it all about?
5. Exploratory Factor Analysis
6. Confirmatory Factor Analysis
7. Multiple groups/multiple occasions
8. Further topics

Data = Model + Residual

- The fundamental equations of statistics are that
  - Data = Model + Residual
  - Residual = Data - Model
- The problem is to specify the model and then evaluate the fit of the model to the data as compared to other models
  - Fit = f(Data, Residual)
  - Typically: $Fit \propto 1 - \frac{Residual^2}{Data^2}$
  - $Fit = \frac{(Data - Model)^2}{Data^2}$
- This is a course in developing, evaluating, and comparing models of data.
- This is not a course in how to use any particular program (e.g., MPlus, LISREL, AMOS, or even R) to do latent variable analysis, but rather in how and why to think about latent variables when thinking about data.

Latent Variable Modeling

- Two kinds of variables
    1. Observed Variables (X, Y)
    2. *Latent Variables*(ξ η ϵ ζ)
- Three kinds of variance/covariances
    1. Observed with Observed C_{xy} or σ_{xy}
    2. Observed with Latent λ
    3. Latent with Latent ϕ
- Three kinds of algebra
    1. Path algebra
    2. Linear algebra
    3. Computer syntax
        - R packages e.g., *psych*, *lavaan*, *sem*, and *OpenMx* and associated functions
        - Commercial packages: MPlus (available through SSCC)
        - AMOS, EQS (if licensed)

Latent and Observed variables

- The distinction between what we see (observe) versus what is *really* there goes back at least to Plato in the *Allegory of the Cave*.
  - Prisoners in a cave observe shadows on the walls of the cave.
  - These are caused by people and objects behind them, but in front of a fire.
  - Movements of the shadows are caused by, but not the same as the movements of the people and objects.
- In psychology we sometimes make the distinction between *surface* traits and *source* traits.
- A major breakthrough in psychological theorizing was the willingness to consider latent constructs.
  - Operational definitions are associated with the observed (surface) measures.
  - Unobserved, latent constructs are now part of our theoretical armamentarium.

Observed Variables

X	Y
$\boxed{X_1}$	$\boxed{Y_1}$
$\boxed{X_2}$	$\boxed{Y_2}$
$\boxed{X_3}$	$\boxed{Y_3}$
$\boxed{X_4}$	$\boxed{Y_4}$
$\boxed{X_5}$	$\boxed{Y_5}$
$\boxed{X_6}$	$\boxed{Y_6}$

Latent Variables

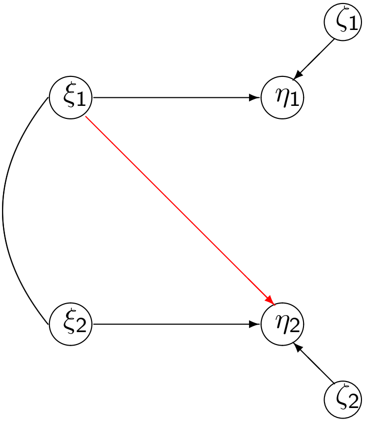
ξ η

ξ_1 η_1

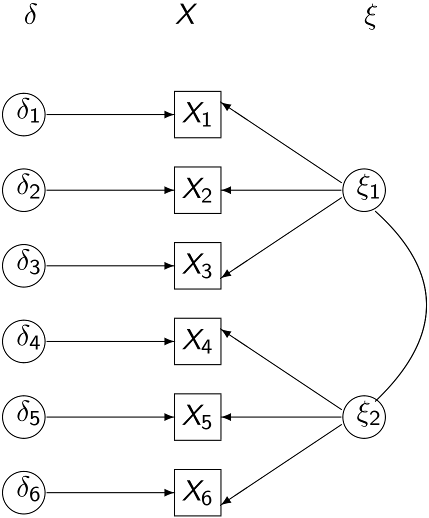
ξ_2 η_2

Theory: A regression model of latent variables

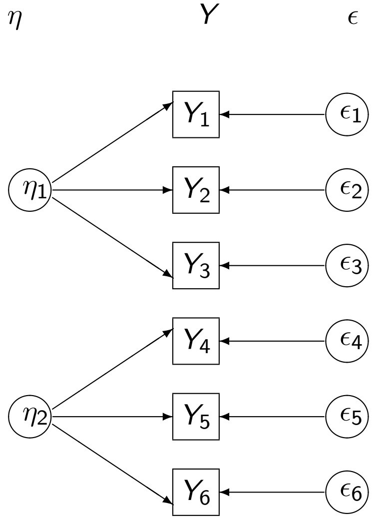
ξ η



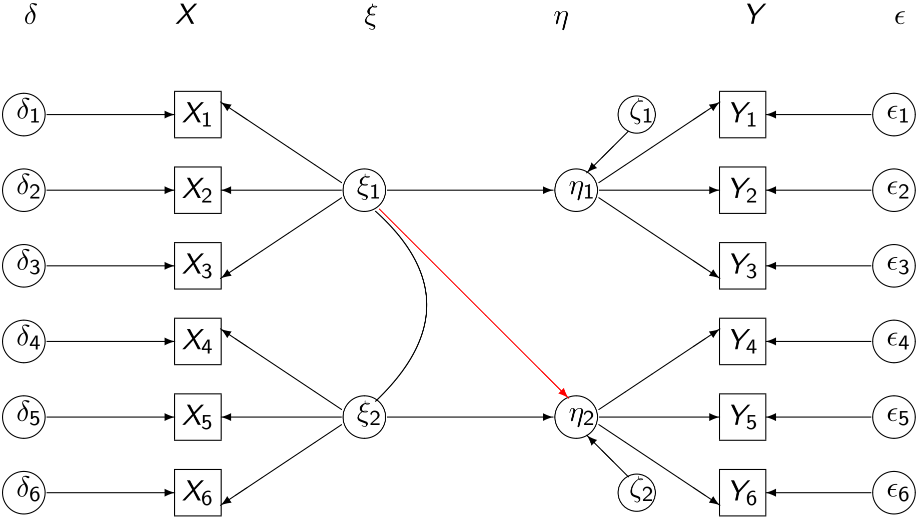
A measurement model for X



A measurement model for Y



A complete structural model



Latent Variable Modeling

1. Requires measuring observed variables
   - Requires defining what is relevant and irrelevant to our theory.
   - Issues in quality of scale information, levels of measurement.
2. Formulating a measurement model of the data: estimating latent constructs
   - Perhaps based upon exploratory and then confirmatory factor analysis, definitely based upon theory.
   - Includes understanding the reliability of the measures.
3. Modeling the structure of the constructs
   - This is a combination of theory and fitting. Do the data fit the theory.
   - Comparison of models. Does one model fit better than alternative models?

Data Analysis: using R to analyze Graduate School Applicants

The data are taken from an Excel file downloaded from the Graduate School. They were then copied into R and saved as a .rds file. First, get the file and download to your machine. Use your browser: http://personality-project.org/revelle/syllabi/454/grad.rds Then, load the file using read.file.

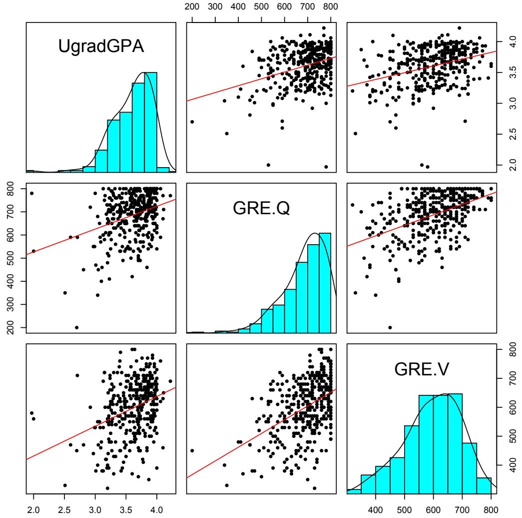
First, examine the data to remove outliers.

```
> library(psych) #necessary first time you use psych package
> grad <- read.file()   #find the grad.rds file using your finder
> describe(grad)
       vars   n   mean     sd median trimmed    mad    min max  range  skew kurtosis   se
V.Gre     1 256 587.23 103.54  590.0  591.41 103.78 300.00 800 500.00 -0.33    -0.43 6.47
Q.Gre     2 256 626.48 113.10  640.0  636.65 103.78 220.00 800 580.00 -1.04     1.49 7.07
A.Gre     3 256 626.84 116.31  650.0  635.44 118.61 310.00 800 490.00 -0.60    -0.37 7.27
P.Gre     4 213 617.79  82.42  640.0  622.46  74.13 350.00 780 430.00 -0.55    -0.12 5.65
X2.GPA    5 217   3.61   0.37    3.7    3.66   0.34   2.15   4   1.85 -1.25     1.47 0.03
X4.GPA    6 218   3.45   0.38    3.5    3.47   0.43   2.17   4   1.83 -0.62    -0.10 0.03
#clean up the data, remove GPA > 5
> grad1 <- scrub(grad,1,max=5)
> describe(grad1)
```

Then, draw a Scatter Plot Matrix of the data.

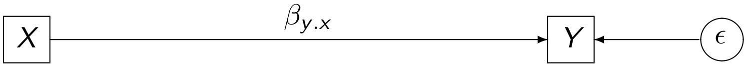
Scatter Plot Matrix of Psychology Graduate Applicants

```
pairs.panels(grad1,cor=FALSE,ellipses=FALSE,smooth=FALSE,lm=TRUE)
```



Bivariate Regression

X Y ϵ

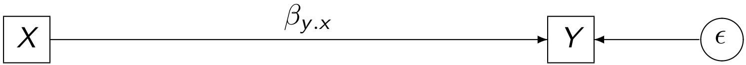


$$\hat{y} = \beta_{y.x} x + \epsilon$$

$$\beta_{y.x} = \frac{\sigma_{xy}}{\sigma_x^2}$$

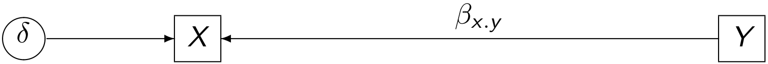
Bivariate Regression

δ X Y ϵ



$$\hat{y} = \beta_{y.x}x + \epsilon$$

$$\beta_{y.x} = \frac{\sigma_{xy}}{\sigma_x^2}$$

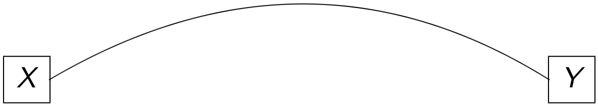


$$\hat{x} = \beta_{x.y}y + \delta$$

$$\beta_{y.x} = \frac{\sigma_{xy}}{\sigma_y^2}$$

Bivariate Correlation

X Y



$\hat{x} = \beta_{x.y} y + \delta$

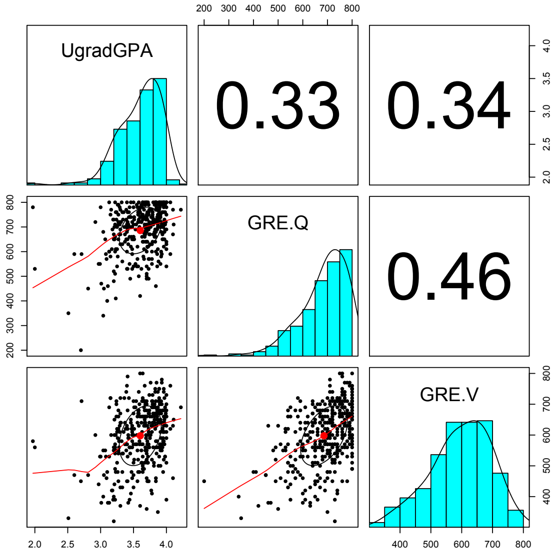
$\hat{y} = \beta_{y.x} x + \epsilon$

$\beta_{y.x} = \frac{\sigma_{xy}}{\sigma_y^2}$

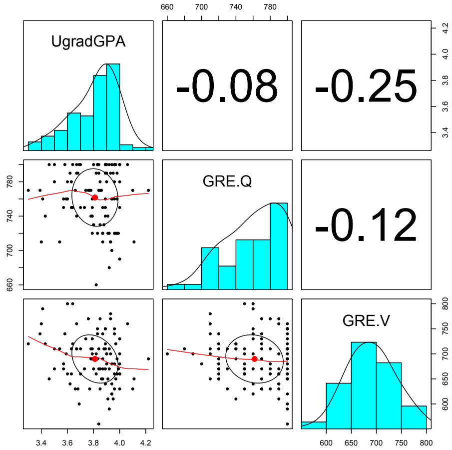
$\beta_{y.x} = \frac{\sigma_{xy}}{\sigma_x^2}$

$$r_{xy} = \frac{\sigma_{xy}}{\sqrt{\sigma^{2x}\sigma_y^2}}$$

Scatter Plot Matrix showing correlation and LOESS regression

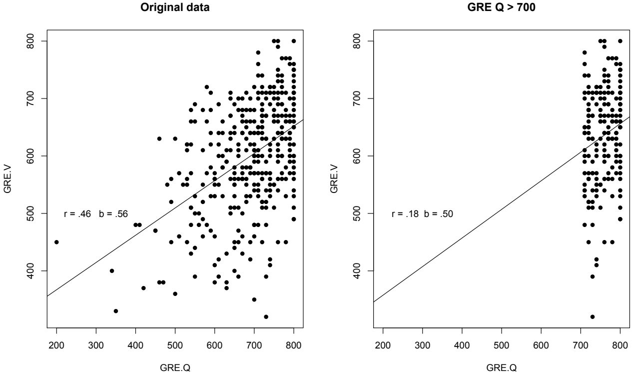


The effect of selection on the correlation



- Consider what happens if we select a subset
    - The "Oregon" model
    - (GPA + (V+Q)/200) > 11.6
- The range is truncated, but even more important, by using a compensatory selection model, we have changed the sign of the correlations.

Regression and restriction of range



Although the correlation is very sensitive, regression slopes are relatively insensitive to restriction of range.

R code for regression figures

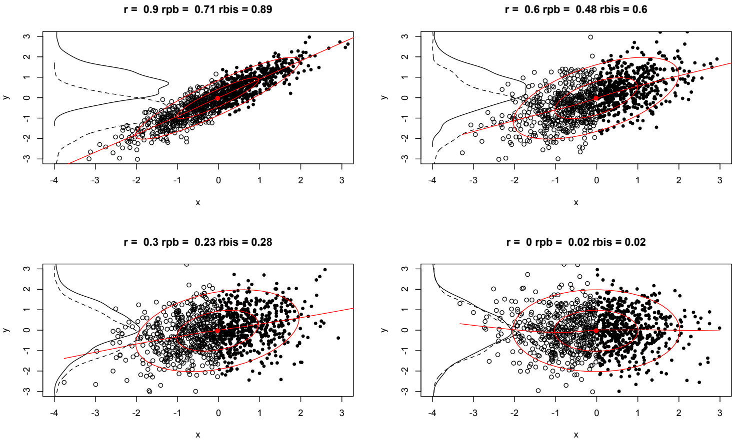
```
gradq <- subset(gradf,gradf[2]>700) #choose the subset
with(gradq,lm(GRE.V ~ GRE.Q)) #do the regression
Call:
lm(formula = GRE.V ~ GRE.Q)
Coefficients:
(Intercept)        GRE.Q
   258.1549       0.4977

#show the graphic
op <- par(mfrow=c(1,2))  #two panel graph
 with(gradf,{
 plot(GRE.V ~ GRE.Q,xlim=c(200,800), main="Original data", pch=16)
 abline(lm(GRE.V ~ GRE.Q))
 } )
 text(300,500,"r = .46   b = .56")
 with(gradq,{
 plot(GRE.V ~ GRE.Q,xlim=c(200,800),main="GRE Q > 700",pch=16)
 abline(lm(GRE.V ~ GRE.Q))
} )
text(300,500,"r = .18  b = .50")
 op <- par(mfrow=c(1,1)) #switch back to one panel
```

Alternative versions of the correlation coefficient

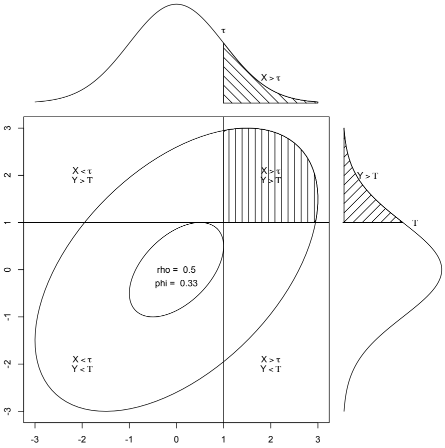
Table: A number of correlations are Pearson r in different forms, or with particular assumptions. If $r = \frac{\sum x_i y_i}{\sqrt{\sum x_i^2 \sum y_i^2}}$, then depending upon the type of data being analyzed, a variety of correlations are found.

Coefficient	symbol	X	Y	Assumptions
Pearson	r	continuous	continuous	
Spearman	rho (ρ)	ranks	ranks	
Point bi-serial	r_{pb}	dichotomous	continuous	
Phi	ϕ	dichotomous	dichotomous	
Bi-serial	r_{bis}	dichotomous	continuous	normality
Tetrachoric	r_{tet}	dichotomous	dichotomous	bivariate normality
Polychoric	r_{pc}	categorical	categorical	bivariate normality

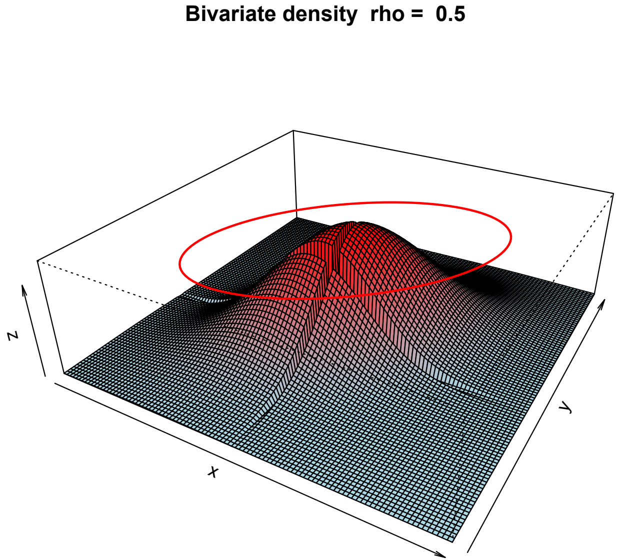
The biserial correlation estimates the latent correlation



The tetrachoric correlation estimates the latent correlation



The tetrachoric correlation estimates the latent correlation



Cautions about correlations–The Anscombe data set

Consider the following 8 variables

```
 describe(anscombe)
   vars  n mean   sd median trimmed  mad  min   max range  skew kurtosis   se
x1    1 11  9.0 3.32   9.00    9.00 4.45 4.00 14.00 10.00  0.00    -1.53 1.00
x2    2 11  9.0 3.32   9.00    9.00 4.45 4.00 14.00 10.00  0.00    -1.53 1.00
x3    3 11  9.0 3.32   9.00    9.00 4.45 4.00 14.00 10.00  0.00    -1.53 1.00
x4    4 11  9.0 3.32   8.00    8.00 0.00 8.00 19.00 11.00  2.47     4.52 1.00
y1    5 11  7.5 2.03   7.58    7.49 1.82 4.26 10.84  6.58 -0.05    -1.20 0.61
y2    6 11  7.5 2.03   8.14    7.79 1.47 3.10  9.26  6.16 -0.98    -0.51 0.61
y3    7 11  7.5 2.03   7.11    7.15 1.53 5.39 12.74  7.35  1.38     1.24 0.61
y4    8 11  7.5 2.03   7.04    7.20 1.90 5.25 12.50  7.25  1.12     0.63 0.61

summary(lm(y1~x1,data=anscombe))  #show one of the regressions
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   3.0001     1.1247   2.667  0.02573 *
x1            0.5001     0.1179   4.241  0.00217 **

Residual standard error: 1.237 on 9 degrees of freedom
Multiple R-squared:  0.6665,       Adjusted R-squared:  0.6295
F-statistic: 17.99 on 1 and 9 DF,  p-value: 0.00217
```

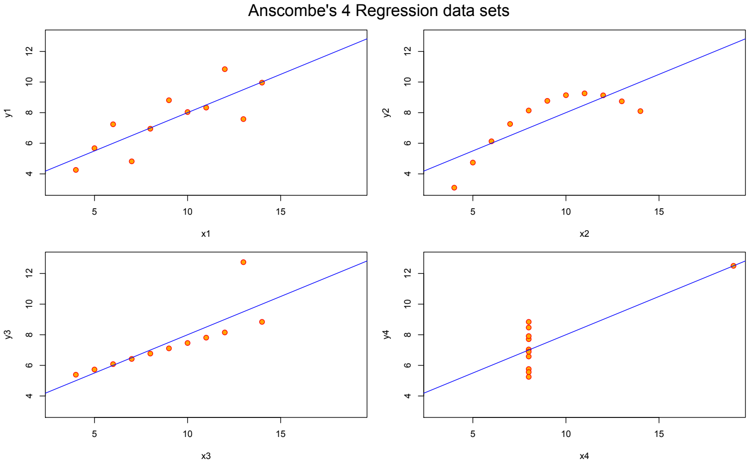
Anscombe (1973)

Cautions, Anscombe continued

With regressions of

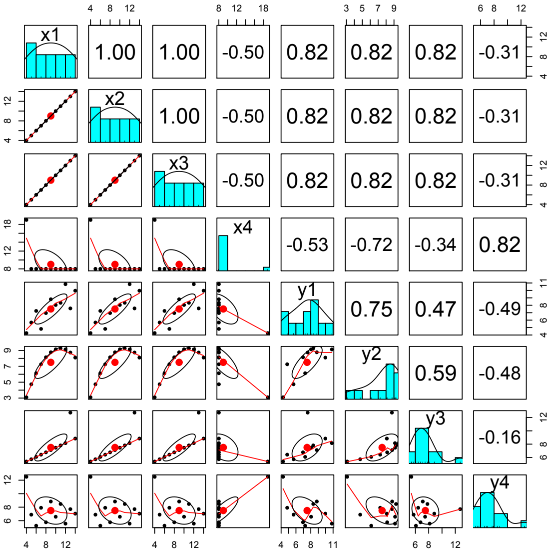
```
              Estimate Std. Error  t value    Pr(>|t|)
(Intercept) 3.0000909  1.1247468 2.667348 0.025734051
x1          0.5000909  0.1179055 4.241455 0.002169629

[[2]]
             Estimate Std. Error  t value    Pr(>|t|)
(Intercept) 3.000909  1.1253024 2.666758 0.025758941
x2          0.500000  0.1179637 4.238590 0.002178816

[[3]]
              Estimate Std. Error  t value    Pr(>|t|)
(Intercept) 3.0024545  1.1244812 2.670080 0.025619109
x3          0.4997273  0.1178777 4.239372 0.002176305

[[4]]
              Estimate Std. Error  t value    Pr(>|t|)
(Intercept) 3.0017273  1.1239211 2.670763 0.025590425
x4          0.4999091  0.1178189 4.243028 0.002164602
```

Cautions about correlations: Anscombe data set



Moral: Always plot your data!

Cautions about correlations: Anscombe data set

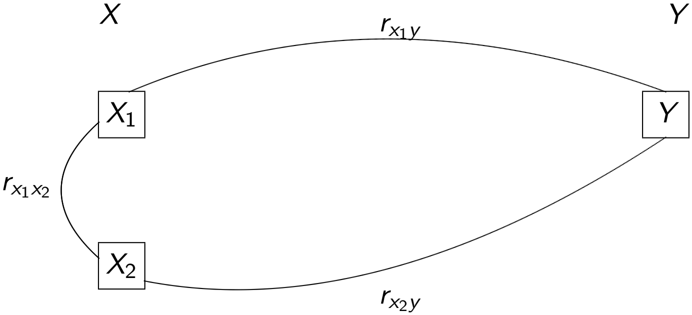


The ubiquitous correlation coefficient

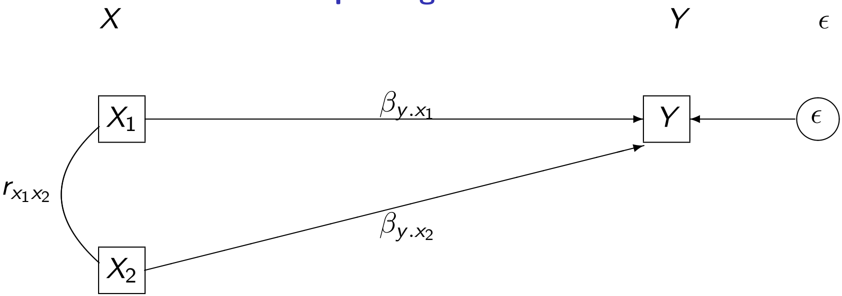
Table: Alternative Estimates of effect size. Using the correlation as a scale free estimate of effect size allows for combining experimental and correlational data in a metric that is directly interpretable as the effect of a standardized unit change in x leads to r change in standardized y.

Statistic	Estimate	r equivalent	as a function of r
Pearson correlation	$r_{xy} = \frac{C_{xy}}{\sigma_x \sigma_y}$	r_{xy}	
Regression	$b_{y.x} = \frac{C_{xy}}{\sigma_x^2}$	$r = b_{y.x} \frac{\sigma_y}{\sigma_x}$	$b_{y.x} = r \frac{\sigma_x}{\sigma_y}$
Cohen's d	$d = \frac{X_1 - X_2}{\sigma_x}$	$r = \frac{d}{\sqrt{d^2+4}}$	$d = \frac{2r}{\sqrt{1-r^2}}$
Hedge's g	$g = \frac{X_1 - X_2}{s_x}$	$r = \frac{g}{\sqrt{g^2+4(df/N)}}$	$g = \frac{2r\sqrt{df/N}}{\sqrt{1-r^2}}$
t - test	$t = \frac{d\sqrt{df}}{2}$	$r = \sqrt{t^2/(t^2+df)}$	$t = \sqrt{\frac{r^2 df}{1-r^2}}$
F-test	$F = \frac{d^2 df}{4}$	$r = \sqrt{F/(F+df)}$	$F = \frac{r^2 df}{1-r^2}$
Chi Square		$r = \sqrt{\chi^2/n}$	$\chi^2 = r^2 n$
Odds ratio	$d = \frac{ln(OR)}{1.81}$	$r = \frac{ln(OR)}{1.81\sqrt{(ln(OR)/1.81)^2+4}}$	$ln(OR) = \frac{3.62r}{\sqrt{1-r^2}}$
$r_{equivalent}$	r with probability p	$r = r_{equivalent}$	

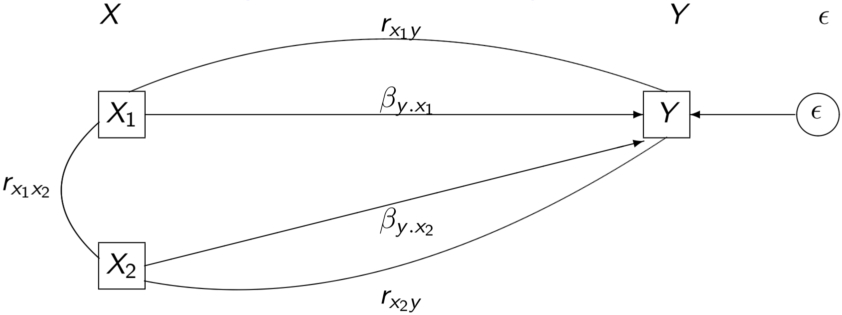
Multiple correlations



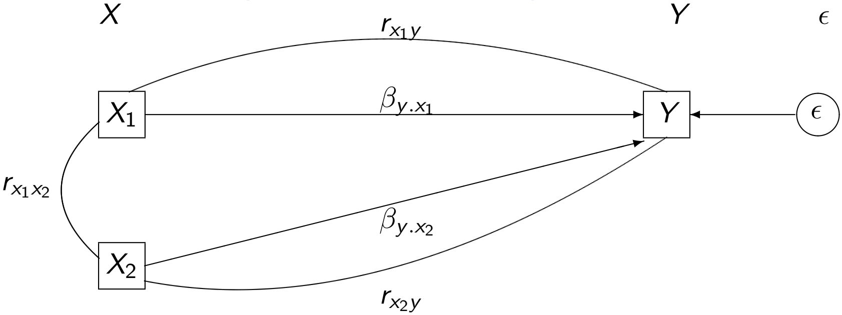
Multiple Regression



Multiple Regression: decomposing correlations



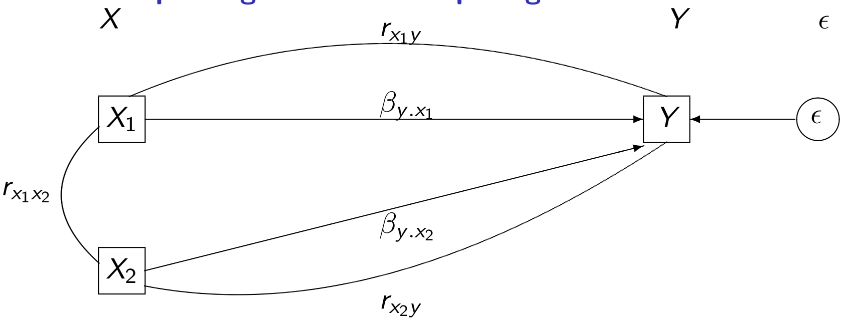
Multiple Regression: decomposing correlations



$$r_{x_1y} = \overbrace{\beta_{y.x_1}}^{\color{blue}{direct}} + \overbrace{r_{x_1x_2}\beta_{y.x_2}}^{\color{red}{indirect}}$$

$$r_{x_2y} = \underbrace{\beta_{y.x_2}}_{\color{blue}{direct}} + \underbrace{r_{x_1x_2}\beta_{y.x_1}}_{\color{red}{indirect}}$$

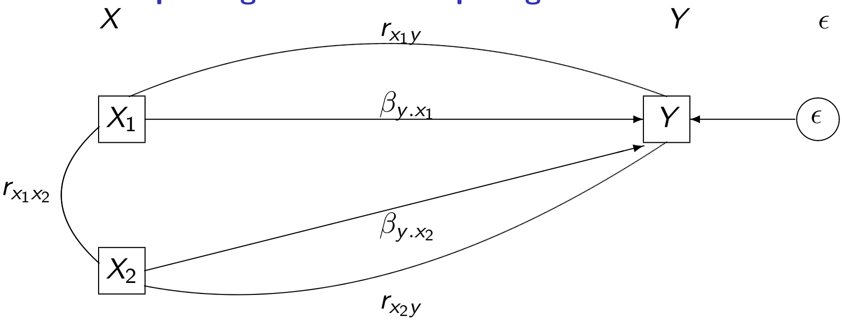
Multiple Regression: decomposing correlations



$$r_{x_1 y} = \overbrace{\beta_{y.x_1}}^{direct} + \overbrace{r_{x_1 x_2}\beta_{y.x_2}}^{indirect} \qquad \beta_{y.x_1} = \frac{r_{x_1 y} - r_{x_1 x_2} r_{x_2 y}}{1 - r^2_{x_1 x_2}}$$

$$r_{x_2 y} = \underbrace{\beta_{y.x_2}}_{direct} + \underbrace{r_{x_1 x_2}\beta_{y.x_1}}_{indirect} \qquad \beta_{y.x_2} = \frac{r_{x_2 y} - r_{x_1 x_2} r_{x_1 y}}{1 - r^2_{x_1 x_2}}$$

Multiple Regression: decomposing correlations



$$r_{x_1 y} = \overbrace{\beta_{y.x_1}}^{direct} + \overbrace{r_{x_1 x_2}\beta_{y.x_2}}^{indirect}$$

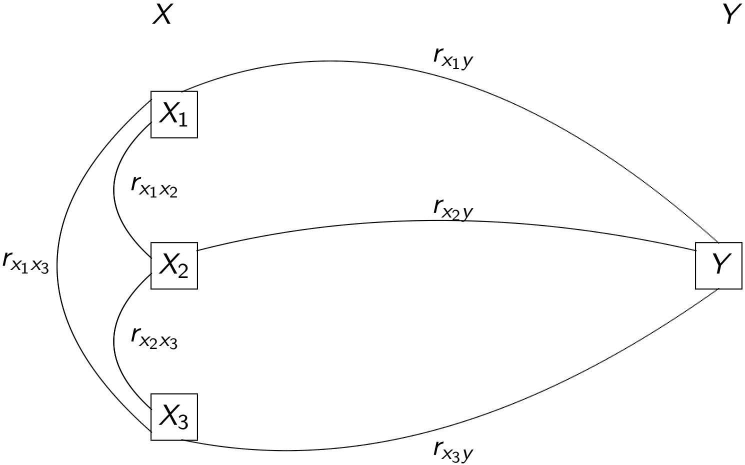
$$r_{x_2 y} = \underbrace{\beta_{y.x_2}}_{direct} + \underbrace{r_{x_1 x_2}\beta_{y.x_1}}_{indirect}$$

$$\beta_{y.x_1} = \frac{r_{x_1 y} - r_{x_1 x_2} r_{x_2 y}}{1 - r^2_{x_1 x_2}}$$

$$\beta_{y.x_2} = \frac{r_{x_2 y} - r_{x_1 x_2} r_{x_1 y}}{1 - r^2_{x_1 x_2}}$$

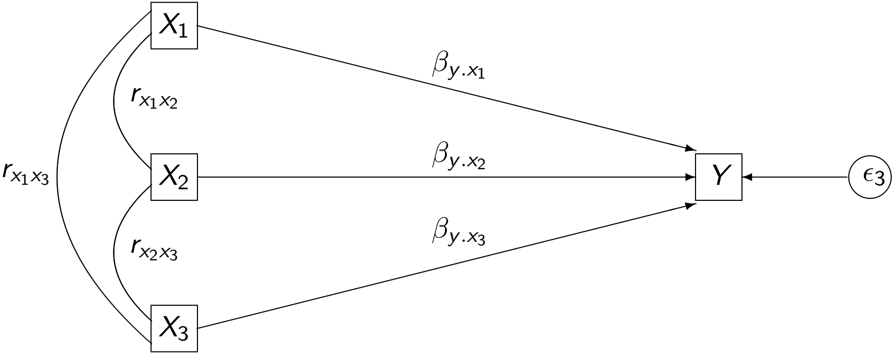
$$R^2 = r_{x_1 y}\beta_{y.x_1} + r_{x_2 y}\beta_{y.x_2}$$

What happens with 3 predictors? The correlations

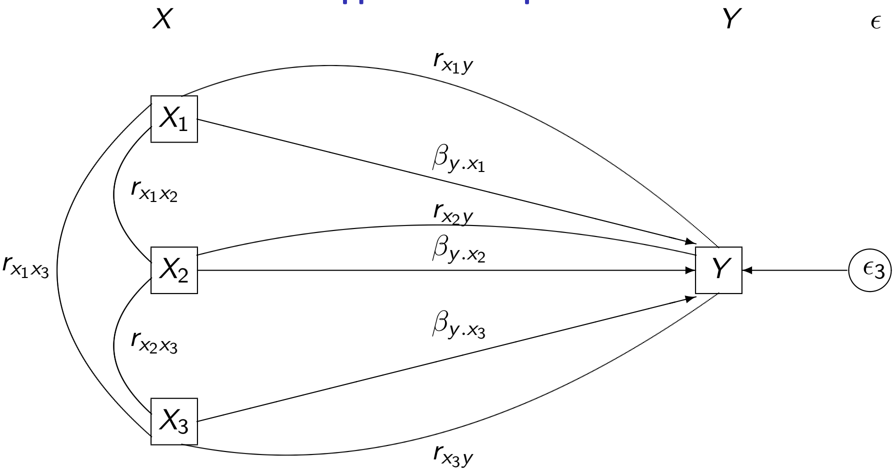


What happens with 3 predictors? β weights

X Y ϵ



What happens with 3 predictors?



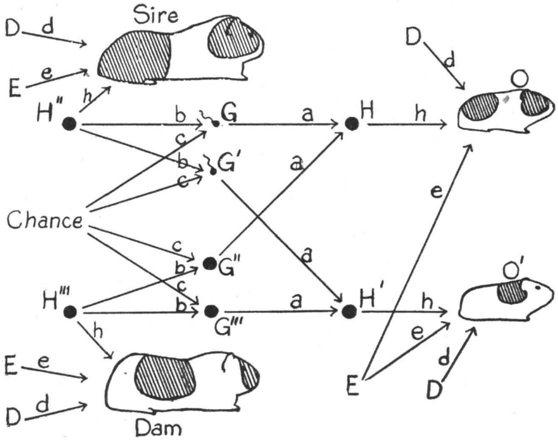
$$r_{x_1 y} = \underbrace{\beta_{y.x_1}}_{\textit{direct}} + \underbrace{r_{x_1 x_2}\beta_{y.x_1} + r_{x_1 x_3}\beta_{y.x_3}}_{\textit{indirect}} \quad r_{x_2 y} = \ldots \quad r_{x_3 y} = \ldots$$

The math gets tedious

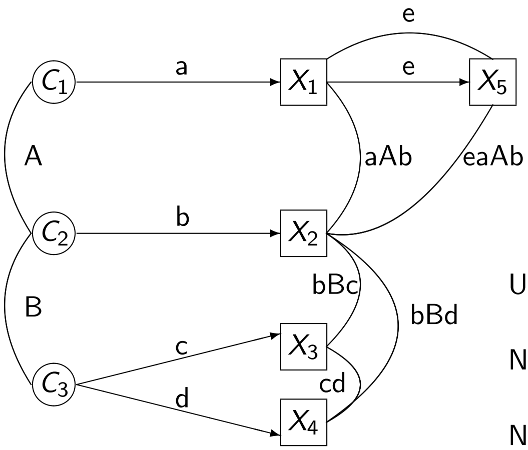
Multiple regression and linear algebra

- Multiple regression requires solving multiple, simultaneous equations to estimate the direct and indirect effects.
  - Each equation is expressed as a $r_{x_i y}$ in terms of direct and indirect effects.
  - Direct effect is $\beta_{y.x_i}$
  - Indirect effect is $\sum_{j \neq i} beta_{y.x_j} r_{x_j y}$
- How to solve these equations?
- Tediously, or just use linear algebra

Wright's Path model of inheritance in the Guinea Pig (Wright, 1921)



The basic rules of path analysis–think genetics



Up ... and over and down ...

No down and up

No double overs

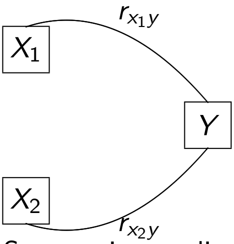
Up ... and down ...

Parents cause children

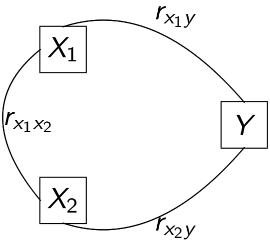
children do not cause parents

3 special cases of regression

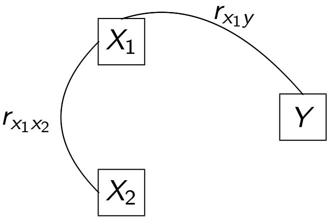
Orthogonal predictors



Correlated predictors

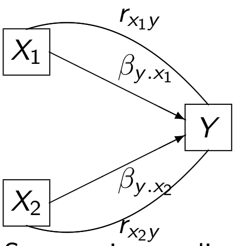


Suppressive predictors

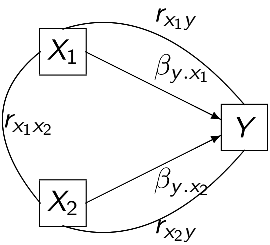


3 special cases of regression

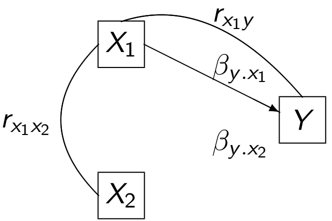
Orthogonal predictors



Correlated predictors



Suppressive predictors

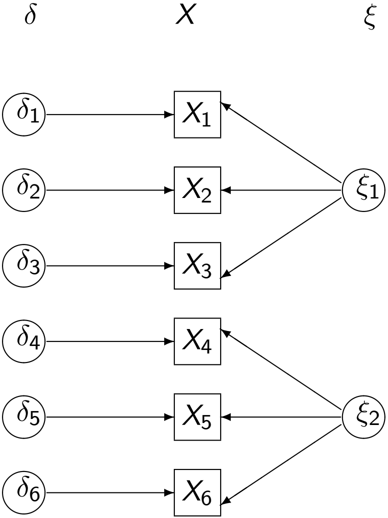


$$\beta_{y.x_1} = \frac{r_{x_1y} - r_{x_1x_2} r_{x_2y}}{1 - r^2_{x_1x_2}}$$

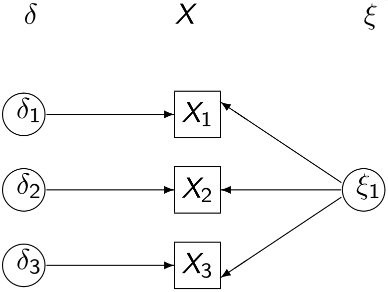
$$\beta_{y.x_2} = \frac{r_{x_2y} - r_{x_1x_2} r_{x_1y}}{1 - r^2_{x_1x_2}}$$

$$R^2 = r_{x_1y}\beta_{y.x_1} + r_{x_2y}\beta_{y.x_2}$$

A measurement model for X



Congeneric Reliability



Creating a congeneric model

```
> cong <- sim.congeneric()
> cong
```

```
     V1   V2   V3   V4
V1 1.00 0.56 0.48 0.40
V2 0.56 1.00 0.42 0.35
V3 0.48 0.42 1.00 0.30
V4 0.40 0.35 0.30 1.00
```

Factor a 1 factor model

```
> cong <- sim.congeneric()
> cong

     V1   V2   V3   V4
V1 1.00 0.56 0.48 0.40
V2 0.56 1.00 0.42 0.35
V3 0.48 0.42 1.00 0.30
V4 0.40 0.35 0.30 1.00
```

```
> f <- fa(cong)
> f

Factor Analysis using method =  minres
Call: fa(r = cong)
Standardized loadings based upon
               correlation matrix
   MR1   h2   u2 com
V1 0.8 0.64 0.36   1
V2 0.7 0.49 0.51   1
V3 0.6 0.36 0.64   1
V4 0.5 0.25 0.75   1
                 MR1
SS loadings     1.74
Proportion Var 0.44
```

Factoring ≠ components !

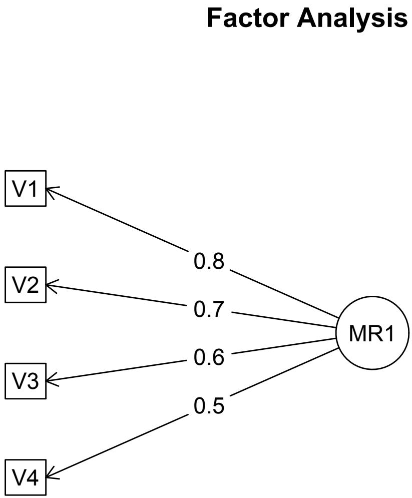
```
> f <- fa(cong)
> f

Factor Analysis using method = minres
Call: fa(r = cong)
Standardized loadings based
      upon correlation matrix
   MR1   h2   u2 com
V1 0.8 0.64 0.36   1
V2 0.7 0.49 0.51   1
V3 0.6 0.36 0.64   1
V4 0.5 0.25 0.75   1

                MR1
SS loadings    1.74
Proportion Var 0.44
```

```
> p <- principal(cong)
> p

Principal Components Analysis
Call: principal(r = cong)
Standardized loadings (pattern matrix) based
        upon correlation matrix
    PC1   h2   u2 com
V1 0.83 0.69 0.31   1
V2 0.79 0.62 0.38   1
V3 0.73 0.53 0.47   1
V4 0.65 0.43 0.57   1

                PC1
SS loadings    2.27
Proportion Var 0.57
```

Exploratory factoring a congeneric model

```
Factor Analysis using method =  minres
Call: fa(r = cong)
Standardized loadings based upon correlation matrix
   MR1   h2   u2
V1 0.8 0.64 0.36
V2 0.7 0.49 0.51
V3 0.6 0.36 0.64
V4 0.5 0.25 0.75
                MR1
SS loadings    1.74
Proportion Var 0.44
Test of the hypothesis that 1 factor is sufficient.
The degrees of freedom for the null model are  6  and the objective function wa
The degrees of freedom for the model are 2  and the objective function was  0
The root mean square of the residuals is  0
The df corrected root mean square of the residuals is  0
Fit based upon off diagonal values = 1
Measures of factor score adequacy
                                                 MR1
Correlation of scores with factors              0.89
Multiple R square of scores with factors        0.78
Minimum correlation of possible factor scores   0.57
```

Show the structure



Congeneric with some noise

```
> cong1 <- sim.congeneric(N=500,short=FALSE)
> cong1


Call: NULL

 $model (Population correlation matrix)
     V1   V2   V3   V4
V1 1.00 0.56 0.48 0.40
V2 0.56 1.00 0.42 0.35
V3 0.48 0.42 1.00 0.30
V4 0.40 0.35 0.30 1.00

$r  (Sample correlation matrix  for sample size =  500 )
     V1   V2   V3   V4
V1 1.00 0.55 0.49 0.42
V2 0.55 1.00 0.45 0.38
V3 0.49 0.45 1.00 0.35
V4 0.42 0.38 0.35 1.00
```

Add noise to the model

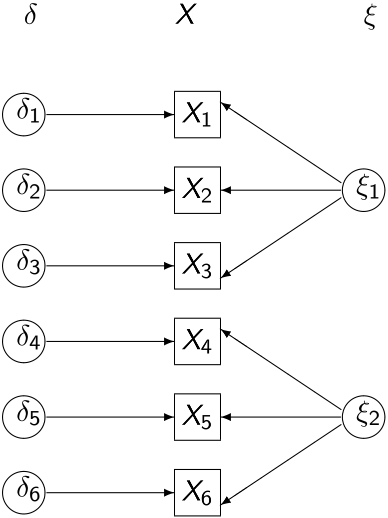
```
> cong1 <- sim.congeneric(N=500,short=FALSE)
> cong1
Call: NULL

 $model (Population correlation matrix)
     V1   V2   V3   V4
V1 1.00 0.56 0.48 0.40
V2 0.56 1.00 0.42 0.35
V3 0.48 0.42 1.00 0.30
V4 0.40 0.35 0.30 1.00

$r  (Sample correlation matrix
for sample size =  500 )
     V1   V2   V3   V4
V1 1.00 0.55 0.49 0.42
V2 0.55 1.00 0.45 0.38
V3 0.49 0.45 1.00 0.35
V4 0.42 0.38 0.35 1.00
```

```
> f <- fa(cong1$observed)
> f

Factor Analysis using method =  minres
Call: fa(r = cong1$observed)
Standardized loadings based upon correlation m
    MR1   h2   u2
V1 0.77 0.60 0.40
V2 0.71 0.50 0.50
V3 0.63 0.40 0.60
V4 0.54 0.29 0.71

                MR1
SS loadings    1.79
Proportion Var 0.45
```

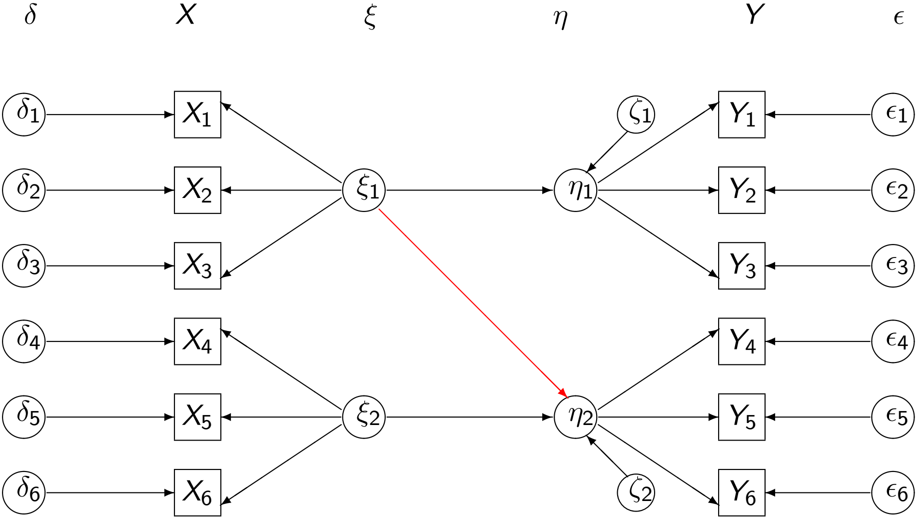
Factor a 1 dimensional model

```
Factor Analysis using method =  minres
Call: fa(r = cong1$observed)
Standardized loadings based upon correlation matrix
    MR1   h2   u2
V1 0.77 0.60 0.40
V2 0.71 0.50 0.50
V3 0.63 0.40 0.60
V4 0.54 0.29 0.71
                MR1
SS loadings    1.79
Proportion Var 0.45
Test of the hypothesis that 1 factor is sufficient.
The degrees of freedom for the null model are  6  and the objective function was  0.95 with Chi Square of
The degrees of freedom for the model are 2  and the objective function was  0
The root mean square of the residuals is  0
The df corrected root mean square of the residuals is  0.01
The number of observations was  500  with Chi Square =  0.11  with prob <  0.95
Tucker Lewis Index of factoring reliability =  1.012
RMSEA index =  0  and the 90 % confidence intervals are  0 0.034
BIC =  -12.32
Fit based upon off diagonal values = 1
Measures of factor score adequacy
                                                MR1
Correlation of scores with factors             0.88
Multiple R square of scores with factors       0.78
Minimum correlation of possible factor scores  0.56
```

Factor Analysis



A complete structural model



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