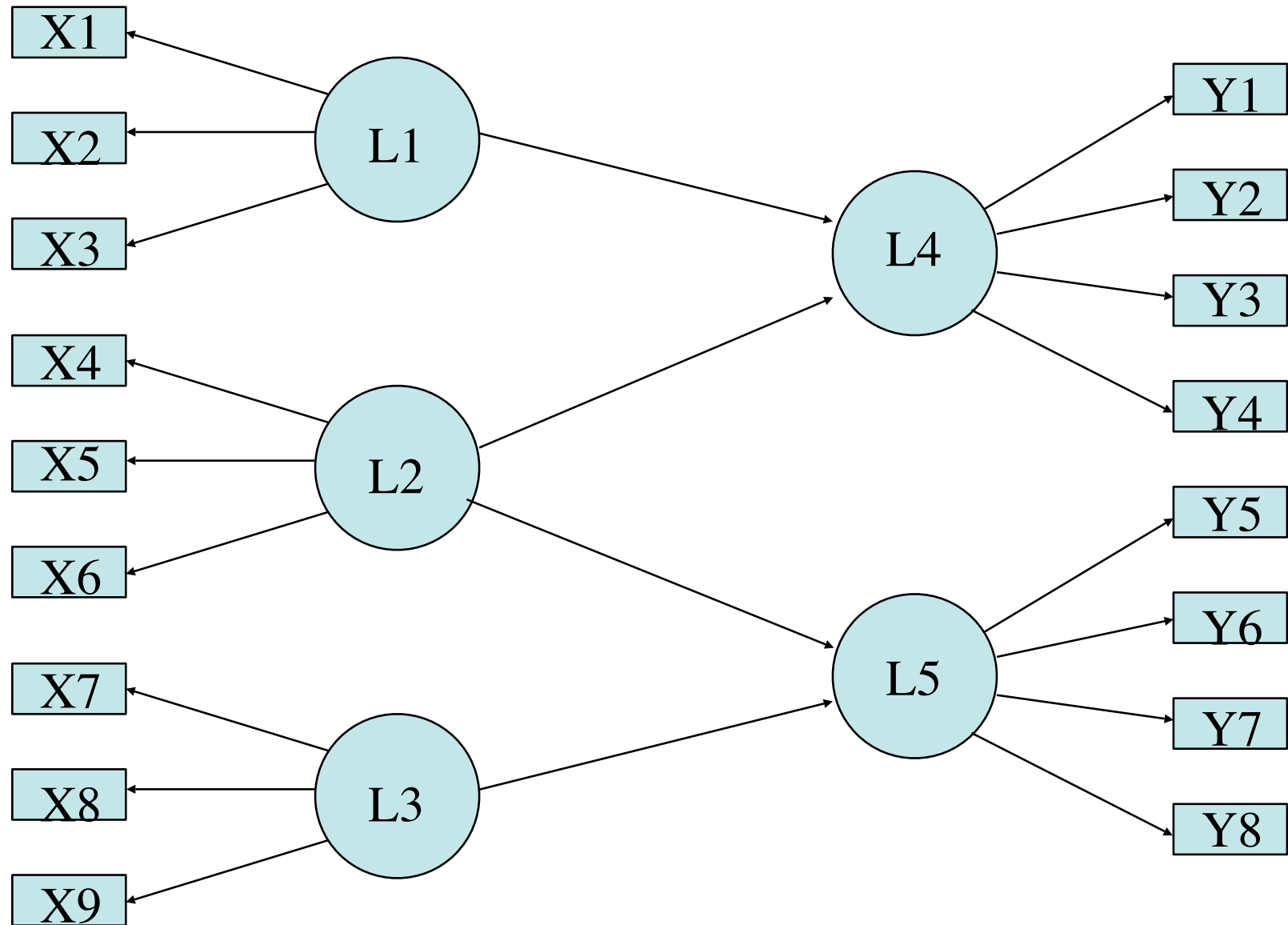


Psychometric Theory: A conceptual Syllabus



Model Fitting

Structural Equation Models

Reliability + Validity

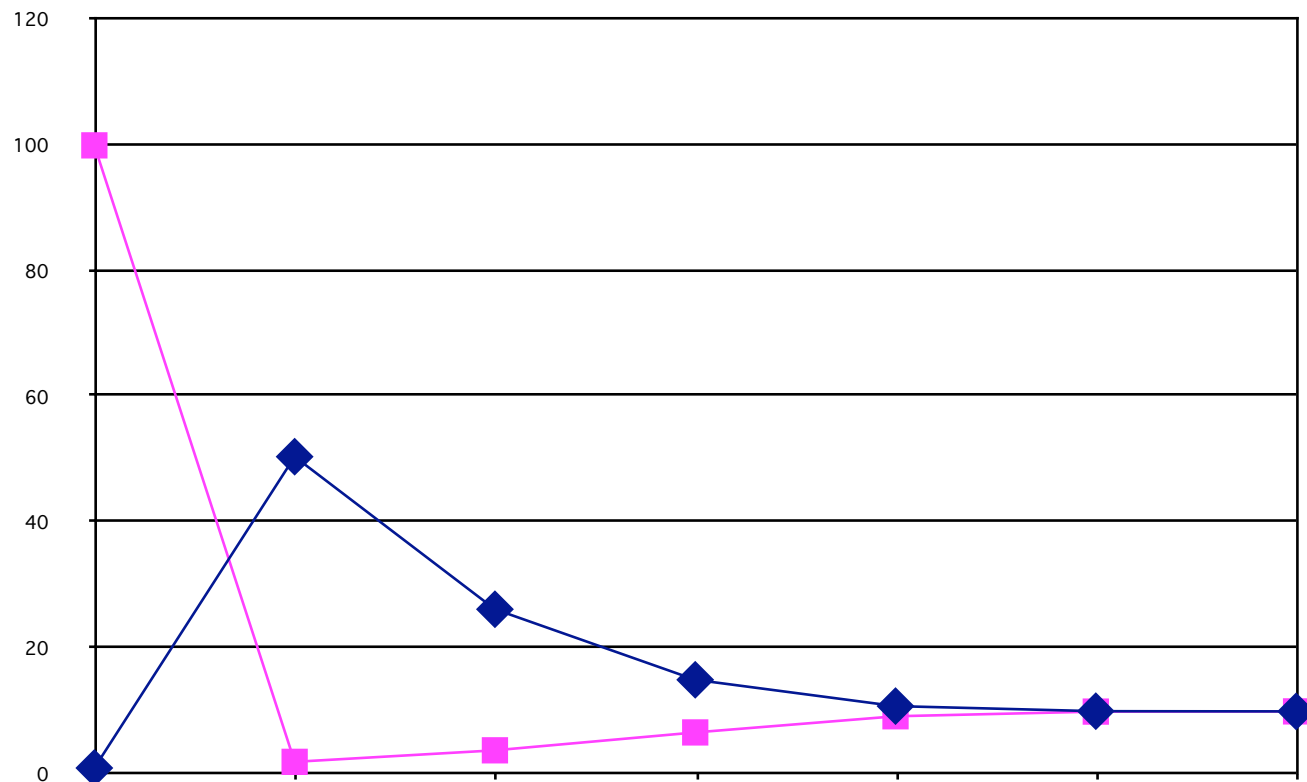
Basic concepts of iterative fit

- Most classical statistics (e.g. means, variances, regression slopes) may be found by algebraic solutions of closed form expressions
- More recent statistics are the results of iteratively fitting a model until some criterion is either minimized or maximized.

Simple example: the square root

Target	100			
Trial	guess	fit	diff	
1	1.0	100.0	-99.0	
2	50.5	2.0	48.5	
3	26.2	3.8	22.4	
4	15.0	6.7	8.4	
5	10.8	9.2	1.6	
6	10.0	10.0	0.1	
7	10.0	10.0	0.0	

Iteratively estimating the square root of 100



Applications: Factor Analysis

x1	1.00		
x2	0.70	1.00	
x3	0.60	0.58	1.00

Find F such that $FF' = R$ for the off diagonal elements of R

Iterative Fit by trial and error

x1	x2	x3	fit
1.000	1.000	1.000	0.4264000
1.000	0.800	1.000	0.2180000
1.000	0.800	0.800	0.0536000
0.800	0.800	0.800	0.0088000
0.800	0.800	0.700	0.0056000
0.800	0.800	0.750	0.0040000
0.850	0.800	0.750	0.0022000
0.850	0.800	0.700	0.0008250
0.850	0.800	0.710	0.0005625
0.851	0.823	0.705	0.0000000

Fitted model

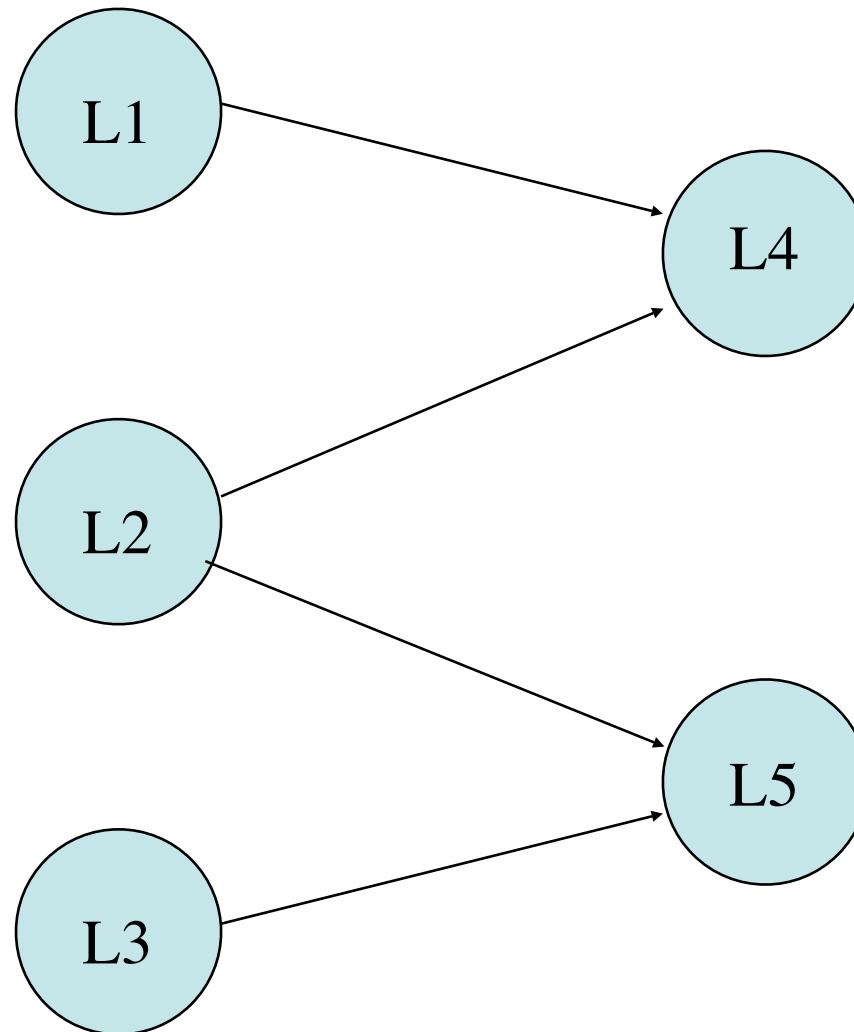
	F1
x1	0.851
x2	0.823
x3	0.705

0.72	0.70	0.60
0.70	0.68	0.58
0.60	0.58	0.50

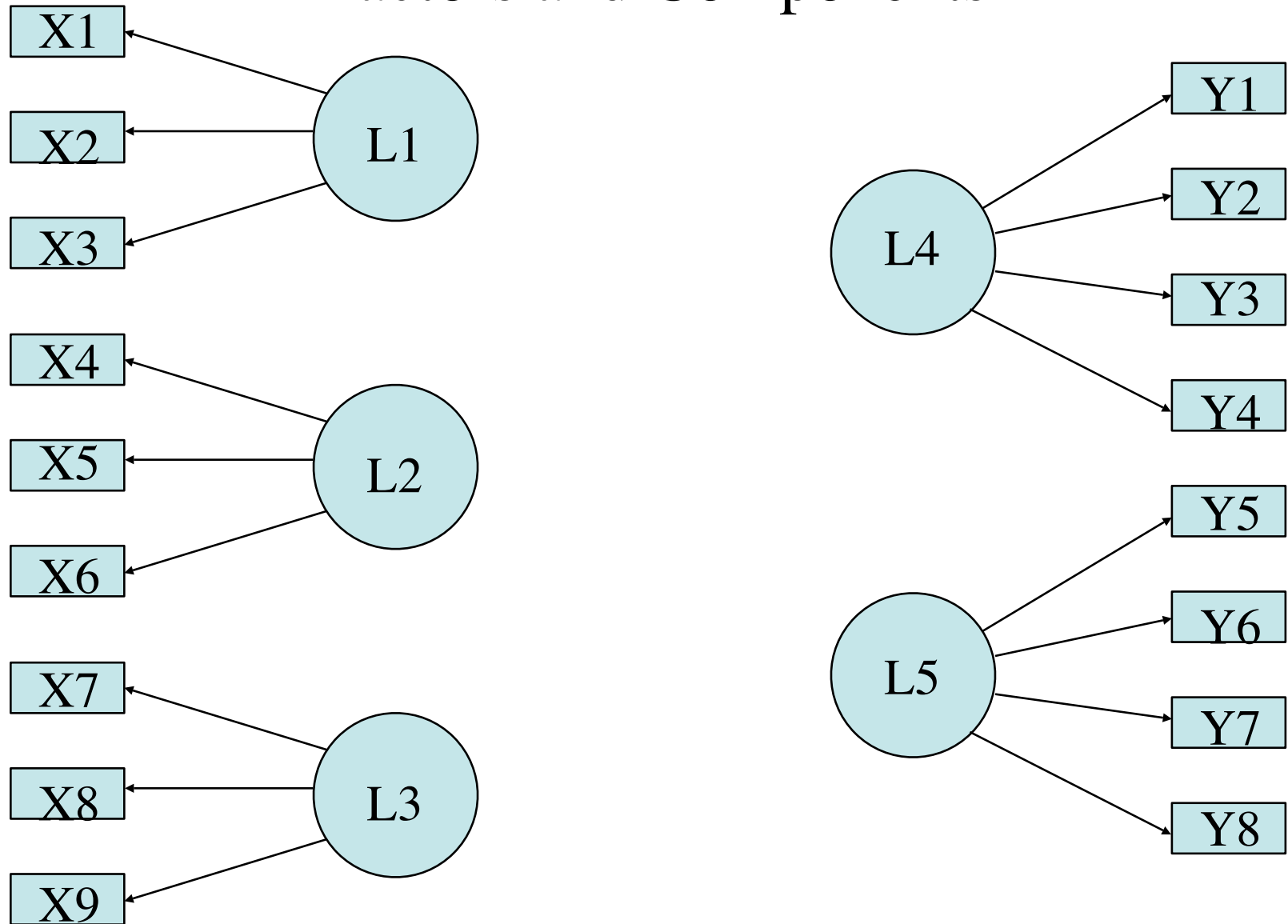
Structural Equation Models

- Basic concept is to apply a measurement model to a structure (regression) model
- Generically known as SEM, particular programs are LISREL, EQS, RAMONA, RAM-path, Mx, sem
- May be used for confirmatory factor analysis as well as sem.

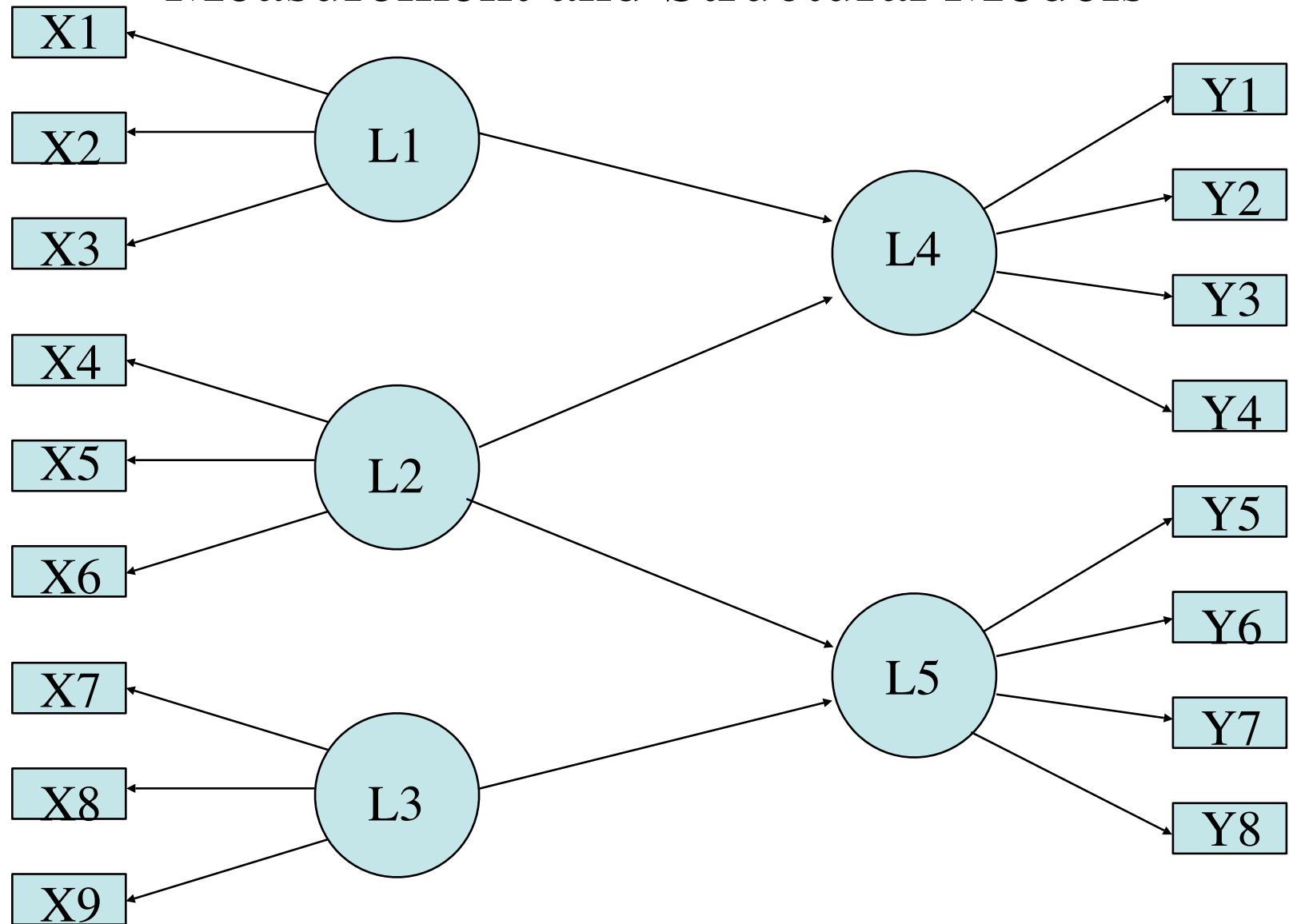
Theory as organization of constructs



Techniques of Data Reduction: Factors and Components



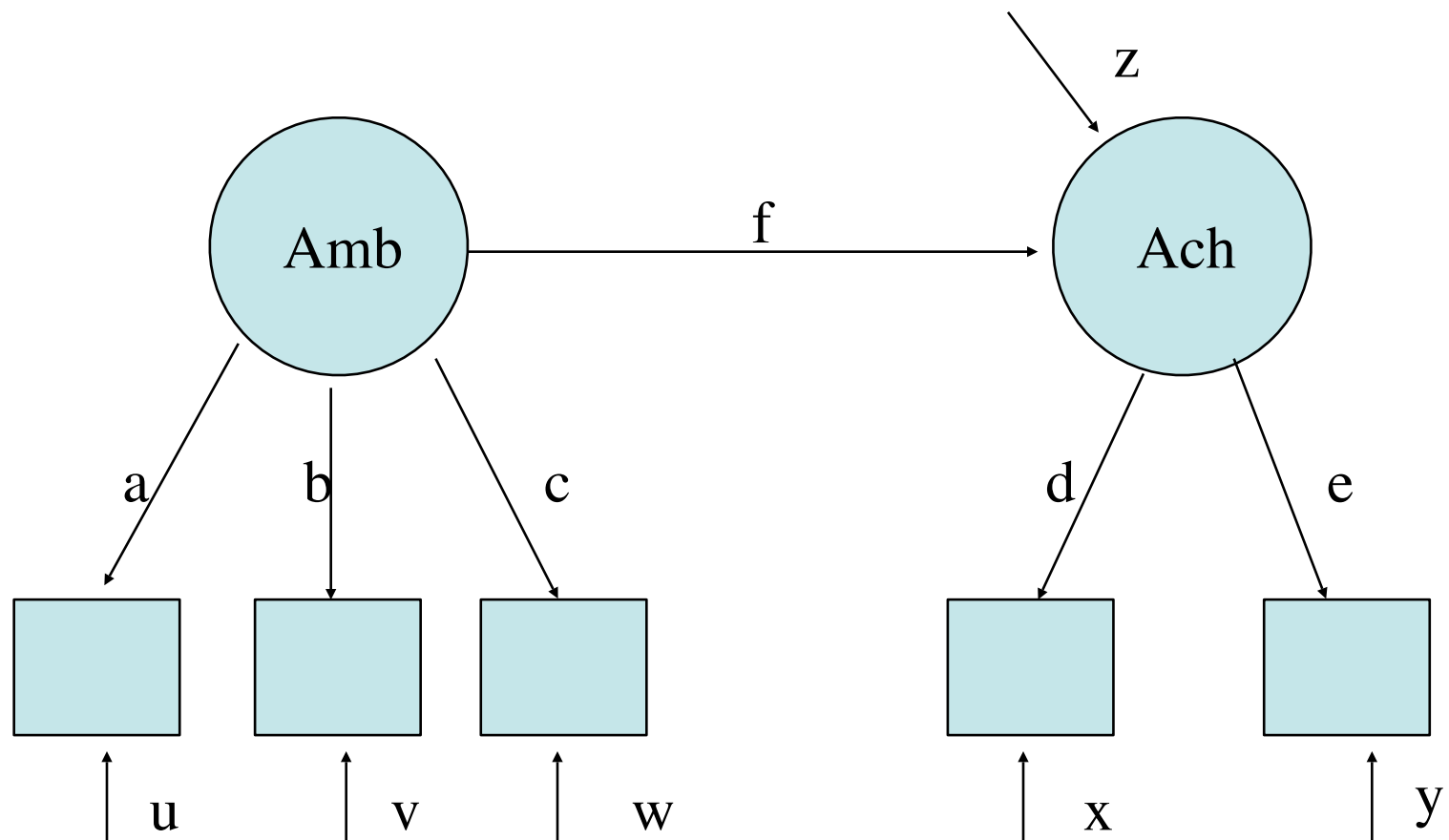
Structural Equation Modeling: Combining Measurement and Structural Models



SEM problem (Loehlin 2.5)

Ach1	Ach2	Amb1	Amb2	Amb3
1	0.6	0.3	0.2	0.2
0.6	1	0.2	0.3	0.1
0.3	0.2	1	0.7	0.6
0.2	0.3	0.7	1	0.5
0.2	0.1	0.6	0.5	1

Ambition and Achievement



R code for sem for Loehlin 2.5

First enter the correlation matrix

```
#Loehlin problem 2.5
```

```
obs.var2.5 = c('Ach1', 'Ach2', 'Amb1', 'Amb2', 'Amb3')
```

```
R.prob2.5 = matrix(c(
```

```
1.00 , .60 , .30, .20, .20,
```

```
.60, 1.00, .20, .30, .10,
```

```
.30, .20, 1.00, .70, .60 ,
```

```
.20, .30, .70, 1.00, .50,
```

```
.20, .10, .60, .50, 1.00), ncol=5, byrow=TRUE)
```

R code for sem -Ram notation

```
model2.51=matrix(c(
  'Ambit -> Amb1',    'a', NA,
  'Ambit -> Amb2' ,   'b', NA,
  'Ambit -> Amb3' ,   'c', NA,
  'Achieve -> Ach1',  'd', NA,
  'Achieve -> Ach2',  'e', NA,
  'Ambit -> Achieve', 'f', NA,
  'Amb1 <-> Amb1' ,    'u', NA,
  'Amb2 <-> Amb2' ,    'v', NA,
  'Amb3 <-> Amb3' ,    'w', NA,
  'Ach1 <-> Ach1' ,    'x', NA,
  'Ach2 <-> Ach2' ,    'y', NA,
  'Achieve <-> Achieve', NA, 1,
  'Ambit <-> Ambit',    NA, 1),
ncol=3, byrow=TRUE)
```


Run the R code and show results

```
sem2.5= sem(model2.5,R.prob2.5,60, obs.var2.5)  
summary(sem2.5,digits=3)
```

Model Chisquare = 9.74 Df = 4 Pr(>Chisq) = 0.0450

Goodness-of-fit index = 0.964

Adjusted goodness-of-fit index = 0.865

RMSEA index = 0.120 90 % CI: (0.0164, 0.219)

BIC = -15.1

Normalized Residuals

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-5.77e-01	-3.78e-02	-2.04e-06	4.85e-03	3.87e-05	1.13e+00

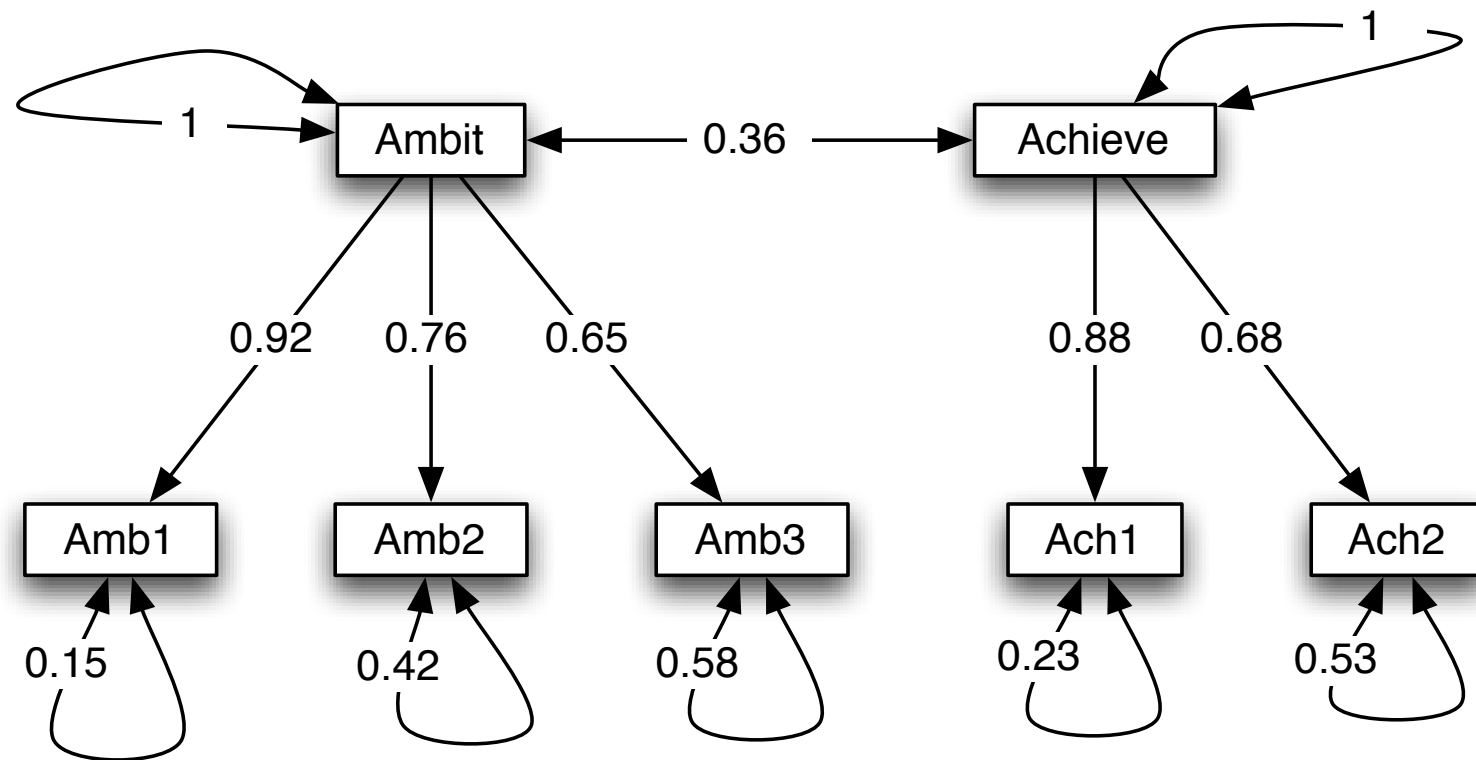
What are the parameters?

Parameter Estimates

	Estimate	Std Error	z value	Pr(> z)	
a	0.920	0.0924	9.966	0.00e+00	Amb1 <--- Ambit
b	0.761	0.0955	7.974	1.55e-15	Amb2 <--- Ambit
c	0.652	0.0965	6.753	1.45e-11	Amb3 <--- Ambit
d	0.879	0.1762	4.986	6.16e-07	Ach1 <--- Achieve
e	0.683	0.1509	4.525	6.03e-06	Ach2 <--- Achieve
f	0.356	0.1138	3.127	1.76e-03	Achieve <--> Ambit
u	0.153	0.0982	1.557	1.20e-01	Amb1 <--> Amb1
v	0.420	0.0898	4.679	2.88e-06	Amb2 <--> Amb2
w	0.575	0.0949	6.061	1.35e-09	Amb3 <--> Amb3
x	0.228	0.2791	0.816	4.15e-01	Ach1 <--> Ach1
y	0.534	0.1837	2.905	3.67e-03	Ach2 <--> Ach2

Iterations = 26

A simple sem



Problems in interpretation

- Model fit does not imply best model
- Consider alternative models
 - Reverse the arrows of causality, nothing happens
 - Range of alternative models
 - Nested models can be compared
 - Non-nested alternative models might be better

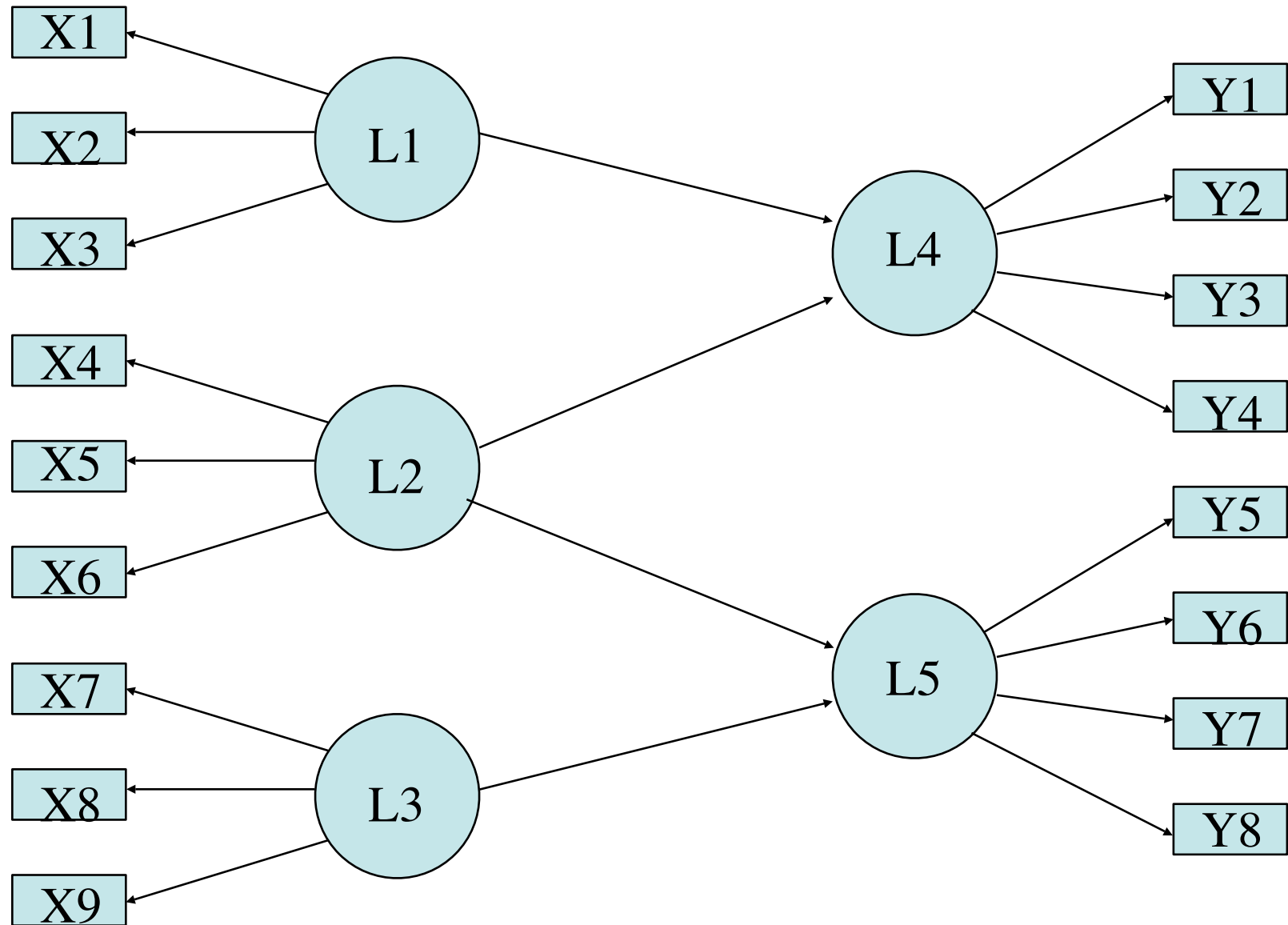
SEM points to consider

- Goodness of fit statistics
 - Statistical indices of size of residuals as compared to null model are sensitive to sample size
 - Comparisons of nested models
- Fits get better with more parameters -> development of df corrected fits
- Inspect residuals to see what is not being fit
- Avoid temptation to 'fix' model based upon results, or, at least be less confident in meaning of good fit

Applications of SEM techniques

- Confirmatory factor analysis
 - Does a particular structure fit the data
- Growth models (growth curve analysis)
- Multiple groups
 - Is the factor structure the same across groups
 - Is the factor structure the same across time

Psychometric Theory: A conceptual Syllabus



Factor analysis

I. The original latent variable model

A.Exploratory factor analysis

B.Confirmatory factor analysis

Methods of Homogeneous Keying

- Latent variable techniques
 - Factor Analysis
- Observed variable analysis
 - Principal Components Analysis
 - Cluster Analysis

Factor Analysis

Consider the following r matrix

	X_1	X_2	X_3	X_4	X_5
X_1	1.00				
X_2	0.72	1.00			
X_3	0.63	0.56	1.00		
X_4	0.54	0.48	0.42	1.00	
X_5	0.45	0.40	0.35	0.30	1.00

Factors as causes; variables as indicators of factors

- Can we recreate the correlation matrix R (of rank n) with a matrix F of rank 1 + a diagonal matrix of uniqueness U^2
- $R \approx FF' + U^2$
- Residual Matrix $R^* = R - (FF' + U^2)$
- Try to minimize the residual

Create an example factor matrix

```
x <- c(.9,.8,.7,.6,.5,.4)
r <- x %*% t(x)
colnames(r) <- rownames(r) <- paste("V",
1:6,sep="")
diag(r) <- 1
r
f1 <- factor.pa(r,1,TRUE,max.iter=1)
```

Observed Correlation matrix

	V1	V2	V3	V4	V5	V6
V1	1.00	0.72	0.63	0.54	0.45	0.36
V2	0.72	1.00	0.56	0.48	0.40	0.32
V3	0.63	0.56	1.00	0.42	0.35	0.28
V4	0.54	0.48	0.42	1.00	0.30	0.24
V5	0.45	0.40	0.35	0.30	1.00	0.20
V6	0.36	0.32	0.28	0.24	0.20	1.00

Find the characteristic roots

- Eigen vectors as characteristic roots of a matrix.
(The vectors unchanged by the matrix.)
- Eigen values as a scaling of the roots
- Given a matrix R , the eigen vectors solve the equation
 - $xR = vx$ or $RX = XD$ where X is a matrix of eigen vectors and D is a diagonal matrix of the eigen values
 - Thus since $XX' = I$, then $R = XDX'$
 - $xR - vxI = 0 \iff x(R-vI) = 0$ and we solve for x

Eigen vectors and eigen values

\$values

ev1	ev2	ev3	ev4	ev5	ev6
3.16	0.82	0.72	0.59	0.44	0.26

\$vectors

	ev1	ev2	ev3	ev4	ev5	ev6
ev1	-0.50	0.061	0.092	0.14	0.238	0.816
ev2	-0.47	0.074	0.121	0.21	0.657	-0.533
ev3	-0.43	0.096	0.182	0.53	-0.675	-0.184
ev4	-0.39	0.142	0.414	-0.78	-0.201	-0.104
ev5	-0.34	0.299	-0.860	-0.20	-0.108	-0.067
ev6	-0.28	-0.934	-0.178	-0.10	-0.067	-0.045

Principal components

- I. Merely a transformation of original matrix to show the eigen vectors and eigen values
- II. The vectors provide the basis space of the original matrix
- III. The lengths of the vectors (the eigen values) reflect the importance of the vector
- IV. If done on a rectangular (data) matrix, known as singular value decomposition, characteristic roots, characteristic vectors

Principal components as rescaled eigen vectors

- If V = the matrix of eigen vectors and v = vector of eigen values and D = diagonal with elements(v)
 - $V \leftarrow \text{eigen}(r)\$vectors$
 - $D \leftarrow \text{diag}(e\$values)$
- then $R = V D V'$ $R \leftarrow V \%*\% D \%*\% t(V)$
- For symmetric matrices $I = VV' = V'V$
- if we let $C = V * \text{sqrt}(D)$ $C \leftarrow V \%*\% \text{sqrt}(D)$
- $R = CC'$ $R \leftarrow C \%*\% t(C)$

Principal Components

```
e <- eigen(r)
c <- e$vectors %*% diag(sqrt(e$values))
rownames(c) <- paste('V',1:6,sep='')
colnames(c) <- paste("C", seq(1:6),sep="")
round(c,2)
```

	C1	C2	C3	C4	C5	C6
V1	-0.88	0.06	0.08	0.11	0.16	0.42
V2	-0.83	0.07	0.10	0.16	0.44	-0.27
V3	-0.77	0.09	0.15	0.41	-0.45	-0.09
V4	-0.69	0.13	0.35	-0.60	-0.13	-0.05
V5	-0.60	0.27	-0.73	-0.15	-0.07	-0.03
V6	-0.50	-0.85	-0.15	-0.08	-0.04	-0.02

$$R = CC'$$

`round(c %*% t(c),2)`

	V1	V2	V3	V4	V5	V6
V1	1.00	0.72	0.63	0.54	0.45	0.36
V2	0.72	1.00	0.56	0.48	0.40	0.32
V3	0.63	0.56	1.00	0.42	0.35	0.28
V4	0.54	0.48	0.42	1.00	0.30	0.24
V5	0.45	0.40	0.35	0.30	1.00	0.20
V6	0.36	0.32	0.28	0.24	0.20	1.00

Take just the first n principal components

I. $R = CC'$ for all components

II. $R \approx CC'$ for first n components

III. Evaluate $R^* = R - CC'$

IV. `pc <- principal(r,1,residuals=TRUE)`

V. notice that the sign of the column
may be reversed

\$loadings

PC I

V1 0.88

V2 0.83

V3 0.77

V4 0.69

V5 0.60

V6 0.50

R for 1 component

```
c1 <- c[,1]
```

```
> round(c1 %*% t(c1),2)
```

	V1	V2	V3	V4	V5	V6
[1,]	0.78	0.74	0.68	0.61	0.53	0.44
[2,]	0.74	0.69	0.64	0.58	0.50	0.42
[3,]	0.68	0.64	0.59	0.53	0.47	0.39
[4,]	0.61	0.58	0.53	0.48	0.42	0.35
[5,]	0.53	0.50	0.47	0.42	0.37	0.30
[6,]	0.44	0.42	0.39	0.35	0.30	0.25

R - CC' for 1 component

	V1	V2	V3	V4	V5	V6
V1	0.22	-0.02	-0.05	-0.07	-0.08	-0.08
V2	-0.02	0.31	-0.08	-0.10	-0.10	-0.10
V3	-0.05	-0.08	0.41	-0.11	-0.12	-0.11
V4	-0.07	-0.10	-0.11	0.52	-0.12	-0.11
V5	-0.08	-0.10	-0.12	-0.12	0.63	-0.10
V6	-0.08	-0.10	-0.11	-0.11	-0.10	0.75

$$\text{Fit} = 1 - R^2/R^2 = .85$$

R for 2 components

```
> c2 <- c[,1:2]
```

```
> round(c2 %*% t(c2),2)
```

	V1	V2	V3	V4	V5	V6
V1	0.78	0.74	0.68	0.62	0.55	0.40
V2	0.74	0.70	0.65	0.59	0.52	0.36
V3	0.68	0.65	0.60	0.55	0.49	0.31
V4	0.62	0.59	0.55	0.50	0.45	0.24
V5	0.55	0.52	0.49	0.45	0.44	0.07
V6	0.40	0.36	0.31	0.24	0.07	0.97

R- CC' for 2 components

```
> round(r-c2 %*% t(c2),2)
```

	V1	V2	V3	V4	V5	V6
V1	0.22	-0.02	-0.05	-0.08	-0.10	-0.04
V2	-0.02	0.30	-0.09	-0.11	-0.12	-0.04
V3	-0.05	-0.09	0.40	-0.13	-0.14	-0.03
V4	-0.08	-0.11	-0.13	0.50	-0.15	0.00
V5	-0.10	-0.12	-0.14	-0.15	0.56	0.13
V6	-0.04	-0.04	-0.03	0.00	0.13	0.03

```
$fit
```

```
[1] 0.9
```


Factor Analysis as eigen value decomposition of reduced matrix

I. Estimate the R as $FF' + U^2$

A. U^2 is the amount not accounted for by the factor and is a diagonal matrix

II. Estimate $(R - U^2) = FF'$ using eigen value model

III. But what is the best value of U^2

IV. Solve for iteratively

The PC model - first pass

```
> round(pc1$loadings %*%  
t(pc1$loadings), 2)
```

	V1	V2	V3	V4	V5	V6
V1	0.77	0.73	0.68	0.61	0.53	0.44
V2	0.73	0.69	0.64	0.57	0.50	0.42
V3	0.68	0.64	0.59	0.53	0.46	0.38
V4	0.61	0.57	0.53	0.48	0.41	0.34
V5	0.53	0.50	0.46	0.41	0.36	0.30
V6	0.44	0.42	0.38	0.34	0.30	0.25

First iteration = PC

\$loadings		\$residual						
PAI		V1	V2	V3	V4	V5	V6	
V1	0.88	V1	0.22	-0.02	-0.05	-0.07	-0.08	-0.08
V2	0.83	V2	-0.02	0.31	-0.08	-0.10	-0.10	-0.10
V3	0.77	V3	-0.05	-0.08	0.41	-0.11	-0.12	-0.11
V4	0.69	V4	-0.07	-0.10	-0.11	0.52	-0.12	-0.11
V5	0.60	V5	-0.08	-0.10	-0.12	-0.12	0.63	-0.10
V6	0.50	V6	-0.08	-0.10	-0.11	-0.11	-0.10	0.75

\$communality
[1] 3.16

\$fit
[1] 0.85

\$fitoff
[1] 0.95

2nd iteration

\$loadings

PAI

V1 0.88

V2 0.81

V3 0.72

V4 0.62

V5 0.52

V6 0.41

\$residual

V1

V2

V3

V4

V5

V6

V1	0.22	0.01	0.00	-0.01	0.00	0.00
V2	0.01	0.35	-0.02	-0.02	-0.02	-0.01
V3	0.00	-0.02	0.48	-0.03	-0.02	-0.01
V4	-0.01	-0.02	-0.03	0.61	-0.02	-0.01
V5	0.00	-0.02	-0.02	-0.02	0.73	-0.01
V6	0.00	-0.01	-0.01	-0.01	-0.01	0.83

\$communality

[1] 3.16 2.77

\$fit

[1] 0.83

\$fitoff

[1] 1

3rd iteration

\$loadings

PAI

V1 0.89

V2 0.80

V3 0.71

V4 0.61

V5 0.50

V6 0.40

\$residual

V1

V2

V3

V4

V5

V6

V1 0.21 0.01 0.00 0.00 0.00 0.00

V2 0.01 0.36 -0.01 -0.01 0.00 0.00

V3 0.00 -0.01 0.50 -0.01 -0.01 0.00

V4 0.00 -0.01 -0.01 0.63 0.00 0.00

V5 0.00 0.00 -0.01 0.00 0.75 0.00

V6 0.00 0.00 0.00 0.00 0.00 0.84

\$communality

[1] 3.16 2.77 2.71

\$fit

[1] 0.82

\$fitoff

[1] 1

4th iteration

\$loadings		\$residual						
PAI			V1	V2	V3	V4	V5	V6
V1	0.89	V1	0.2	0.00	0.00	0.00	0.00	0.00
V2	0.80	V2	0.0	0.36	0.00	0.00	0.00	0.00
V3	0.70	V3	0.0	0.00	0.51	0.00	0.00	0.00
V4	0.60	V4	0.0	0.00	0.00	0.64	0.00	0.00
V5	0.50	V5	0.0	0.00	0.00	0.00	0.75	0.00
V6	0.40	V6	0.0	0.00	0.00	0.00	0.00	0.84

\$communality
[1] 3.16 2.77 2.71 2.71

\$fit
[1] 0.82

\$fitoff
[1] 1

5th iteration

		\$residual					
\$loadings		V1	V2	V3	V4	V5	V6
PAI	V1	0.2	0.00	0.00	0.00	0.00	0.00
V1 0.9	V2	0.0	0.36	0.00	0.00	0.00	0.00
V2 0.8	V3	0.0	0.00	0.51	0.00	0.00	0.00
V3 0.7	V4	0.0	0.00	0.00	0.64	0.00	0.00
V4 0.6	V5	0.0	0.00	0.00	0.00	0.75	0.00
V5 0.5	V6	0.0	0.00	0.00	0.00	0.00	0.84
V6 0.4							
\$communality		\$fit		\$fitoff			
[1]	3.16 2.77 2.71 2.71 2.71	[1] 0.82		[1] 1			

EFA and communalities

- Exploratory Factor analysis vs. Confirmatory Factor Analysis
- The question of communalities and number of factors
- Alternative estimates of starting values for communalities
 - 1
 - max
 - Squared Multiple Correlation (smc)

Squared Multiple Correlation

- What is the amount of variance in a variable predicted by all the other variables in a matrix?
- Residual is the inverse of the diagonal of the inverse of R.
- $SMC = I - (\text{diag}(R^{-1}))^{-1}$
- found by $SMC = \text{smc}(R)$
- $\text{round}(\text{smc}(R), 2)$
- V1 V2 V3 V4 V5 V6
- 0.64 0.55 0.43 0.32 0.22 0.14

Estimates of communality

	load	h2	smc	max	max2
V1	0.9	0.81	0.64	0.72	0.52
V2	0.8	0.64	0.55	0.72	0.52
V3	0.7	0.49	0.43	0.63	0.40
V4	0.6	0.36	0.32	0.54	0.29
V5	0.5	0.25	0.22	0.45	0.20
V6	0.4	0.16	0.14	0.36	0.13

Principal axis and smc

Loadings as a function of iteration

	it1	it2	it3	ite4
V1	0.86	0.88	0.89	0.9
V2	0.79	0.80	0.80	0.8
V3	0.70	0.70	0.70	0.7
V4	0.60	0.60	0.60	0.6
V5	0.50	0.50	0.50	0.5
V6	0.40	0.40	0.40	0.4

\$communality.iterations
1] 2.62 2.68 2.70 2.71 2.71 2.71
compare to using 1s

\$communality.iterations
[1] 3.16 2.77 2.71 2.71 2.71

Why discuss EFA?

- EFA is an latent variable model
- What is latent?
 - Communalities as sum of true variances
 - factor loadings as predictors of communalities and covariances

Consider another matrix

	V1	V2	V3	V4	V5	V6
V1	1.00	0.72	0.63	0.00	0.00	0.00
V2	0.72	1.00	0.56	0.00	0.00	0.00
V3	0.63	0.56	1.00	0.00	0.00	0.00
V4	0.00	0.00	0.00	1.00	0.56	0.48
V5	0.00	0.00	0.00	0.56	1.00	0.42
V6	0.00	0.00	0.00	0.48	0.42	1.00

$$R = FF' \quad (\text{one factor})$$

round(pa1 %*% t(pa1),2)

\$loadings		V1	V2	V3	V4	V5	V6
PA1	V1	0.79	0.72	0.62	0	0	0
V1 0.89	V2	0.72	0.66	0.57	0	0	0
V2 0.81	V3	0.62	0.57	0.49	0	0	0
V3 0.70	V4	0.00	0.00	0.00	0	0	0
V4 0.00	V5	0.00	0.00	0.00	0	0	0
V5 0.00	V6	0.00	0.00	0.00	0	0	0
V6 0.00							

Factor with 1 factor

		\$residual					
\$loadings		V1	V2	V3	V4	V5	V6
PAI	V1	0.21	0.00	0.00	0.00	0.00	0.00
V1 0.89	V2	0.00	0.35	-0.01	0.00	0.00	0.00
V2 0.81	V3	0.00	-0.01	0.50	0.00	0.00	0.00
V3 0.70	V4	0.00	0.00	0.00	1.00	0.56	0.48
V4 0.00	V5	0.00	0.00	0.00	0.56	1.00	0.42
V5 0.00	V6	0.00	0.00	0.00	0.48	0.42	1.00
V6 0.00							

\$communality						\$fit	\$fitoff
[1]	2.28	2.04	1.96	1.94	1.94	[1] 0.51	[1] 0.63

$$R \approx FF' \text{ (2 factors)}$$

			r round(pa2 %*% t(pa2),2)						
\$loadings				V1	V2	V3	V4	V5	V6
	PA1	PA2							
V1	0.89	0.00	V1	0.79	0.71	0.62	0.00	0.00	0.00
V2	0.80	0.00	V2	0.71	0.64	0.56	0.00	0.00	0.00
V3	0.70	0.00	V3	0.62	0.56	0.49	0.00	0.00	0.00
V4	0.00	0.79	V4	0.00	0.00	0.00	0.62	0.55	0.47
V5	0.00	0.70	V5	0.00	0.00	0.00	0.55	0.49	0.42
V6	0.00	0.60	V6	0.00	0.00	0.00	0.47	0.42	0.36

Try 2 factors!

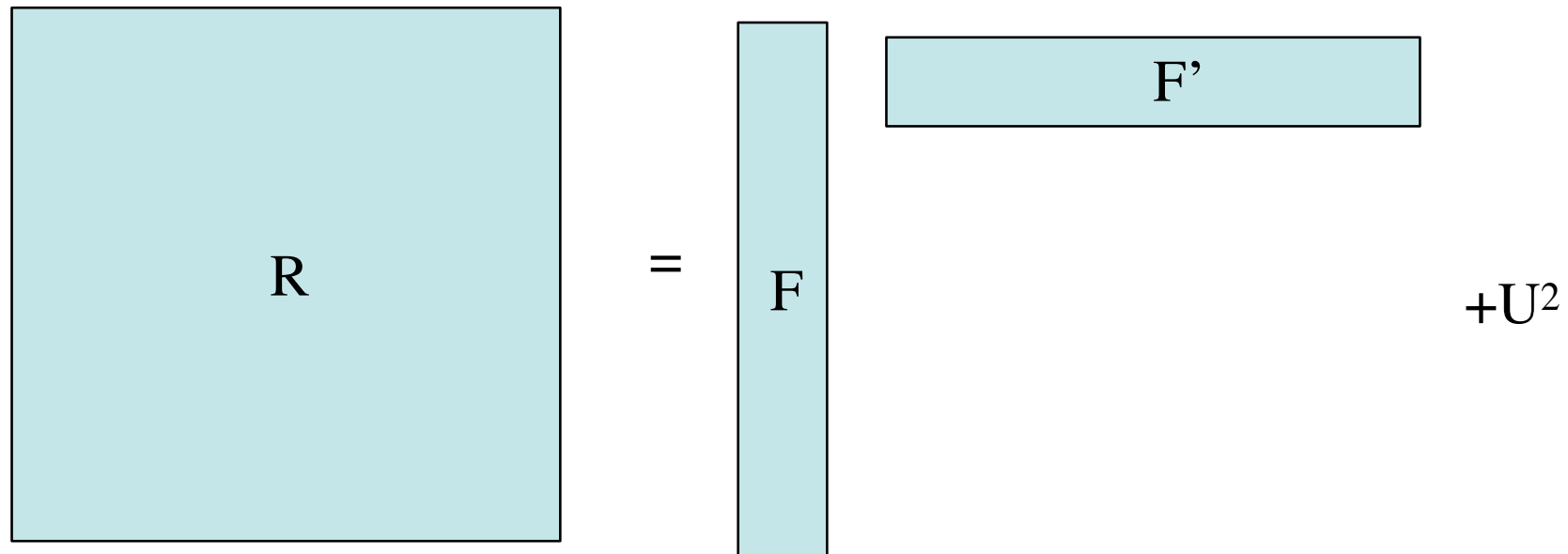
			\$residual						
\$loadings				v1	v2	v3	v4	v5	v6
	PA1	PA2							
V1	0.89	0.00	V1	0.2	0.00	0.00	0.00	0.0	0.00
V2	0.80	0.00	V2	0.0	0.35	-0.01	0.00	0.0	0.00
V3	0.70	0.00	V3	0.0	-0.01	0.51	0.00	0.0	0.00
V4	0.00	0.79	V4	0.0	0.00	0.00	0.37	0.0	0.00
V5	0.00	0.70	V5	0.0	0.00	0.00	0.00	0.5	0.00
V6	0.00	0.60	V6	0.0	0.00	0.00	0.00	0.0	0.64

\$communality							\$fit	\$fitoff
[1]	4.25	3.68	3.50	3.44	3.43	3.42	[1] 0.88	[1] 1

Factors as causes; variables as indicators of factors

- Can we recreate the correlation matrix R (of rank n) with a matrix F of rank $1 +$ a diagonal matrix of uniqueness U^2
- $R \approx FF' + U^2$
- Residual Matrix $R^* = R - (FF' + U^2)$
- Try to minimize the residual

Factor analysis



Factor Analysis: the model

I. $R \approx FF' + U^2$

II. Residual Matrix $R^* = R - (FF' + U^2)$

III. Try to minimize the residual

IV. Variables are linear composites of unknown (latent) factors.

V. Covariance structures of observables in terms of covariance of unobservables

Factor analysis of mood

- What is the structure of affect?
- Limited number of variables taken from a larger set

Structure of mood - how not to display data

	AFRAID	AT_EASE	CALM	ENERGETI	HAPPY	LIVELY	SLEEPY	TENSE	TIRED
AFRAID	1.000								
AT_EASE	-0.209	1.000							
CALM	-0.157	0.586	1.000						
ENERGETI	0.019	0.230	0.056	1.000					
HAPPY	-0.070	0.452	0.294	0.595	1.000				
LIVELY	0.018	0.255	0.073	0.778	0.609	1.000			
SLEEPY	0.087	-0.112	0.031	-0.457	-0.264	-0.405	1.000		
TENSE	0.397	-0.337	-0.332	0.088	-0.103	0.084	0.044	1.000	
TIRED	0.082	-0.141	0.012	-0.484	-0.297	-0.439	0.808	0.044	1.000
UNHAPPY	0.350	-0.283	-0.187	-0.185	-0.314	-0.187	0.202	0.360	0.235

Structure of mood:

“Alabama need not come first”

	AFRAID	AT_EASE	CALM	ENERGETI	HAPPY	LIVELY	SLEEPY	TENSE	TIRED
AFRAID	1.0								
AT_EASE	-0.2	1.0							
CALM	-0.2	0.6	1.0						
ENERGETI	0.0	0.2	0.1	1.0					
HAPPY	-0.1	0.5	0.3	0.6	1.0				
LIVELY	0.0	0.3	0.1	0.8	0.6	1.0			
SLEEPY	0.1	-0.1	0.0	-0.5	-0.3	-0.4	1.0		
TENSE	0.4	-0.3	-0.3	0.1	-0.1	0.1	0.0	1.0	
TIRED	0.1	-0.1	0.0	-0.5	-0.3	-0.4	0.8	0.0	1.0
UNHAPPY	0.3	-0.3	-0.2	-0.2	-0.3	-0.2	0.2	0.4	0.2

Structure of mood data

	ENERGETI	LIVELY	TIRED	SLEEPY	AFRAID	TENSE	AT_EASE	CALM	HAPPY
ENERGETI	1								
LIVELY	0.8	1							
TIRED	-0.5	-0.4	1						
SLEEPY	-0.5	-0.4	0.8	1					
AFRAID	0	0	0.1	0.1	1				
TENSE	0.1	0.1	0	0	0.4	1			
AT_EASE	0.2	0.3	-0.1	-0.1	-0.2	-0.3	1		
CALM	0.1	0.1	0	0	-0.2	-0.3	0.6	1	
HAPPY	0.6	0.6	-0.3	-0.3	-0.1	-0.1	0.5	0.3	1
UNHAPPY	-0.2	-0.2	0.2	0.2	0.3	0.4	-0.3	-0.2	-0.3

NUMBER OF OBSERVATIONS: 3748

Correlation of mood data

possible structure

	ENERGETI	LIVELY	TIRED	SLEEPY	AFRAID	TENSE	AT_EASE	CALM	HAPPY
ENERGETI	1.0								
LIVELY	0.8	1.0							
TIRED	-0.5	-0.4	1.0						
SLEEPY	-0.5	-0.4	0.8	1.0					
AFRAID	0.0	0.0	0.1	0.1	1.0				
TENSE	0.1	0.1	0.0	0.0	0.4	1.0			
AT_EASE	0.2	0.3	-0.1	-0.1	-0.2	-0.3	1.0		
CALM	0.1	0.1	0.0	0.0	-0.2	-0.3	0.6	1.0	
HAPPY	0.6	0.6	-0.3	-0.3	-0.1	-0.1	0.5	0.3	1.0
UNHAPPY	-0.2	-0.2	0.2	0.2	0.3	0.4	-0.3	-0.2	-0.3

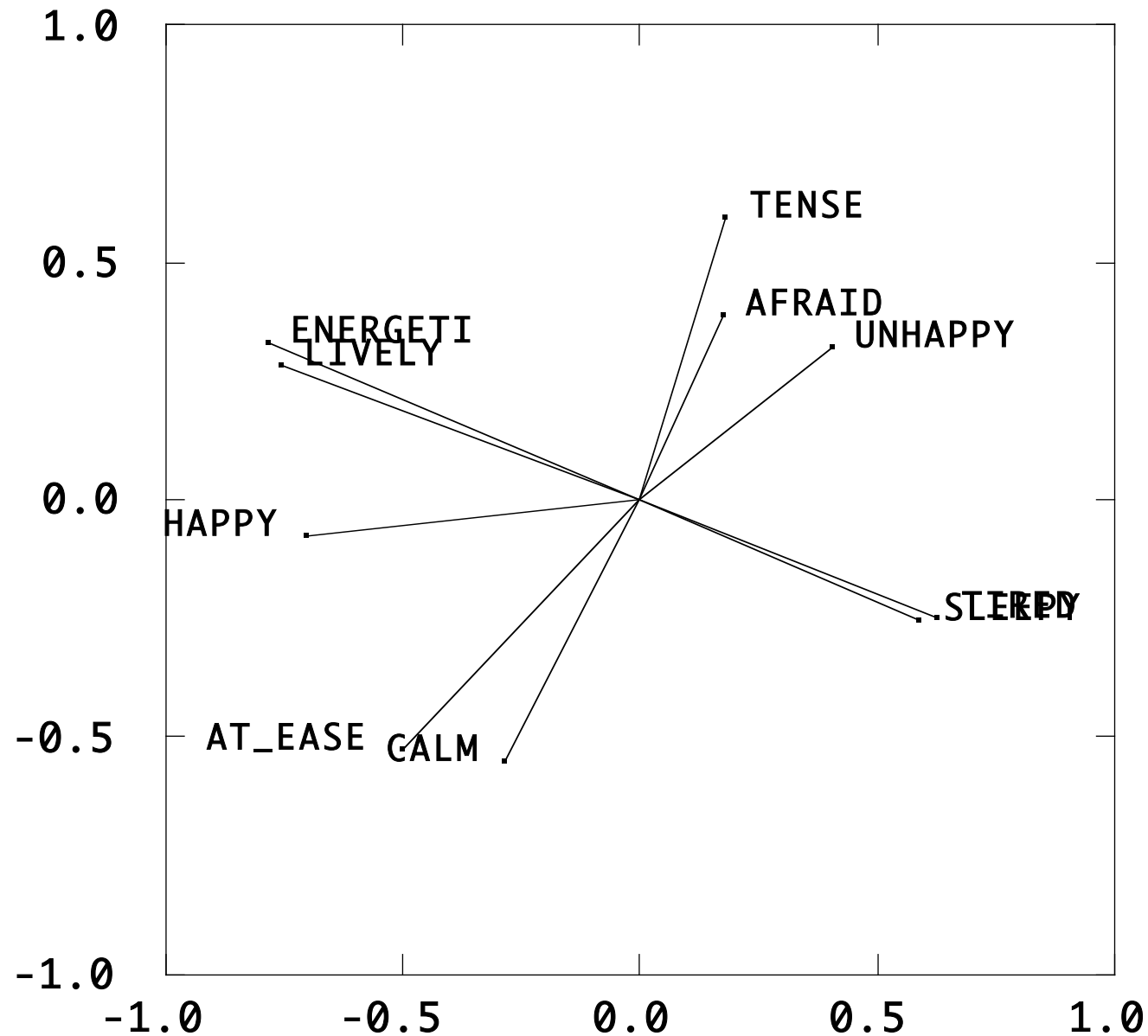
NUMBER OF OBSERVATIONS: 3748

Factor analysis 2 factor solution

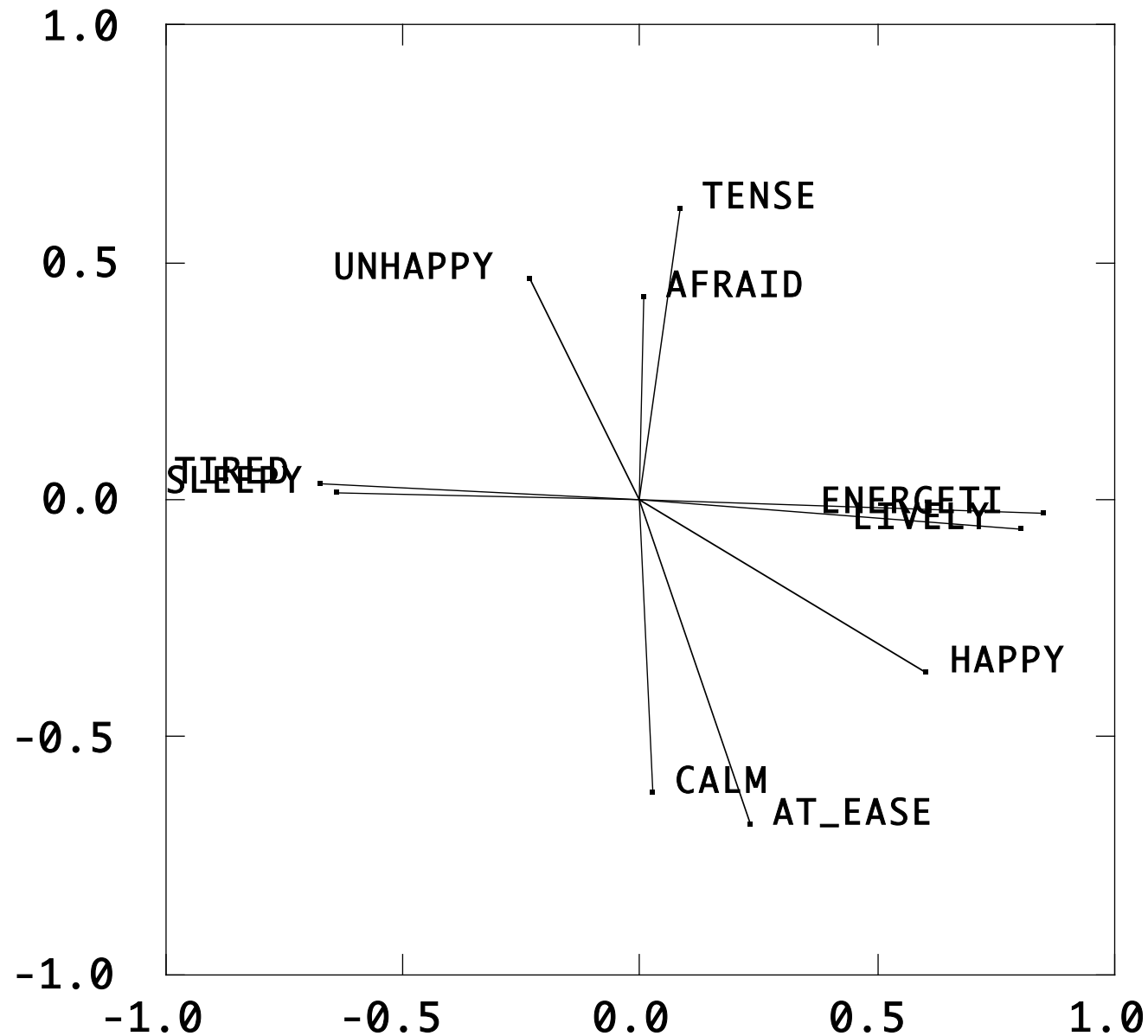
FACTOR PATTERN

	1	2	h2
ENERGETI	-0.8	0.3	0.73
LIVELY	-0.8	0.3	0.73
HAPPY	-0.7	-0.1	0.50
TIRED	0.6	-0.3	0.45
SLEEPY	0.6	-0.3	0.45
TENSE	0.2	0.6	0.40
CALM	-0.3	-0.6	0.45
AT_EASE	-0.5	-0.5	0.50
AFRAID	0.2	0.4	0.20
UNHAPPY	0.4	0.3	0.25
VARIANCE EXP	3.0	1.50	

2 factors of mood



2 factors of mood (rotated)

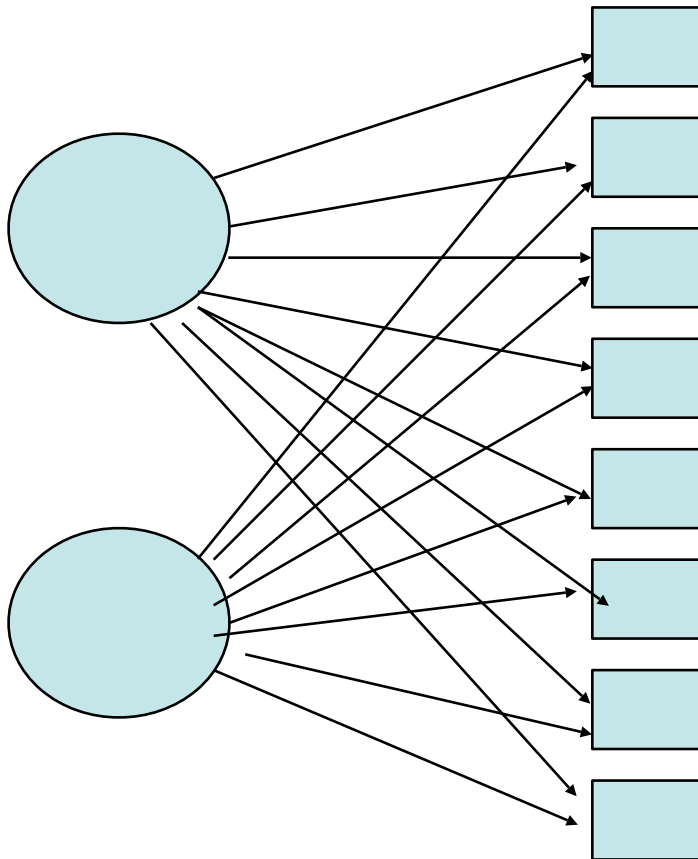


Rotation as orthogonal transformation

	F1	F2	F1'	F2'
ENERGETI	-0.8	0.3	0.8	0.0
LIVELY	-0.8	0.3	0.8	-0.1
HAPPY	-0.7	-0.1	0.6	-0.4
TIRED	0.6	-0.3	-0.7	0.0
SLEEPY	0.6	-0.3	-0.6	0.0
TENSE	0.2	0.6	0.1	0.6
CALM	-0.3	-0.6	0.0	-0.6
AT_EASE	-0.5	-0.5	0.2	-0.7
AFRAID	0.2	0.4	0.0	0.4
UNHAPPY	0.4	0.3	-0.2	0.5
eigen values	3	1.5	2.7	1.8

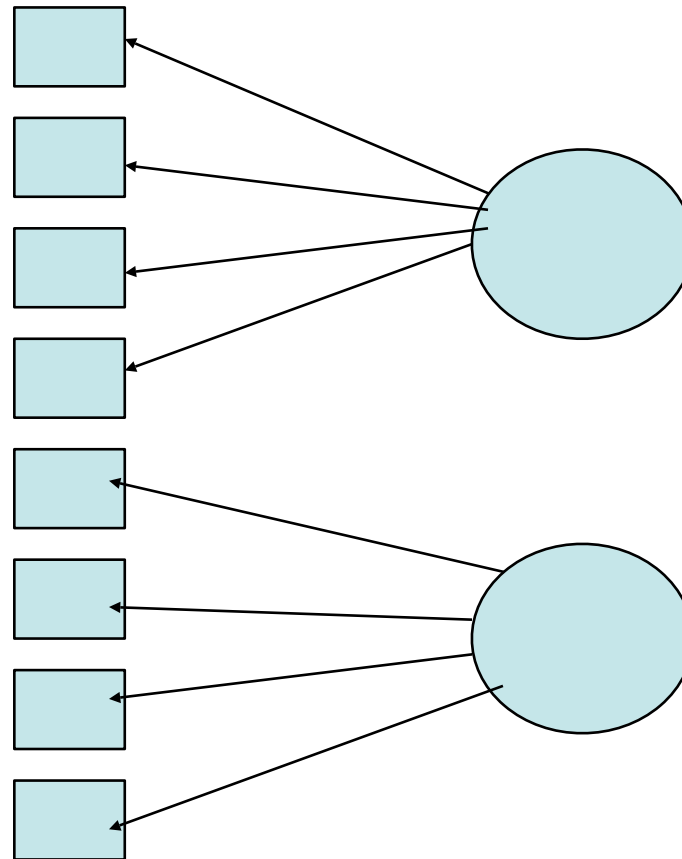
Rotation to simple structure

Original



Rotation to simple structure

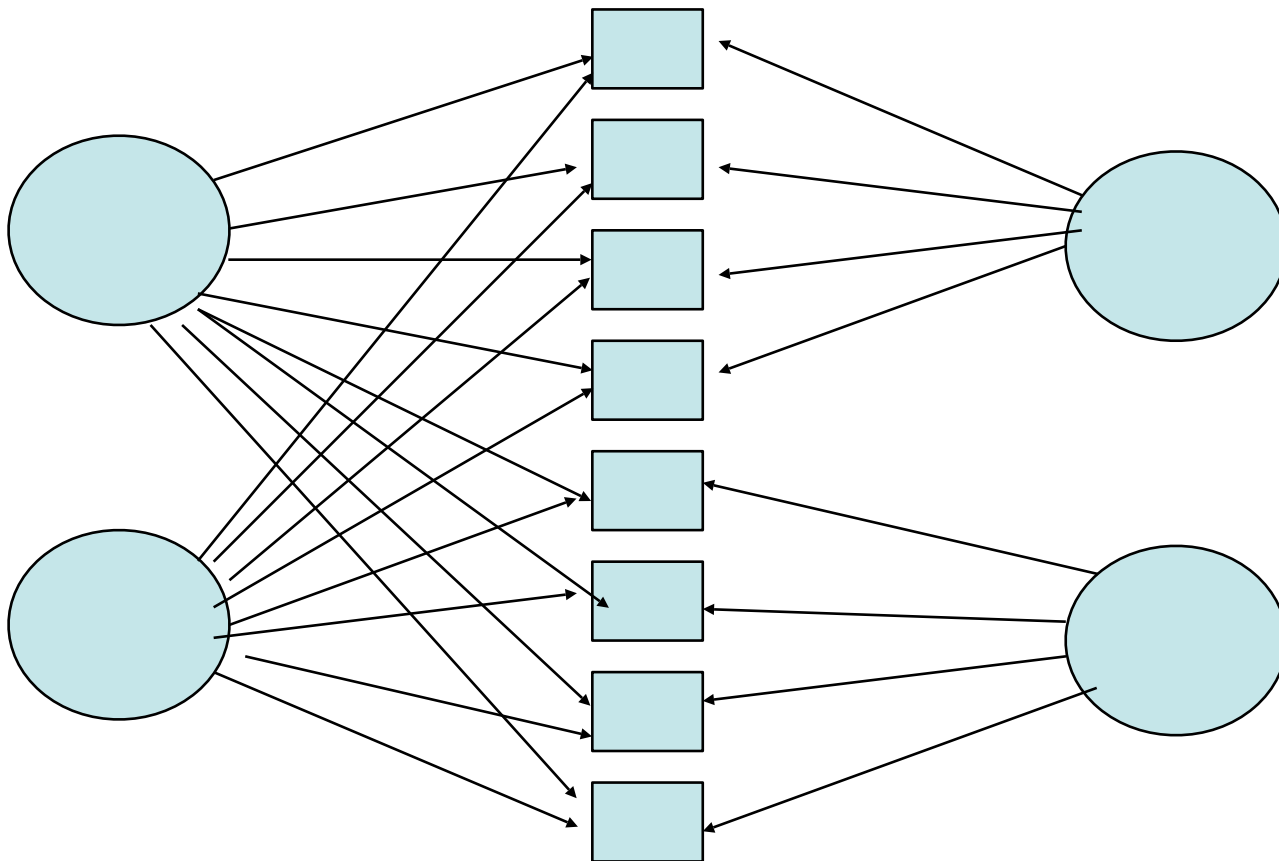
Simple Structure



Rotation to simple structure

Original

Rotated



Principal components

I. $R \approx CC'$

II. Residual Matrix $R^* = R - (CC')$

III. Try to minimize the residual

IV. Components are linear composites of known variables.

V. Covariance structures of observables in terms of covariance of observables

VI. Components account for observed variance

2 Principal Components of mood

	C1	C2
ENERGETI	-0.8	-0.4
LIVELY	-0.8	-0.3
HAPPY	-0.8	0
TIRED	0.7	0.3
SLEEPY	0.7	0.3
AT_EASE	-0.6	0.5
TENSE	0.2	-0.7
CALM	-0.3	0.6
AFRAID	0.2	-0.5
UNHAPPY	0.5	-0.4
eigen values	3.4	2.1

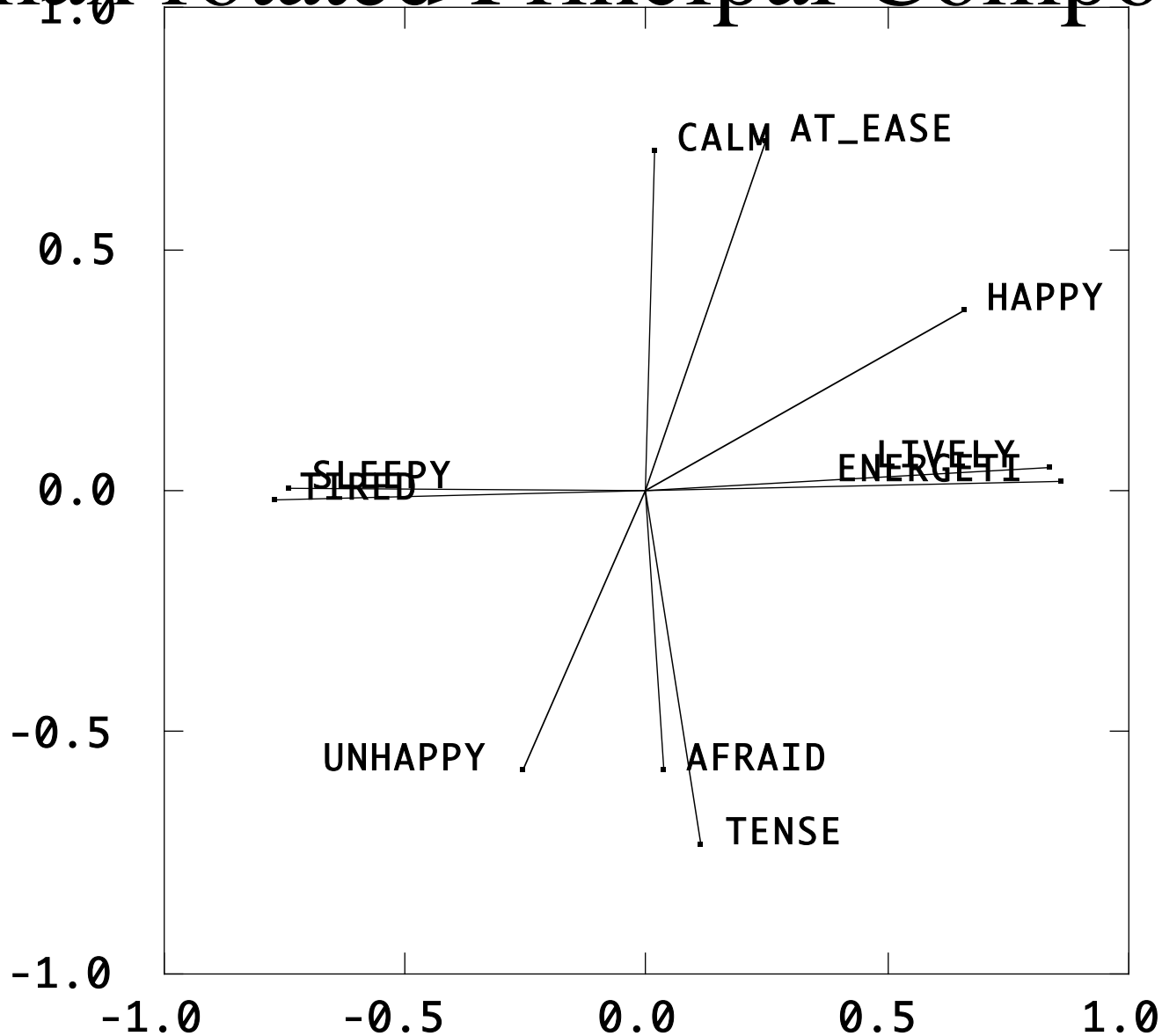
Unrotated and rotated PCs

	C1	C2	C1'	C2'
ENERGETI	-0.8	-0.4	0.9	0
LIVELY	-0.8	-0.3	0.8	0
HAPPY	-0.8	0	0.7	0.4
TIRED	0.7	0.3	-0.8	0
SLEEPY	0.7	0.3	-0.7	0
AT_EASE	-0.6	0.5	0.2	0.7
TENSE	0.2	-0.7	0.1	-0.7
CALM	-0.3	0.6	0	0.7
AFRAID	0.2	-0.5	0	-0.6
UNHAPPY	0.5	-0.4	-0.3	-0.6
eigen values	3.4	2.1	3.2	2.4

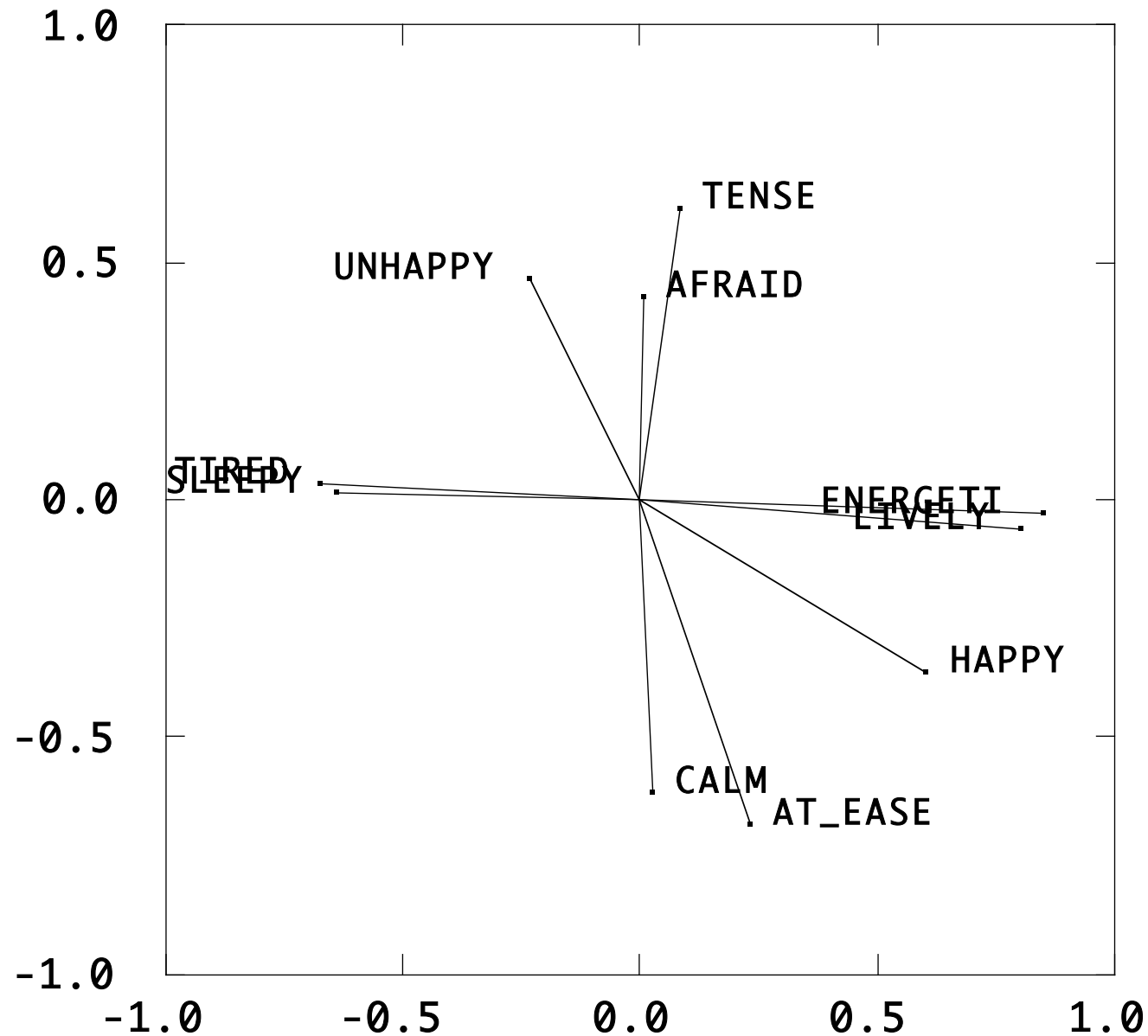
Varimax factors and components

	F1'	F2'	C1'	C2'
ENERGETI	0.8	0.0	0.9	0.0
LIVELY	0.8	-0.1	0.8	0.0
HAPPY	0.6	-0.4	0.7	0.4
TIRED	-0.7	0.0	-0.8	0.0
SLEEPY	-0.6	0.0	-0.7	0.0
AT_EASE	0.2	-0.7	0.2	0.7
TENSE	0.1	0.6	0.1	-0.7
CALM	0.0	-0.6	0.0	0.7
AFRAID	0.0	0.4	0.0	-0.6
UNHAPPY	-0.2	0.5	-0.3	-0.6
eigen values	2.7	1.8	3.2	2.4

Varimax rotated Principal Components



Varimax rotated factors of mood



Components vs. Factors

- Factors are theoretical constructs
- Components are observed linear sums
- Factors do not change when new variables are added
- Components change

Consider this matrix

```
> f <- matrix(c(.9,.8,.7,rep(0,6),.8,.7,.6),ncol=2) #the model  
> rownames(f) <- paste("V",seq(1:6),sep="") #add labels  
> colnames(f) <- c("F1", "F2")  
> R <- f %*% t(f) #create the correlation matrix  
> diag(R) <- 1 #adjust the diagonal of the matrix  
> R
```

	V1	V2	V3	V4	V5	V6
V1	1.00	0.72	0.63	0.00	0.00	0.00
V2	0.72	1.00	0.56	0.00	0.00	0.00
V3	0.63	0.56	1.00	0.00	0.00	0.00
V4	0.00	0.00	0.00	1.00	0.56	0.48
V5	0.00	0.00	0.00	0.56	1.00	0.42
V6	0.00	0.00	0.00	0.48	0.42	1.00


```
factanal(factors = 2, covmat = R)
```

V1	V2	V3	V4	V5	V6
0.19	0.36	0.51	0.36	0.51	0.64

Loadings:

	Factor1	Factor2
--	---------	---------

V1	0.9	
----	-----	--

V2	0.8	
----	-----	--

V3	0.7	
----	-----	--

V4		0.8
----	--	-----

V5		0.7
----	--	-----

V6		0.6
----	--	-----

	Factor1	Factor2
SS loadings	1.940	1.490

Proportion Var	0.323	0.248
----------------	-------	-------

Cumulative Var	0.323	0.572
----------------	-------	-------

```
pc <- principal(R,2)
```

	PC1	PC2
V1	0.90	
V2	0.88	
V3	0.83	
V4		0.85
V5		0.82
V6		0.77

	PC1	PC2
SS loadings	2.273	1.988
Proportion Var	0.379	0.331
Cumulative Var	0.379	0.710

Add two variables

	V1	V2	V3	V4	V5	V6	V7	V8
V1	1.00	0.72	0.63	0.00	0.00	0.00	0.63	0.00
V2	0.72	1.00	0.56	0.00	0.00	0.00	0.56	0.00
V3	0.63	0.56	1.00	0.00	0.00	0.00	0.49	0.00
V4	0.00	0.00	0.00	1.00	0.56	0.48	0.00	0.40
V5	0.00	0.00	0.00	0.56	1.00	0.42	0.00	0.35
V6	0.00	0.00	0.00	0.48	0.42	1.00	0.00	0.30
V7	0.63	0.56	0.49	0.00	0.00	0.00	1.00	0.00
V8	0.00	0.00	0.00	0.40	0.35	0.30	0.00	1.00

Factors don't change

	Factor1	Factor2
V1	0.9	
V2	0.8	
V3	0.7	
V4		0.8
V5		0.7
V6		0.6

	Factor1	Factor2
V1	0.9	
V2	0.8	
V3	0.7	
V4		0.8
V5		0.7
V6		0.6
V7	0.7	
V8		0.5

	Factor1	Factor2
SS loadings	1.940	1.490
Proportion Var	0.323	0.248
Cumulative Var	0.323	0.572

	Factor1	Factor2
SS loadings	2.430	1.740
Proportion Var	0.304	0.218
Cumulative Var	0.304	0.52

Components do

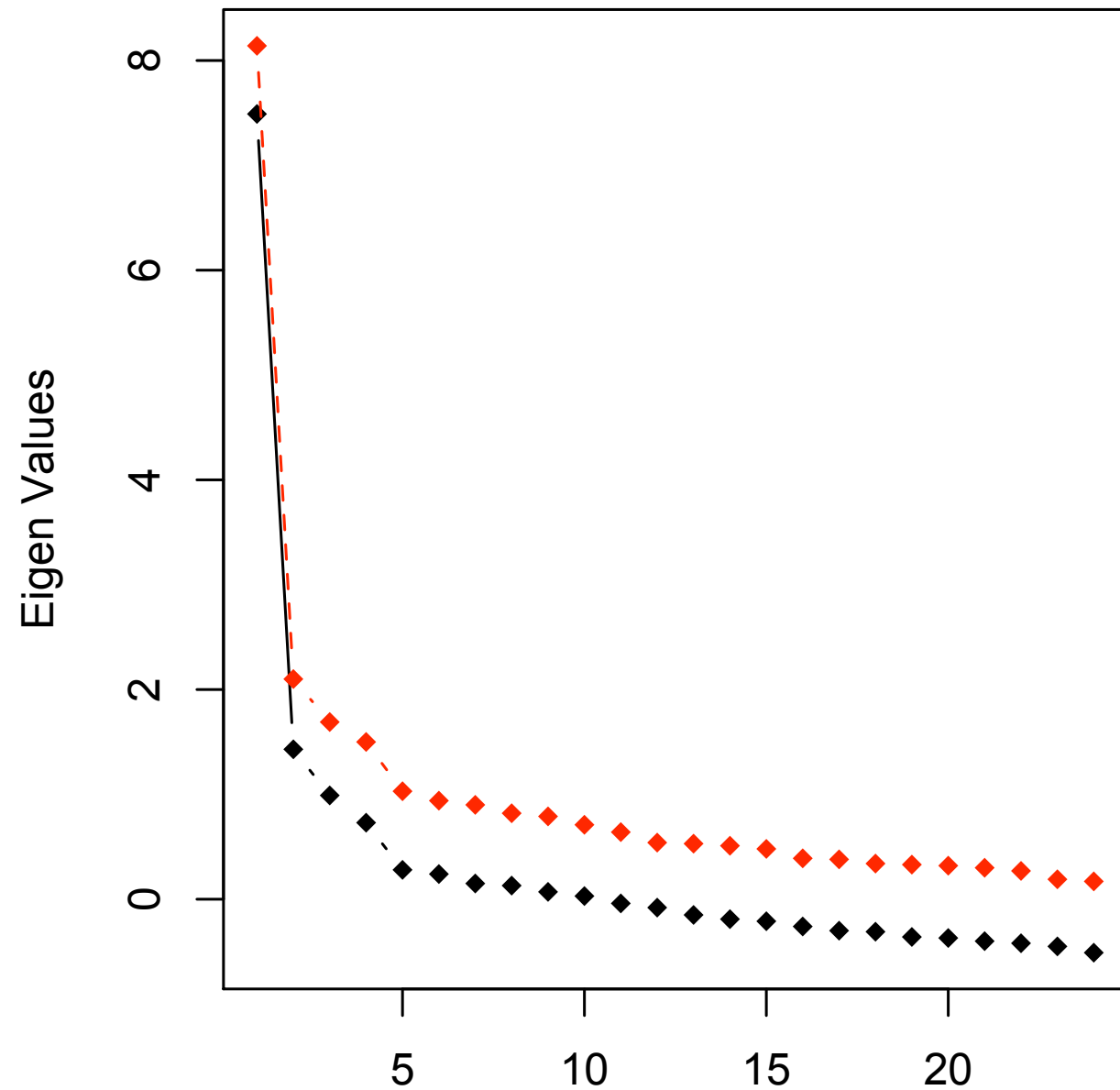
	PC1	PC2
V1	0.90	
V2	0.88	
V3	0.83	
V4		0.85
V5		0.82
V6		0.77

	PC1	PC2
SS loadings	2.273	1.988
Proportion Var	0.379	0.331
Cumulative Var	0.379	0.710

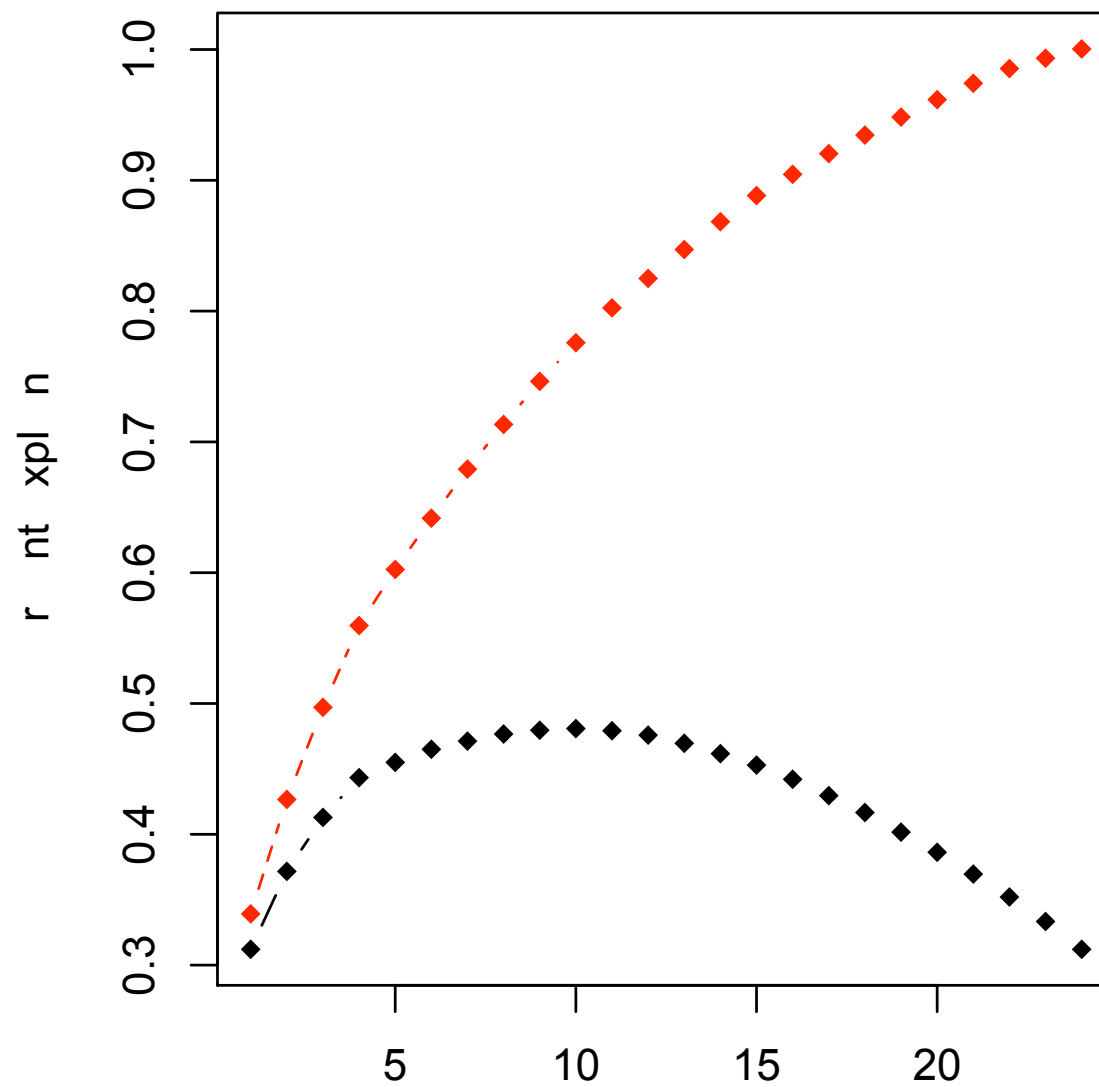
	PC1	PC2
V1	0.90	
V2	0.85	
V3	0.80	
V4		0.83
V5		0.79
V6		0.73
V7	0.80	
V8		0.65

	PC1	PC2
SS loadings	2.812	2.268
Proportion Var	0.352	0.284
Cumulative Var	0.352	0.635

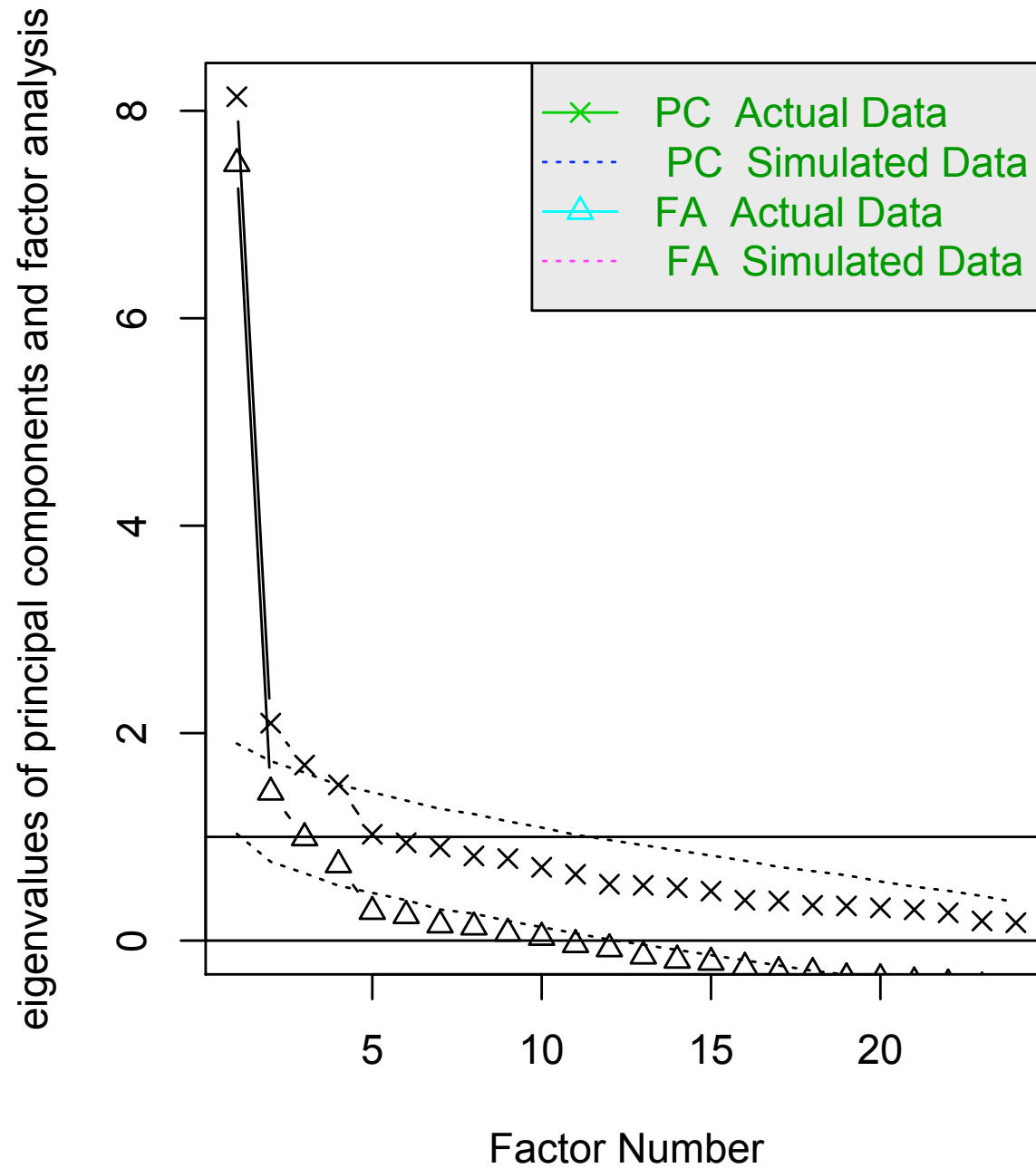
Eigen values of PCs and FAs for 24 mental tests



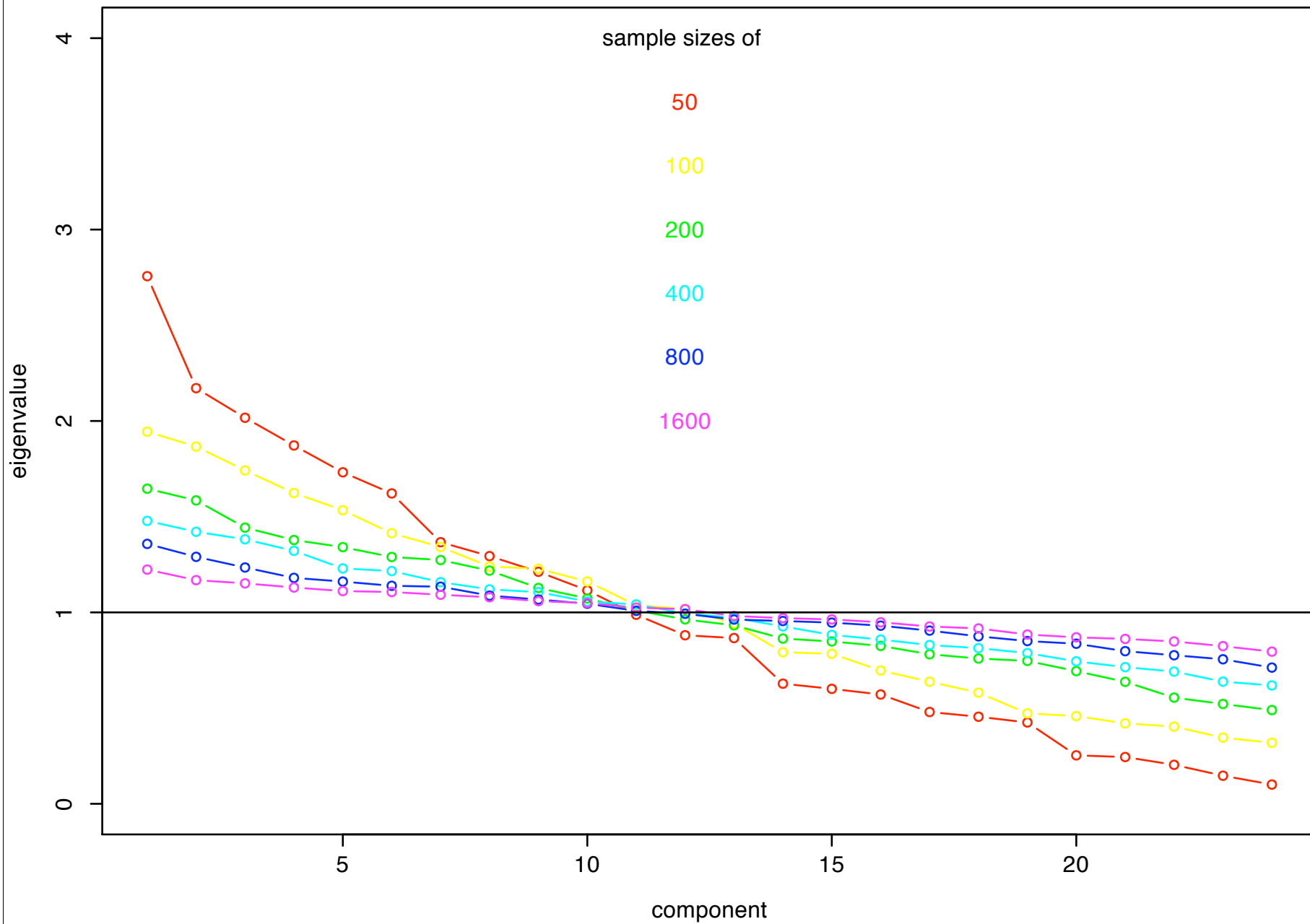
cumulative variance explained 24 mental tests



Parallel Analysis Scree Plots



Eigen values of a random matrix

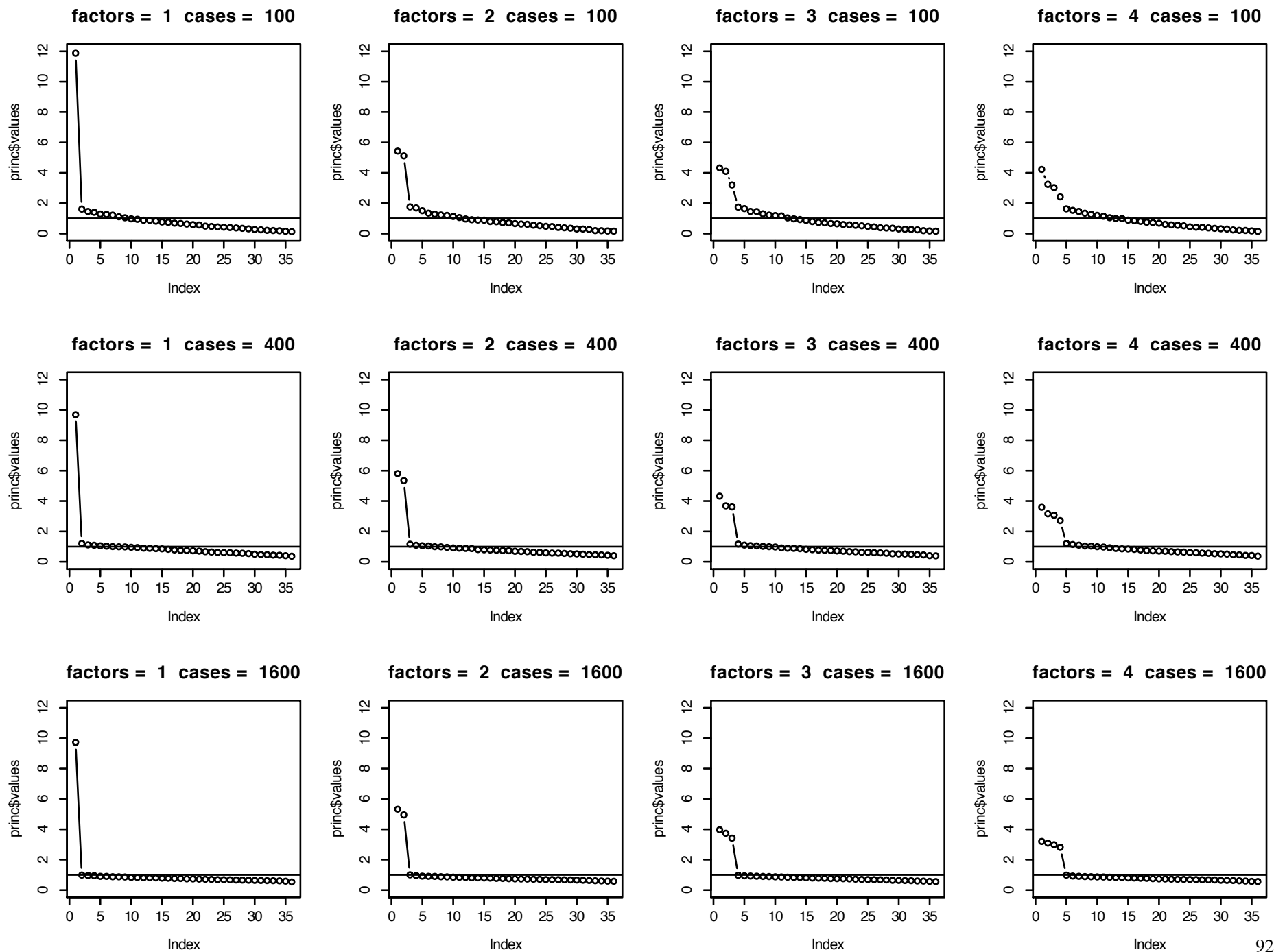


Determining the number of factors/components

- Statistically: extract factors until residual matrix does not differ from 0
 - Sensitive to sample size (large $N \rightarrow$ more factors)
- Theoretically: extract factors that make sense
 - Different theorists differ in their cognitive complexity
- Pragmatically: scree test
- Pragmatically: extract factors with eigen values greater than a random factor matrix
- Pragmatically: extract factors using Very Simple Structure Test or the Minimal Partial Correlation
- Pragmatically: do not use eigen value > 1 rule!

Scree vs. eigen value > 1

- Artificial data with number of factors ranging from 1 to 4
- Number of cases ranges from small (100) to normal (400) to large (1600)
- Item communality set to “typical” = .25
- Simulation using VSS.simulate in VSS “package”
 - `source("http://personality-project.org/r/vss.r")`
 - see <http://personality-project.org/r/vss.html>



Minimum Average Partial (Velicer)

R	C'
C	CC'

 =

R	C'
C	I

Minimize partial R

- $R^* = R - CC'$ is the residual correlation
- $D = \text{diag}(R^*)$ is the variance of residuals
- $R^\# = D^{-.5} R^* D^{-.5}$ (matrix form)

MAP

- $R^* = R - CC'$ is the residual correlation
- $D = \text{diag}(R^*)$ is the variance of residuals
- $R^\# = R^*/\text{sqrt}(D)$ (conceptually)
- $R^\# = D^{-.5} R^* D^{-.5}$ (matrix form)
- Now part of the VSS function

```
>round(harman.vss$map,4)
[1]0.0245 0.0216 0.0175 0.0174 0.0208 0.0237 0.0278 0.0301
>round(circ.vss$map,3)
[1]0.044 0.006 0.008 0.011 0.014 0.017 0.020 0.024
```

Very Simple Structure

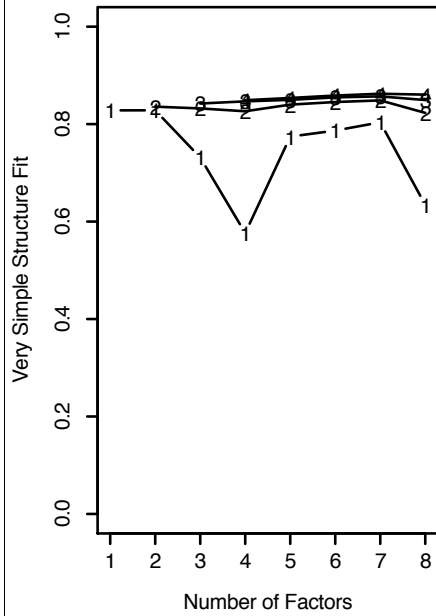
(Revelle and Rocklin)

- Consider the factors as interpreted by user
 - Small loadings are thought to be zero
- How well does this very simple interpretation of the actual structure reproduce the correlation matrix
- $R_{VSS}^* = R - F_{VSS} * F_{VSS}'$
- Consider structures of various levels of simplicity (complexity) (1, 2, ... loadings/item)
- Solution peaks at optimal number of factors

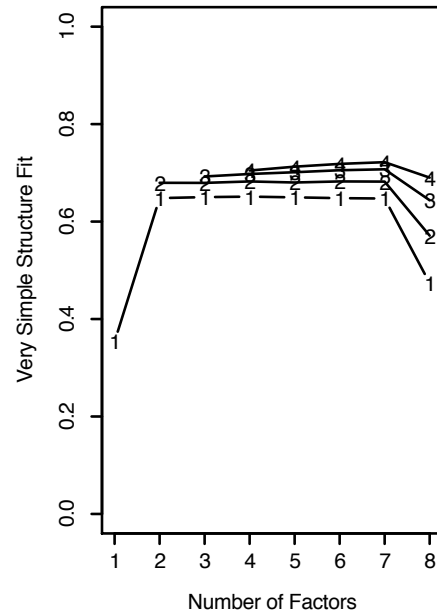
Very Simple Structure

- A quasi confirmatory model
 - Fit the correlation matrix with the simplified structure matrix
 - Simplified $\Rightarrow$ the structure matrix as interpreted
 - small loadings $\rightarrow 0$
 - Could have statistical test, prefer graphical test

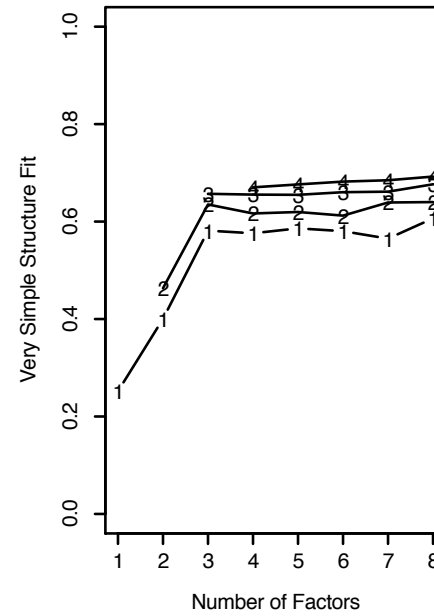
1 factor no rotation



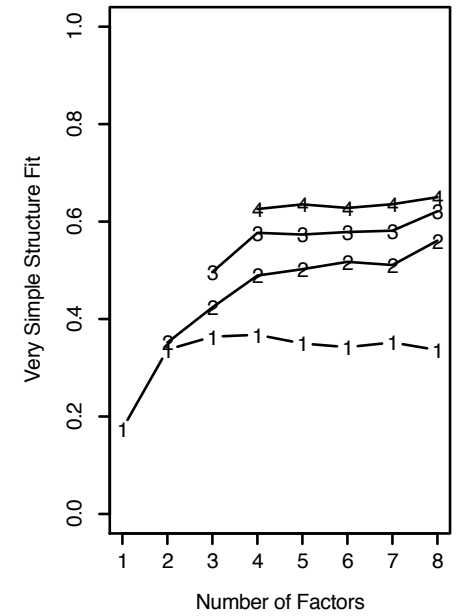
2 factor no rotation



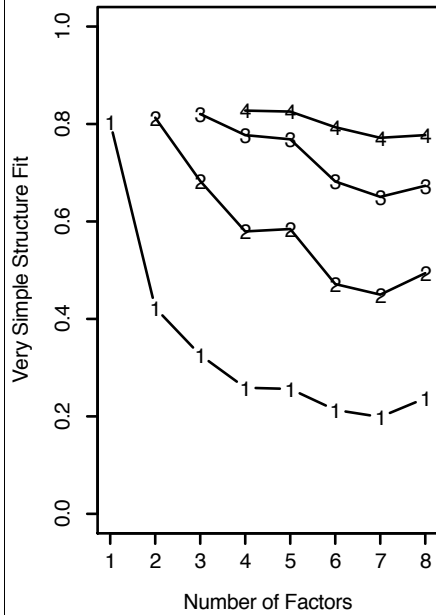
3 factor no rotation



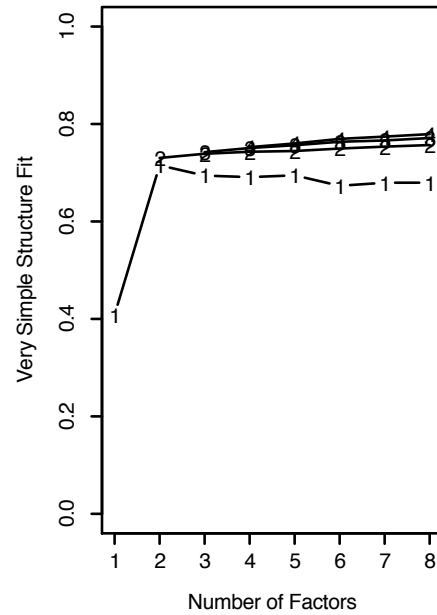
4 factor no rotation



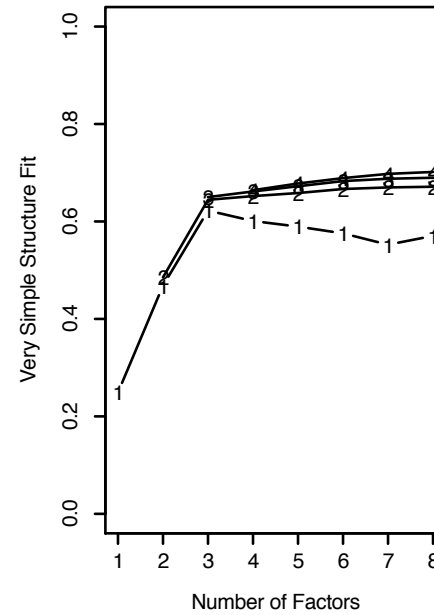
1 factor varimax rotation



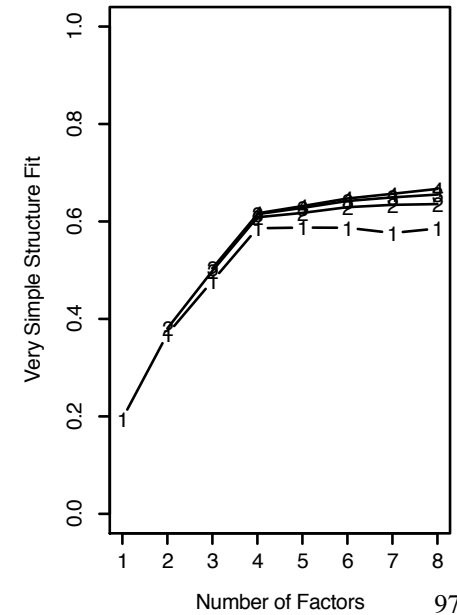
2 factor varimax rotation

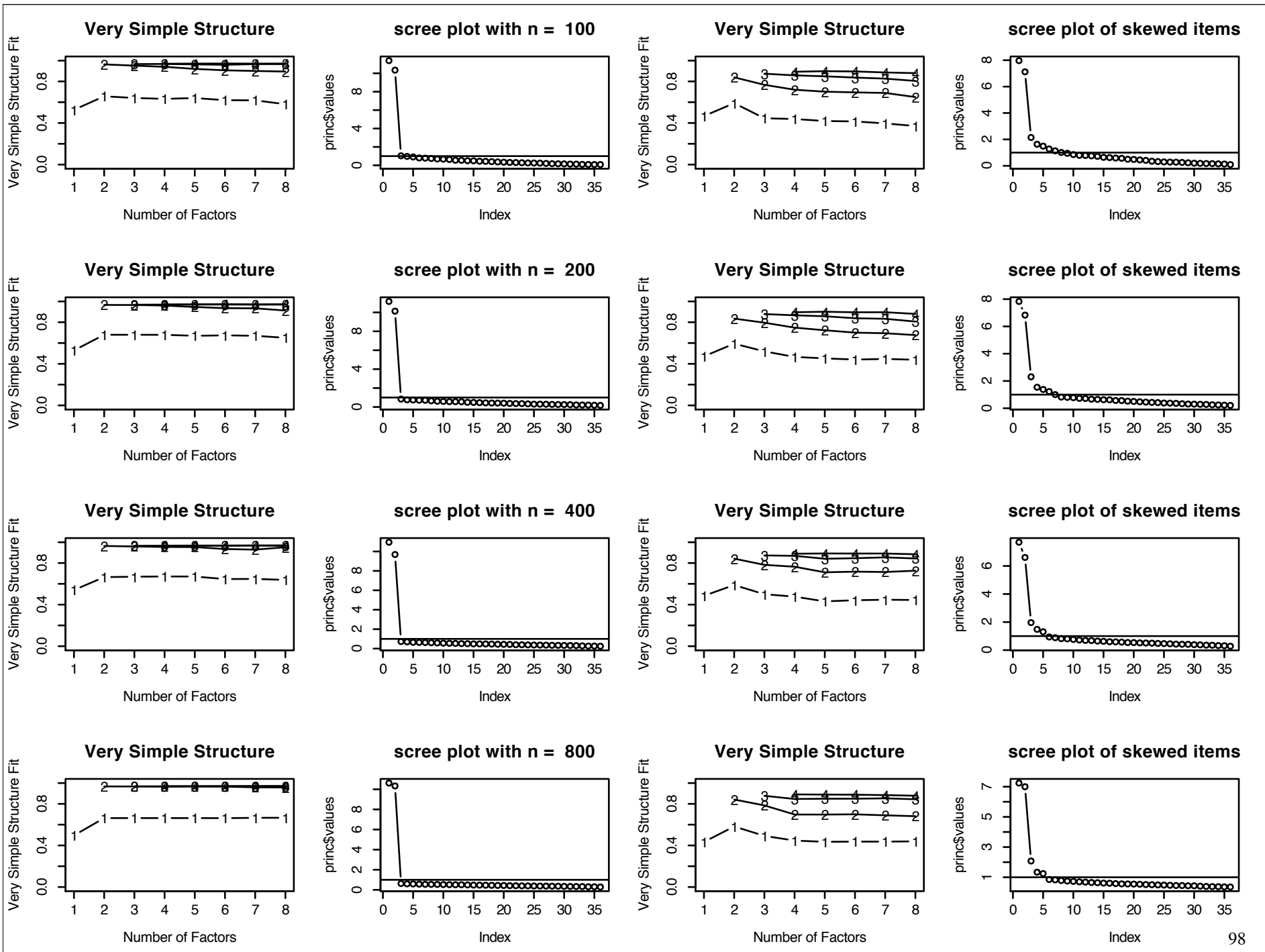


3 factor varimax rotation



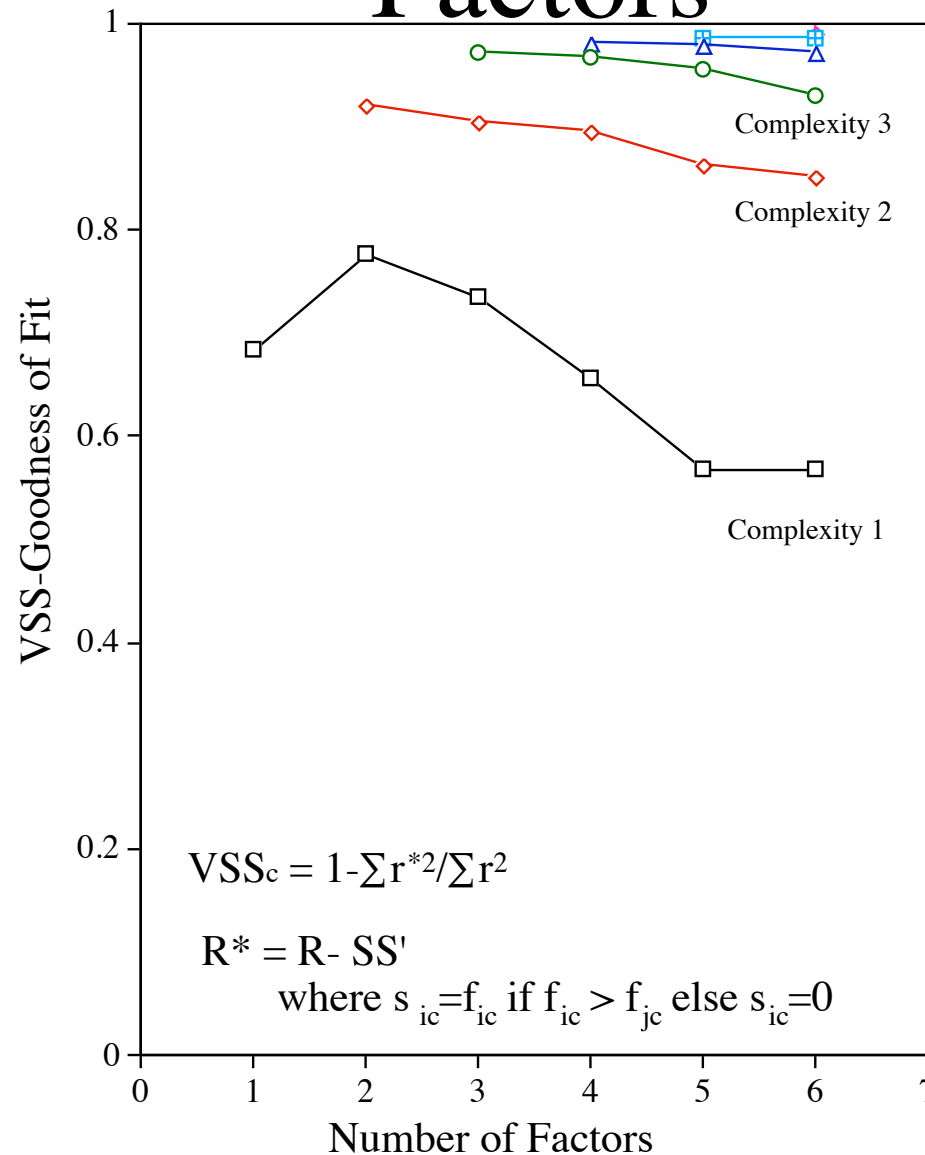
4 factor varimax rotation



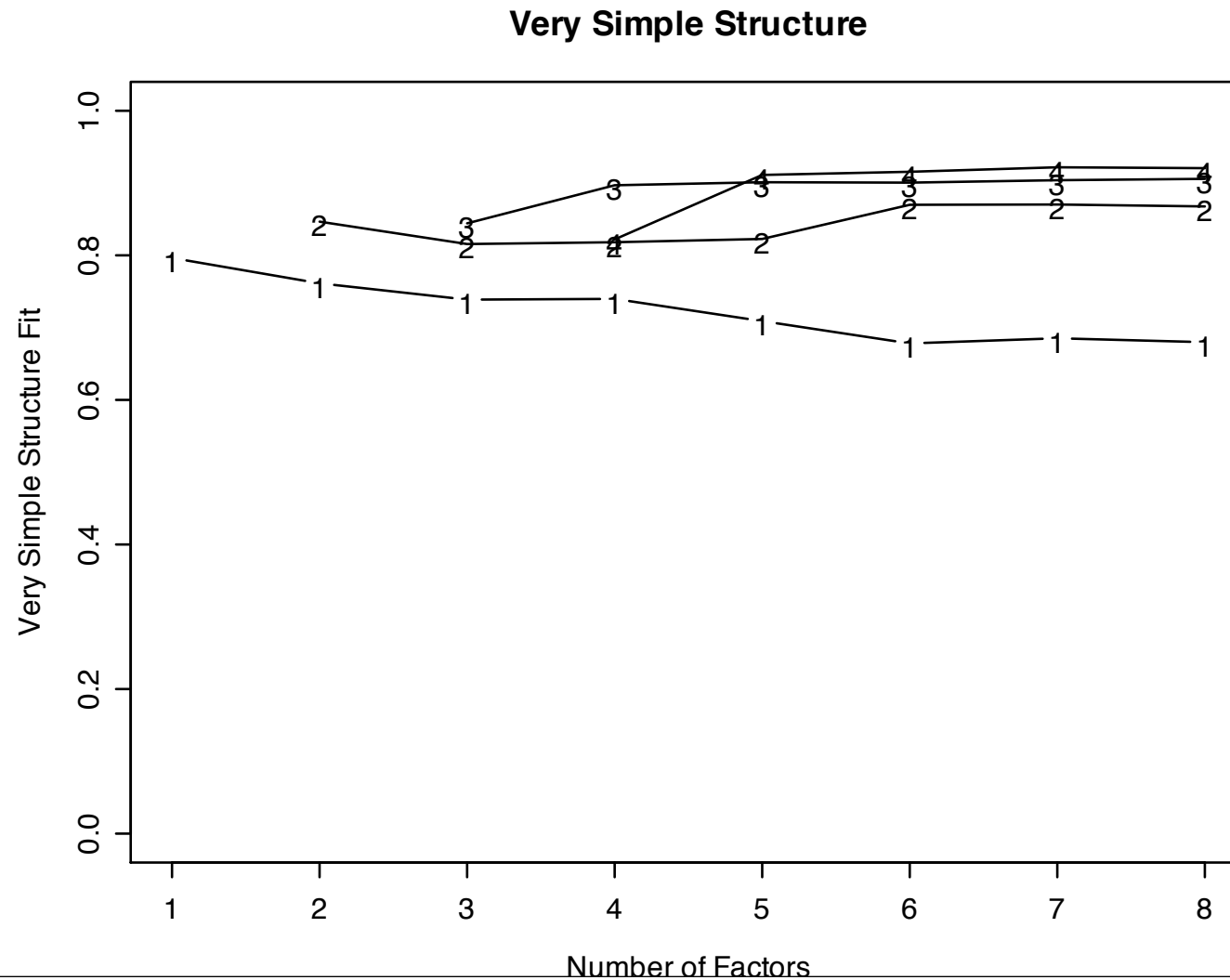


Very Simple Structure of affect => 2

Factors



VSS of Ability

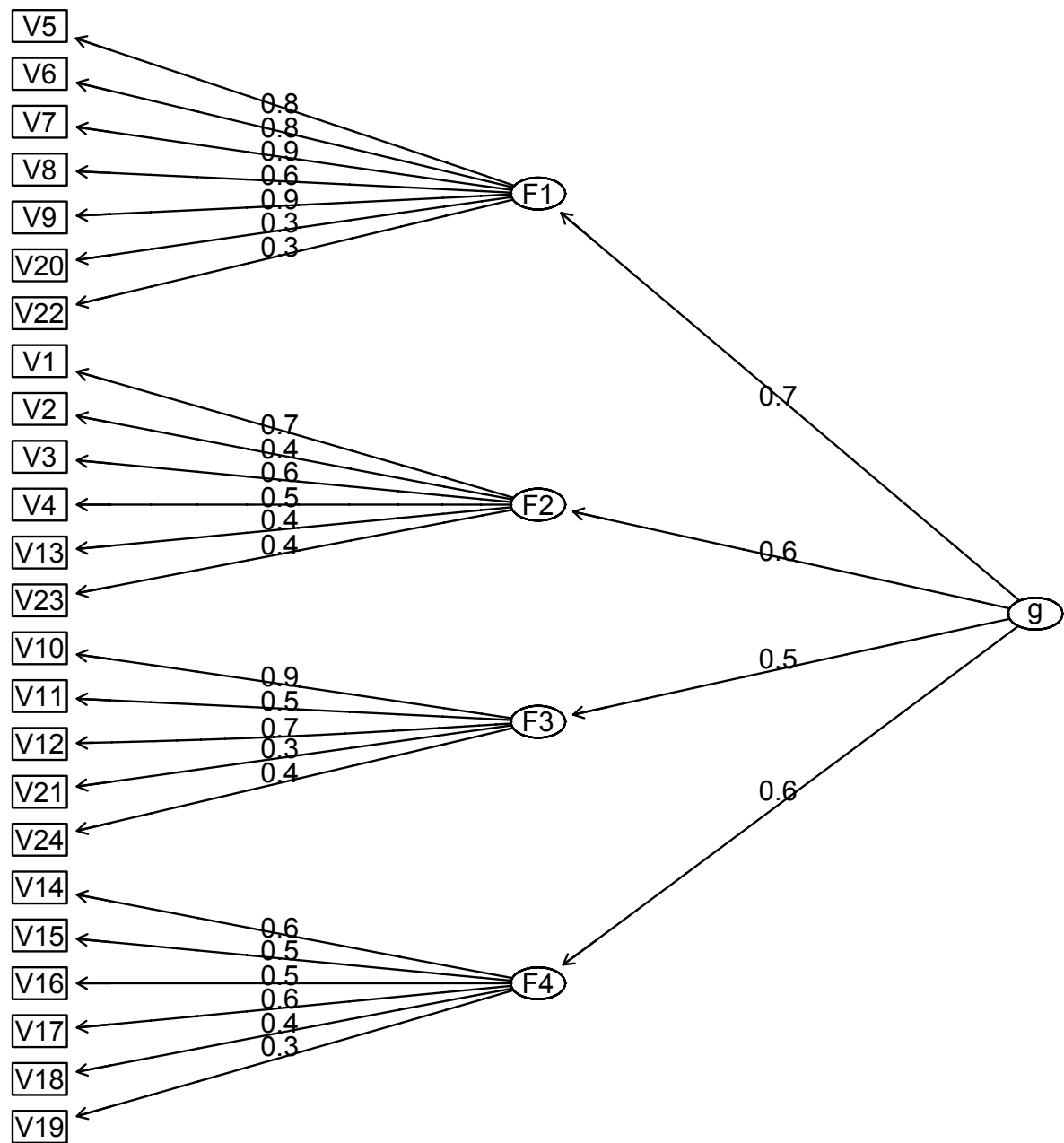


More complicated factor models

- Hierarchical models
- Bifactor models
- Typical in measurements of ability, becoming more used in measures of affective traits

Hierarchical model

	g factor	PA1	PA2	PA3	PA4	h2	u2
VisualPerception	0.50	0.031	0.0417	0.5429	0.042	0.47	0.53
Cubes	0.31	0.037	0.0202	0.3593	0.015	0.21	0.79
PaperFormBoard	0.35	0.068	0.1275	0.4318	0.040	0.33	0.67
Flags	0.40	0.134	0.0327	0.4085	0.017	0.30	0.70
GeneralInformation	0.54	0.576	0.1055	0.0103	0.019	0.59	0.41
ParagraphComprehension	0.55	0.605	0.0485	0.0034	0.072	0.65	0.35
SentenceCompletion	0.52	0.660	0.0410	0.0057	0.083	0.77	0.23
WordClassification	0.54	0.423	0.1000	0.1709	0.007	0.37	0.63
WordMeaning	0.56	0.652	0.0681	0.0134	0.059	0.75	0.25
Addition	0.38	0.057	0.7549	0.1274	0.028	0.76	0.24
Code	0.47	0.052	0.4380	0.0028	0.229	0.34	0.66
CountingDots	0.38	0.081	0.6169	0.1711	0.019	0.55	0.45
StraightCurvedCapitals	0.48	0.055	0.4090	0.3306	0.045	0.40	0.60
WordRecognition	0.39	0.107	0.0023	0.0890	0.447	0.36	0.64
NumberRecognition	0.36	0.027	0.0043	0.0044	0.420	0.29	0.71
FigureRecognition	0.46	0.041	0.0722	0.2656	0.409	0.40	0.60
ObjectNumber	0.41	0.030	0.1297	0.0674	0.457	0.38	0.62
NumberFigure	0.46	0.094	0.2185	0.1974	0.330	0.32	0.68
FigureWord	0.39	0.050	0.0799	0.1342	0.247	0.14	0.86
Deduction	0.51	0.248	0.0357	0.2476	0.160	0.25	0.75
NumericalPuzzles	0.49	0.053	0.2977	0.2886	0.082	0.26	0.74
ProblemReasoning	0.51	0.237	0.0052	0.2415	0.159	0.23	0.77
SeriesCompletion	0.57	0.225	0.0753	0.3514	0.079	0.30	0.70
ArithmeticProblems	0.53	0.213	0.3700	0.0600	0.134	0.29	0.70



9 variable problem

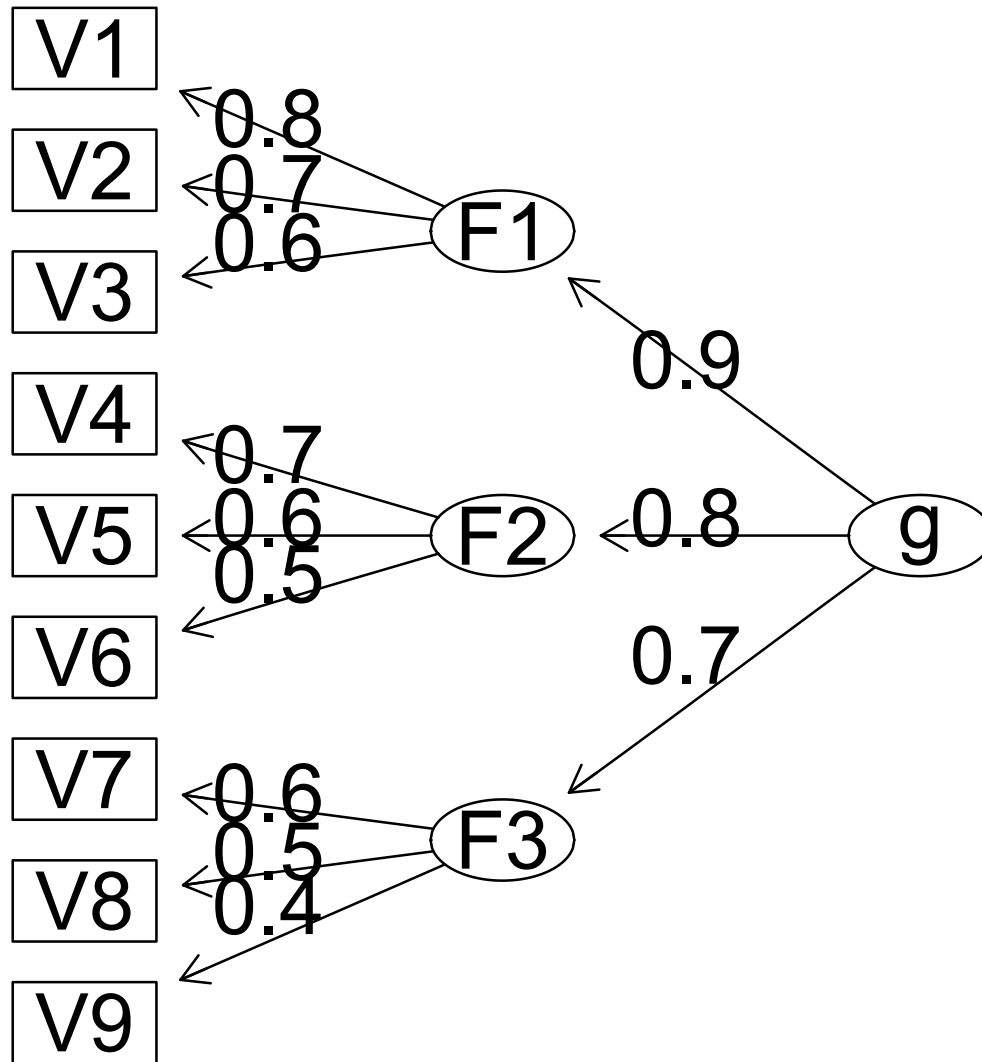
	V1	V2	V3	V4	V5	V6	V7	V8	V9
V1	1.00	0.56	0.48	0.40	0.35	0.29	0.30	0.25	0.20
V2	0.56	1.00	0.42	0.35	0.30	0.25	0.26	0.22	0.18
V3	0.48	0.42	1.00	0.30	0.26	0.22	0.23	0.19	0.15
V4	0.40	0.35	0.30	1.00	0.42	0.35	0.24	0.20	0.16
V5	0.35	0.30	0.26	0.42	1.00	0.30	0.20	0.17	0.13
V6	0.29	0.25	0.22	0.35	0.30	1.00	0.17	0.14	0.11
V7	0.30	0.26	0.23	0.24	0.20	0.17	1.00	0.30	0.24
V8	0.25	0.22	0.19	0.20	0.17	0.14	0.30	1.00	0.20
V9	0.20	0.18	0.15	0.16	0.13	0.11	0.24	0.20	1.00

General + Group

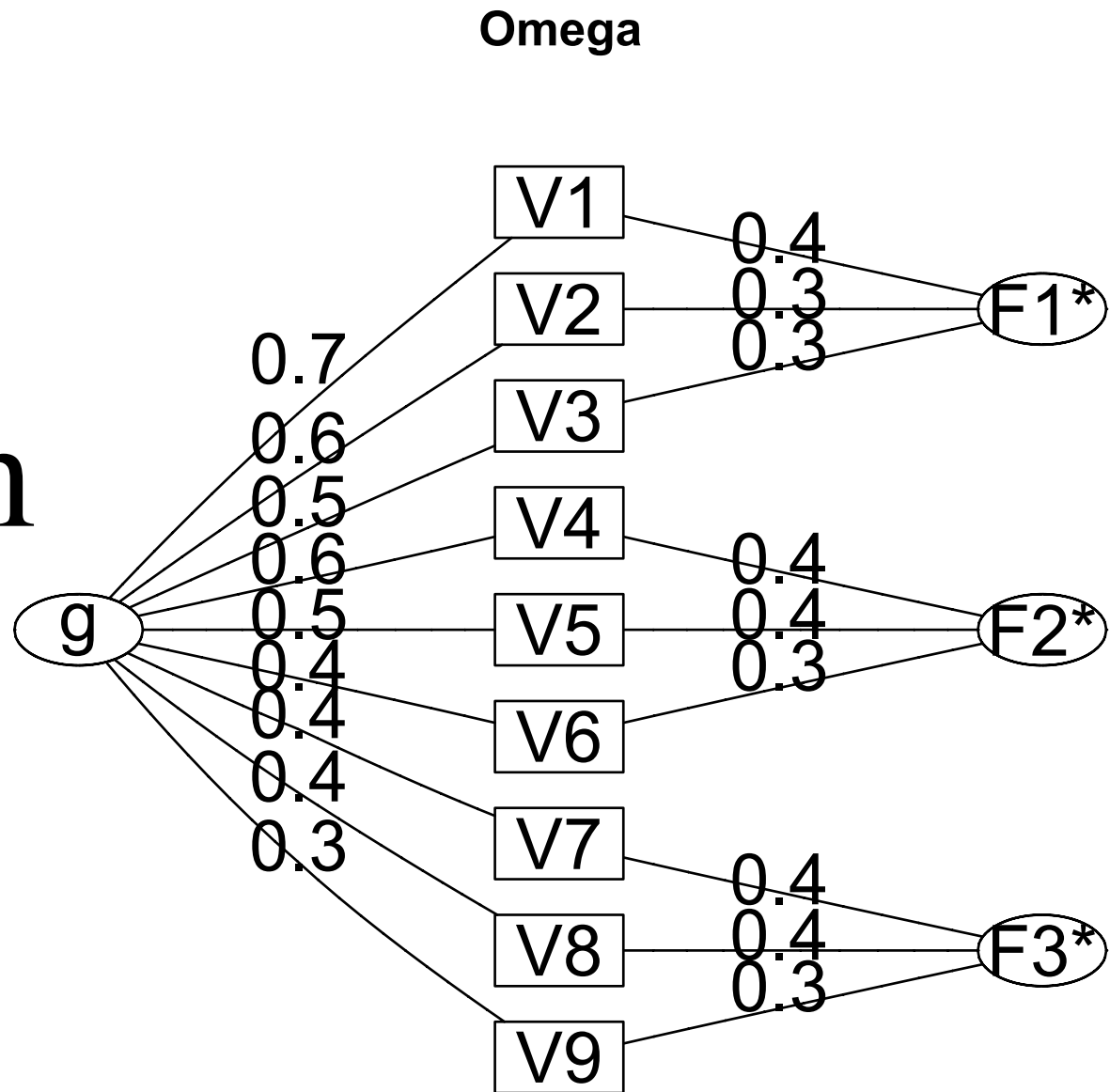
	V1	V2	V3	V4	V5	V6	V7	V8	V9
V1	1	0.6	0.5	0.4	0.4	0.3	0.3	0.3	0.2
V2	0.6	1	0.4	0.4	0.3	0.3	0.3	0.2	0.2
V3	0.5	0.4	1	0.3	0.3	0.2	0.2	0.2	0.2
V4	0.4	0.4	0.3	1	0.4	0.4	0.2	0.2	0.2
V5	0.4	0.3	0.3	0.4	1	0.3	0.2	0.2	0.1
V6	0.3	0.3	0.2	0.4	0.3	1	0.2	0.1	0.1
V7	0.3	0.3	0.2	0.2	0.2	0.2	1	0.3	0.2
V8	0.3	0.2	0.2	0.2	0.2	0.1	0.3	1	0.2
V9	0.2	0.2	0.2	0.2	0.1	0.1	0.2	0.2	1

Omega

Hierarchical



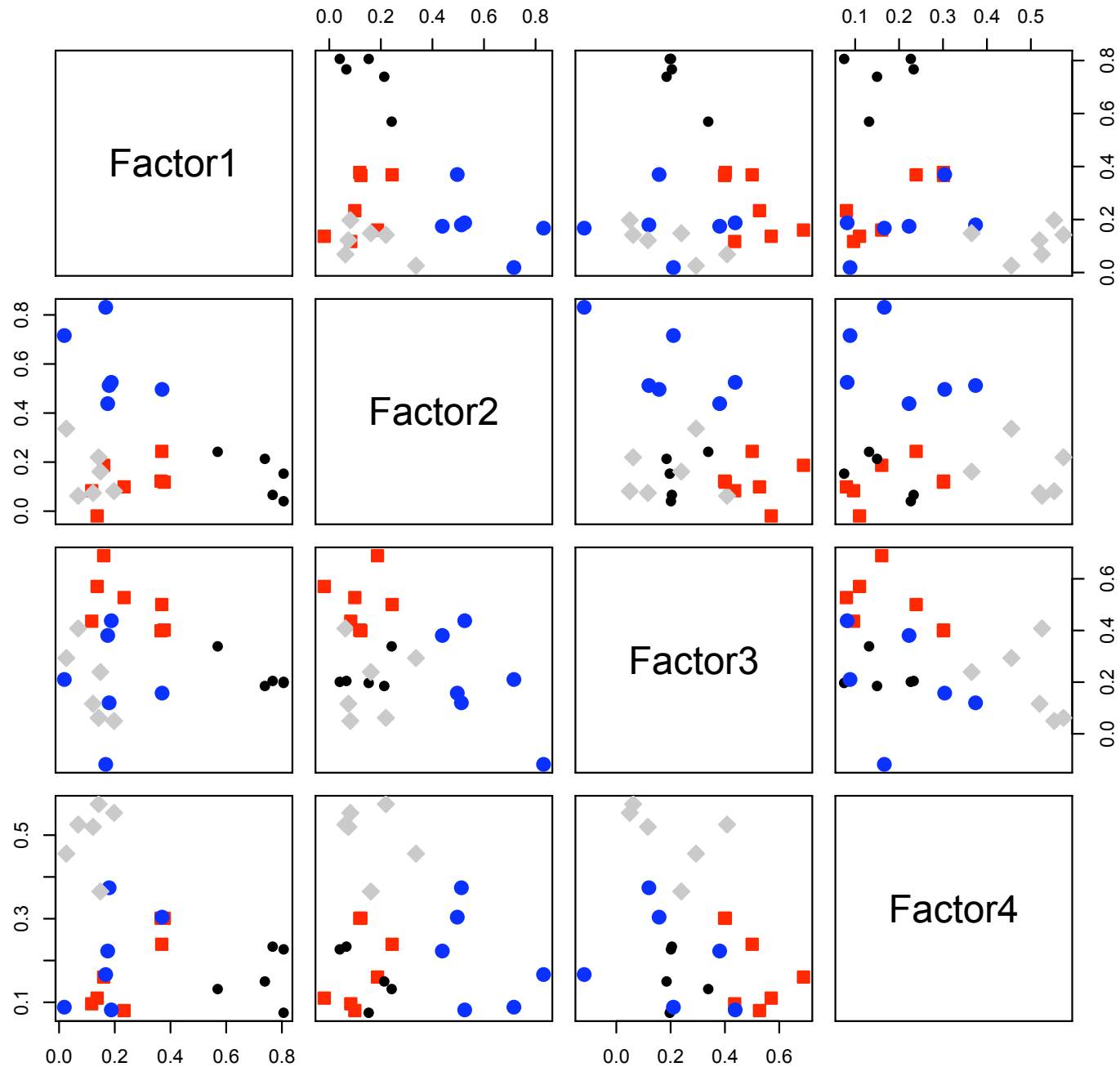
Schmid
Leiman
transform
makes a
bifactor
solution



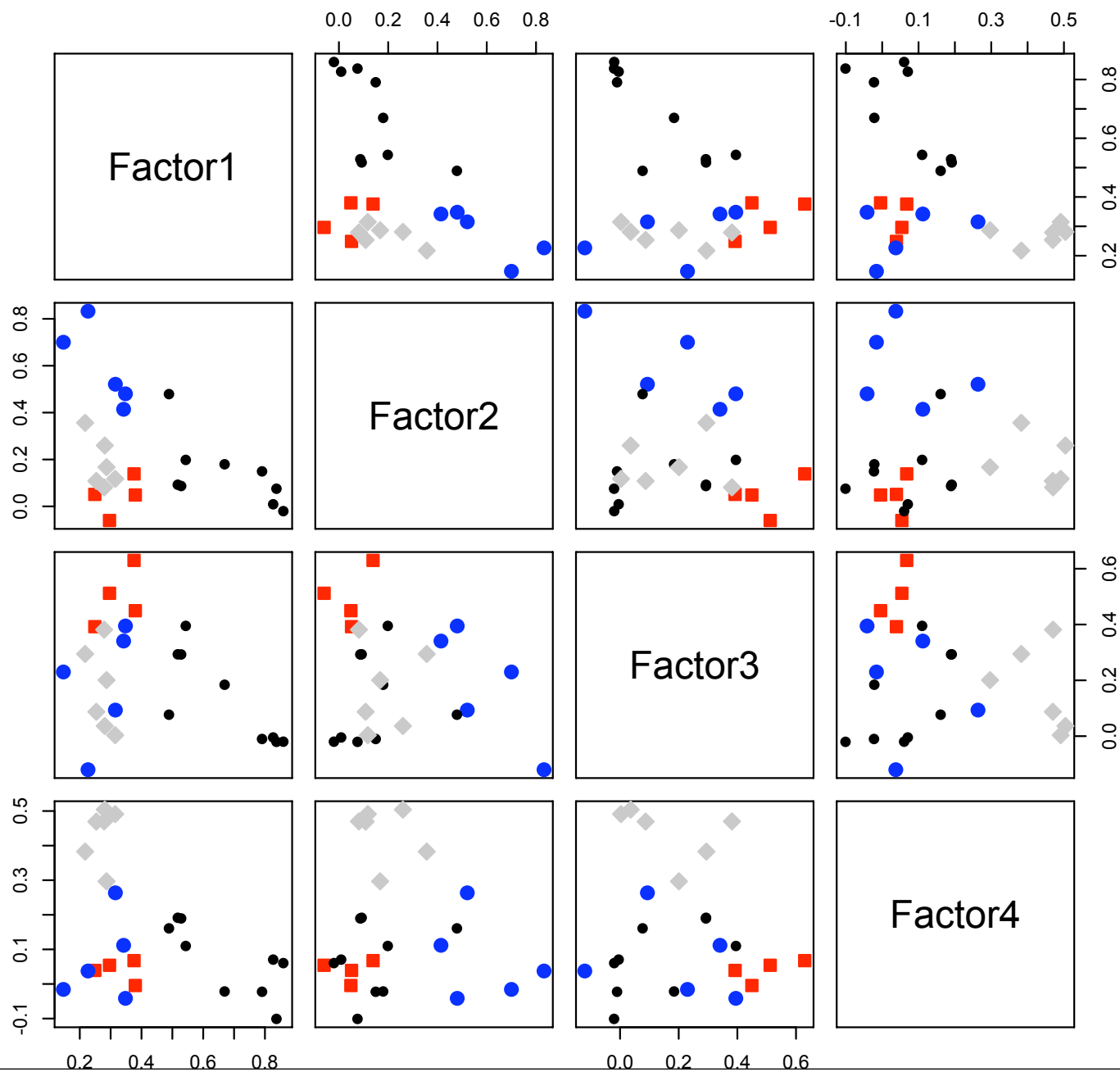
Schmid Leiman

	g factor	Factor1	Factor2	Factor3	h2	u2
V1	0.72	3.5e-01	7.3e-07	1.7e-07	0.64	0.36
V2	0.63	3.1e-01	3.1e-07	5.4e-07	0.49	0.51
V3	0.54	2.6e-01	8.3e-08	3.3e-07	0.36	0.64
V4	0.56	3.2e-07	4.2e-01	1.5e-07	0.49	0.51
V5	0.48	3.0e-07	3.6e-01	1.4e-07	0.36	0.64
V6	0.40	3.6e-07	3.0e-01	1.7e-07	0.25	0.75
V7	0.42	4.4e-06	6.3e-06	4.3e-01	0.36	0.64
V8	0.35	2.3e-06	4.4e-06	3.6e-01	0.25	0.75
V9	0.28	2.1e-06	3.6e-06	2.9e-01	0.16	0.84

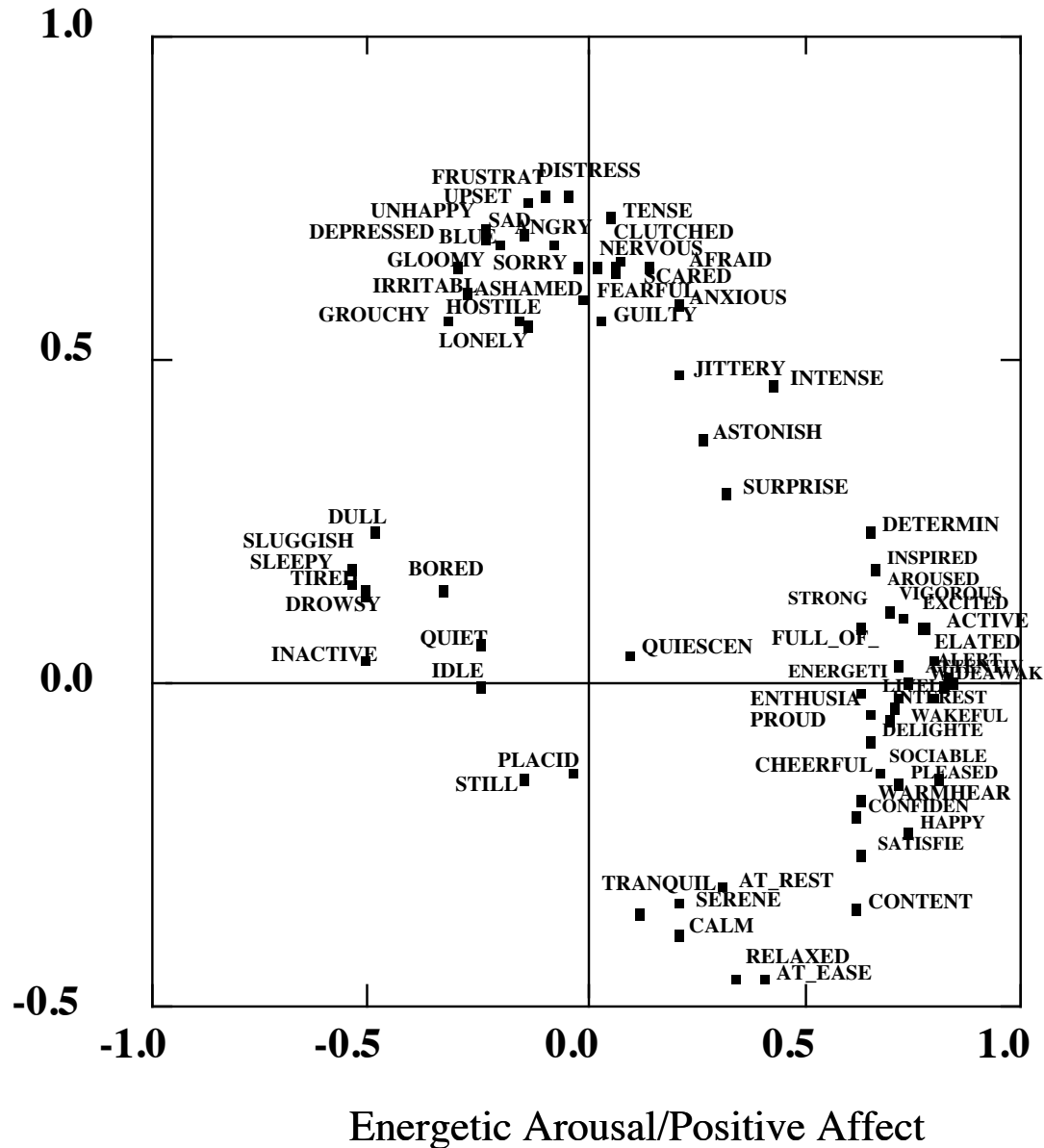
24 mental abilities Varimax

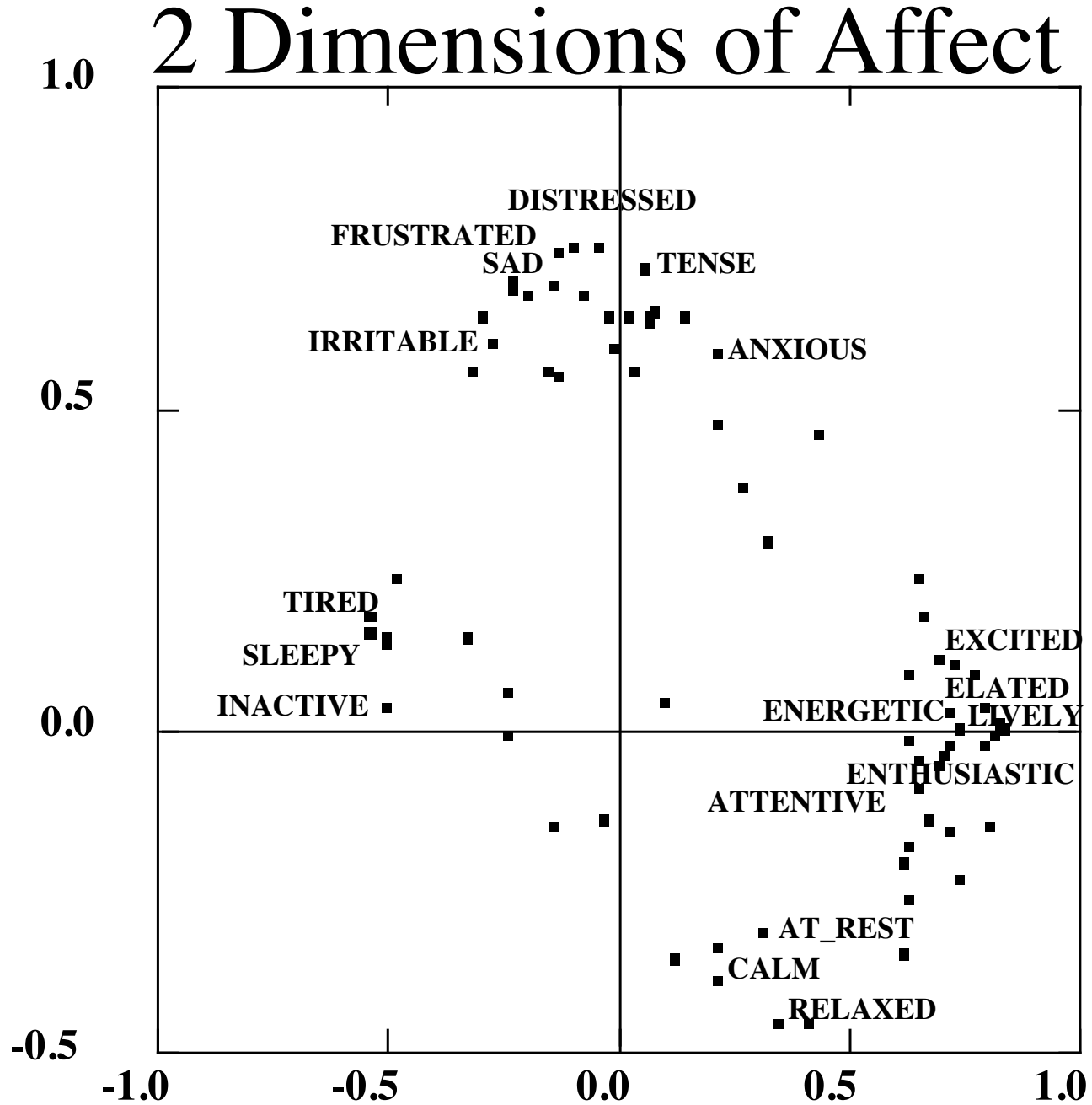


24 mental abilities Quartimax



2 Dimensions of Affect





Representative MSQ items (arranged by angular location)

Item	EA-PA	TA-NA	Angle
energetic	0.8	0.0	1
elated	0.7	0.0	2
excited	0.8	0.1	6
anxious	0.2	0.6	70
tense	0.1	0.7	85
distressed	0.0	0.8	93
frustrated	-0.1	0.8	98
sad	-0.1	0.7	101
irritable	-0.3	0.6	114
sleepy	-0.5	0.1	164
tired	-0.5	0.2	164
inactive	-0.5	0.0	177
calm	0.2	-0.4	298
relaxed	0.4	-0.5	307
at ease	0.4	-0.5	312
attentive	0.7	0.0	357
enthusiastic	0.8	0.0	358
lively	0.9	0.0	360

FA and PCA vocabulary

- Eigen values = $\sum(\text{loading}^2)$ across variables = amount of variance accounted for by factor
- Communalities = $\sum(\text{loading}^2)$ across factors = amount of variance accounted for in a variable by all the factors
- Rotations versus Transformations
 - Rotations are orthogonal transformations
 - Oblique Transformations

Rotations and transformations

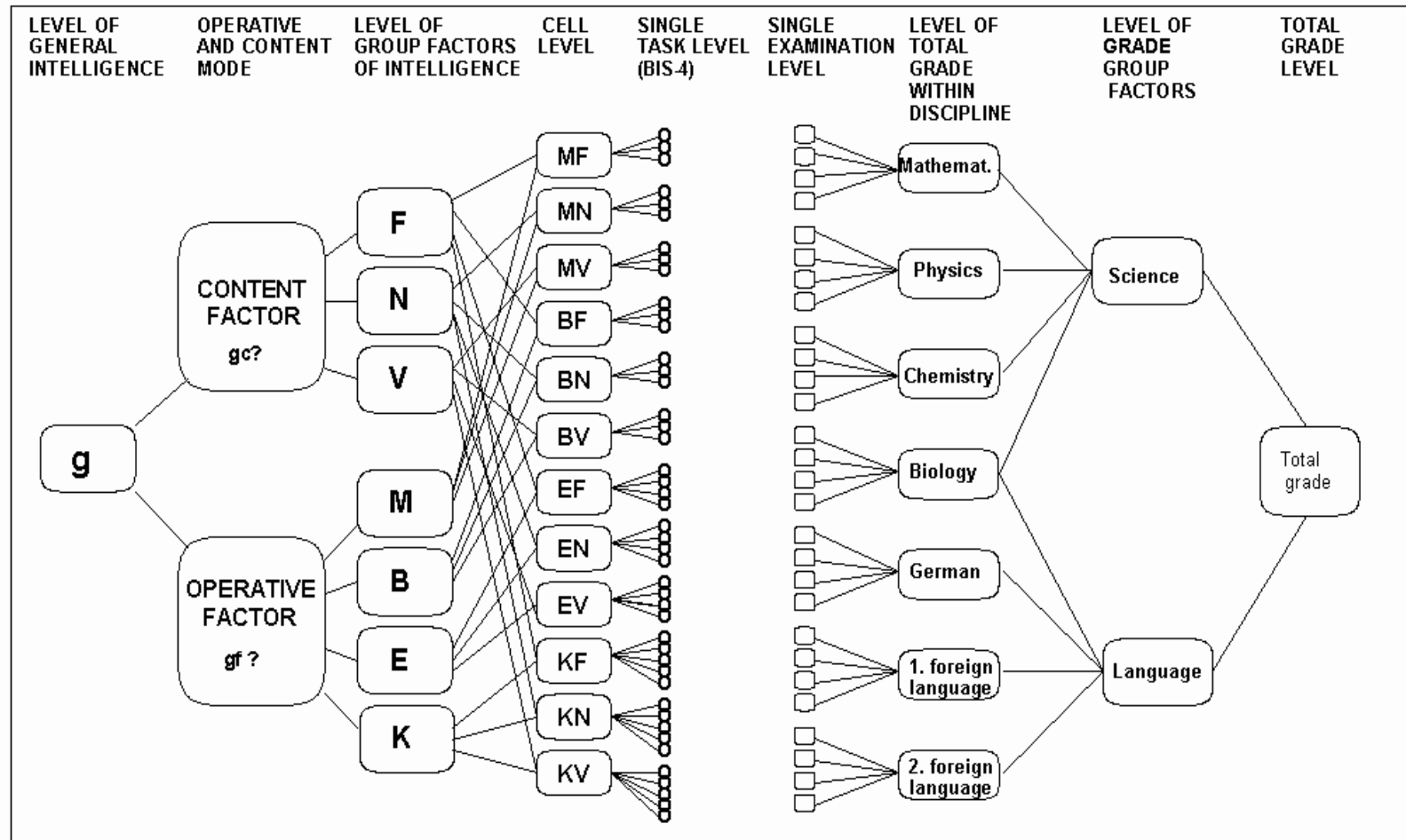
- Simple structure as a criterion for rotation
- Simple structure in the eye of the beholder
- Simple factors (few high, many 0 loadings)
- Simple variables (few high, many 0 loadings)
- VARIMAX, Quartimax, Quartimin
- Procrustes

Rotations and transformations

- Orthogonal rotations
 - Factors are orthogonal, rotated to reduce (or maximize) particular definition of simple structure
- Oblique transformations and higher order factors
 - Allows factors to be correlated (and thus have higher order factors)
 - Many domains have hierarchical structure

Fig. 9:

Hierarchical version of the Berlin model of intelligence and a grade hierarchy model



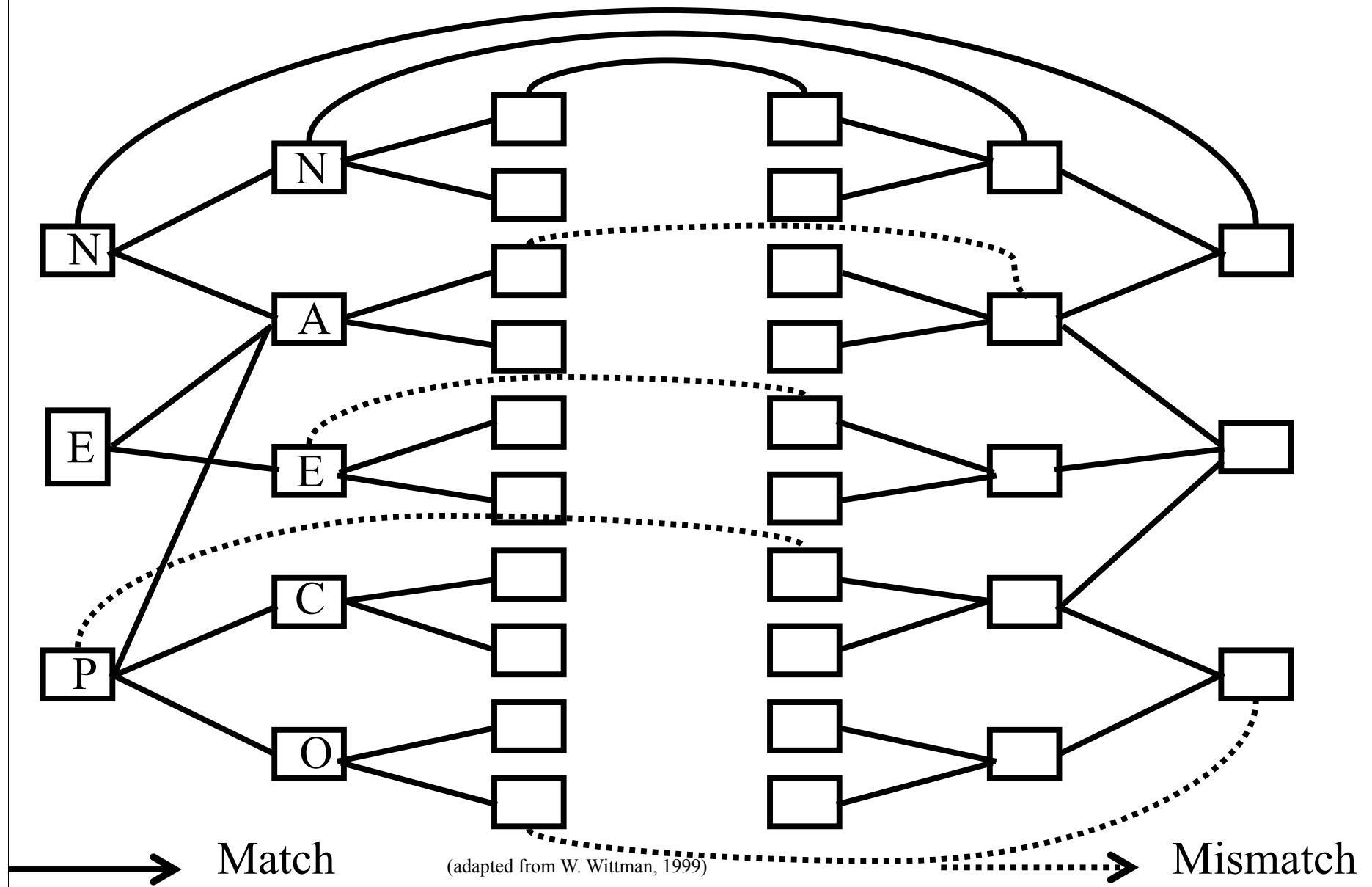
K: Processing capacity for complex information, i.e. reasoning
 E: Creativity
 B: Speed on relatively simple tasks
 M: Memory, i.e. storage capacity for information

F: figural Intelligence
 N: numerical Intelligence
 V: verbal intelligence

Specificity vs. generality --Matching predictors to outcomes

Predictors

Criteria



Exploratory versus Confirmatory

- Exploratory:
 - How many factors and best rotation
 - Extraction
 - How many factors?
 - Algorithm for extraction
 - Rotation to simple structure -- what is best SS?
- Confirmatory: does a particular model fit?
 - Apply statistical test of fit
 - But larger $N \Rightarrow$ less fit
 - Is the model the original one or has it been modified?

Components versus Factors

- Components are linear sums of variables and are thus defined at the data level
- Factors represent the covariances of variables and are only estimated (variables are linear sums of factors)
 - Model is undefined at data level, but is defined at structural level
 - Factor indeterminacy problem

Further adventures in hyperspace

I. Orthogonal PC, FA, MLE

II. Oblique FA

III. Cluster analysis

Harman 8 physical variables

	height	arm.span	forearm	lower.leg	weight	bitro.diameter	chest.girth	chest.width
height	1.00	0.85	0.81	0.86	0.47	0.40	0.30	0.38
arm.span	0.85	1.00	0.88	0.83	0.38	0.33	0.28	0.41
forearm	0.81	0.88	1.00	0.80	0.38	0.32	0.24	0.34
lower.leg	0.86	0.83	0.80	1.00	0.44	0.33	0.33	0.36
weight	0.47	0.38	0.38	0.44	1.00	0.76	0.73	0.63
bitro.diameter	0.40	0.33	0.32	0.33	0.76	1.00	0.58	0.58
chest.girth	0.30	0.28	0.24	0.33	0.73	0.58	1.00	0.54
chest.width	0.38	0.41	0.34	0.36	0.63	0.58	0.54	1.00

Principal Components

\$values

[1] 4.67 1.77 0.48 0.42 0.23 0.19 0.14 0.10

\$fit

[1] 0.98

Rotated

\$loadings

PC1 PC2

PC1 PC2

height	0.86	-0.37	height	0.90	0.25
arm.span	0.84	-0.44	arm.span	0.93	0.19
forearm	0.81	-0.46	forearm	0.92	0.16
lower.leg	0.84	-0.40	lower.leg	0.90	0.22
weight	0.76	0.52	weight	0.26	0.88
bitro.diameter	0.67	0.53	bitro.diameter	0.19	0.84
chest.girth	0.62	0.58	chest.girth	0.11	0.84
chest.width	0.67	0.42	chest.width	0.26	0.75

Principal Factors

	PA1	PA2
height	0.86	-0.32
arm.span	0.85	-0.41
forearm	0.81	-0.41
lower.leg	0.83	-0.34
weight	0.75	0.57
bitro.diameter	0.63	0.49
chest.girth	0.57	0.51
chest.width	0.61	0.35

	PA1	PA2
height	0.87	0.28
arm.span	0.92	0.21
forearm	0.89	0.19
lower.leg	0.86	0.26
weight	0.23	0.91
bitro.diameter	0.18	0.78
chest.girth	0.12	0.76
chest.width	0.25	0.66

Maximum Likelihood

	Factor1	Factor2
height	0.865	0.287
arm.span	0.927	0.181
forearm	0.895	0.179
lower.leg	0.859	0.252
weight	0.233	0.925
bitro.diameter	0.194	0.774
chest.girth	0.134	0.752
chest.width	0.278	0.621

	Factor1	Factor2
SS loadings	3.335	2.617
Proportion Var	0.417	0.327
Cumulative Var	0.417	0.744

Factor congruence

	PC1	PC2
PA1	1.00	0.44
PA2	0.49	1.00

	Factor1	Factor2
PA1	1.0	0.47
PA2	0.5	1.00

Correlations of factor loadings $\neq$ factor congruence

	PC1	PC2
PA1	1.00	-0.99
PA2	-0.96	0.99

	Factor1	Factor2
PA1	1.00	-0.96
PA2	-0.97	1.00

Oblique factors

	PA1	PA2
height	0.88	0.06
arm.span	0.96	-0.03
forearm	0.93	-0.04
lower.leg	0.88	0.04
weight	0.02	0.93
bitro.diameter	0.00	0.80
chest.girth	-0.07	0.80
chest.width	0.10	0.65

Rotating matrix:

	[,1]	[,2]
[1,]	1.10	-0.284
[2,]	-0.26	1.094

Phi:

	[,1]	[,2]
[1,]	1.000	0.467
[2,]	0.467	1.000

Cluster Analysis: a poor person's factor analysis?

I. Clustering as a grouping procedure

A.membership is -1, 0, 1

II.Clusters as partitions of variable space

III.Hierarchical clusters

A.top down

B.bottom up

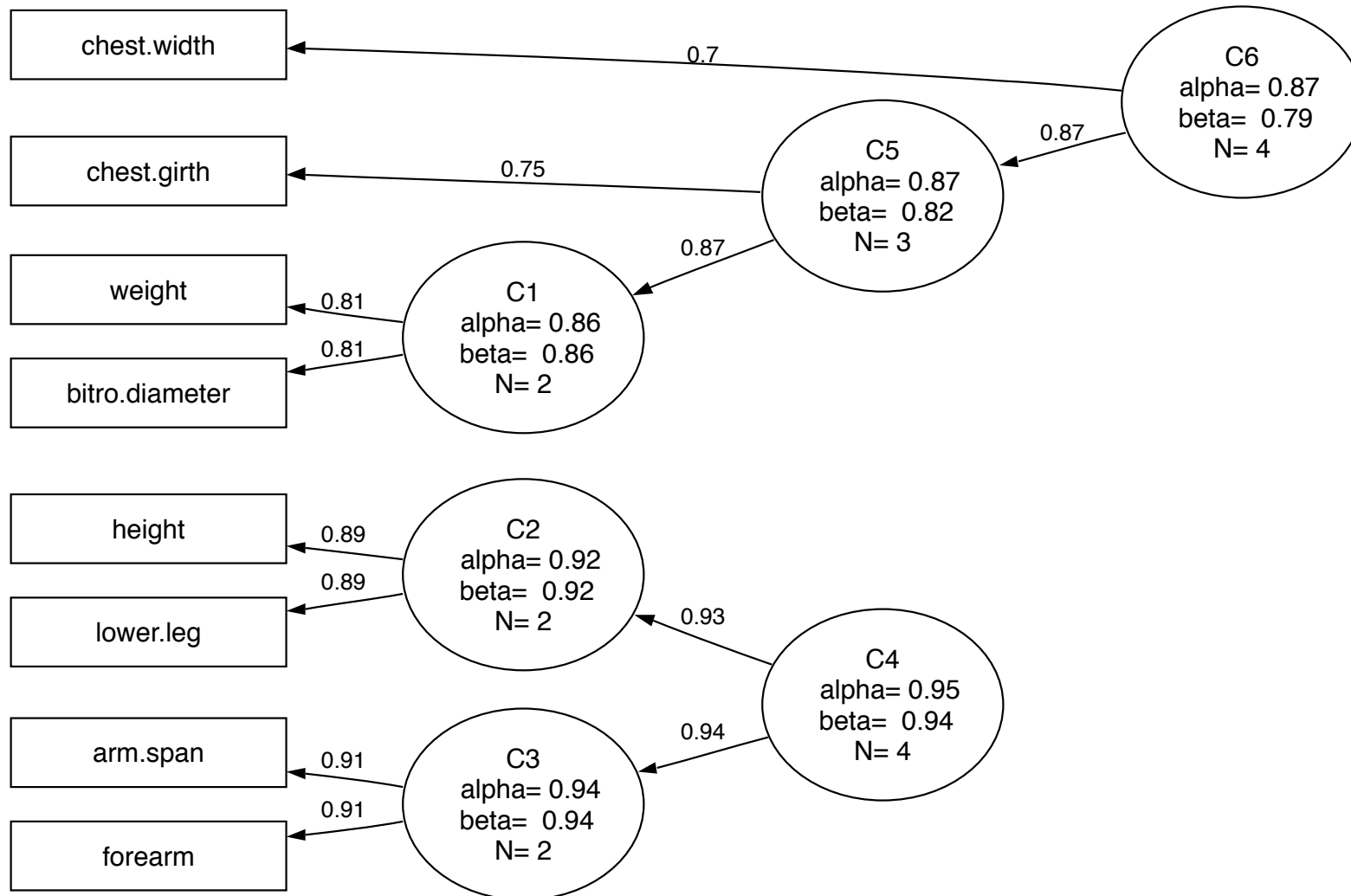
Hierarchical Clustering

- I. Form item proximity matrix
(correlations?)
- II. Search for most similar pair
- III. Combine them (if criteria are reached)
- IV. Re-estimate similarities
- V. Repeat II-IV until stop criteria

ICLUST: hierarchical clustering of items

- I. Proximities = correlations (corrected for reliability?)
- II. Combination rule is matrix addition
- III. Criteria are increase in alpha and beta
- IV. Implemented in R: ICLUST

Harman - cluster



ICLUST

2 cluster solution

	item	content	cluster	C4	C6
arm.span	2	arm.span	1	0.91	0.41
height	1	height	1	0.89	0.46
forearm	3	forearm	1	0.89	0.38
lower.leg	4	lower.leg	1	0.88	0.43
weight	5	weight	2	0.44	0.83
bitro.diameter	6	bitro.diameter	2	0.37	0.75
chest.girth	7	chest.girth	2	0.30	0.72
chest.width	8	chest.width	2	0.40	0.68

Correlated clusters

\$purified\$corrected

C4 C6

C4 0.95 0.49

C6 0.44 0.88

\$p.fit\$clusterfit

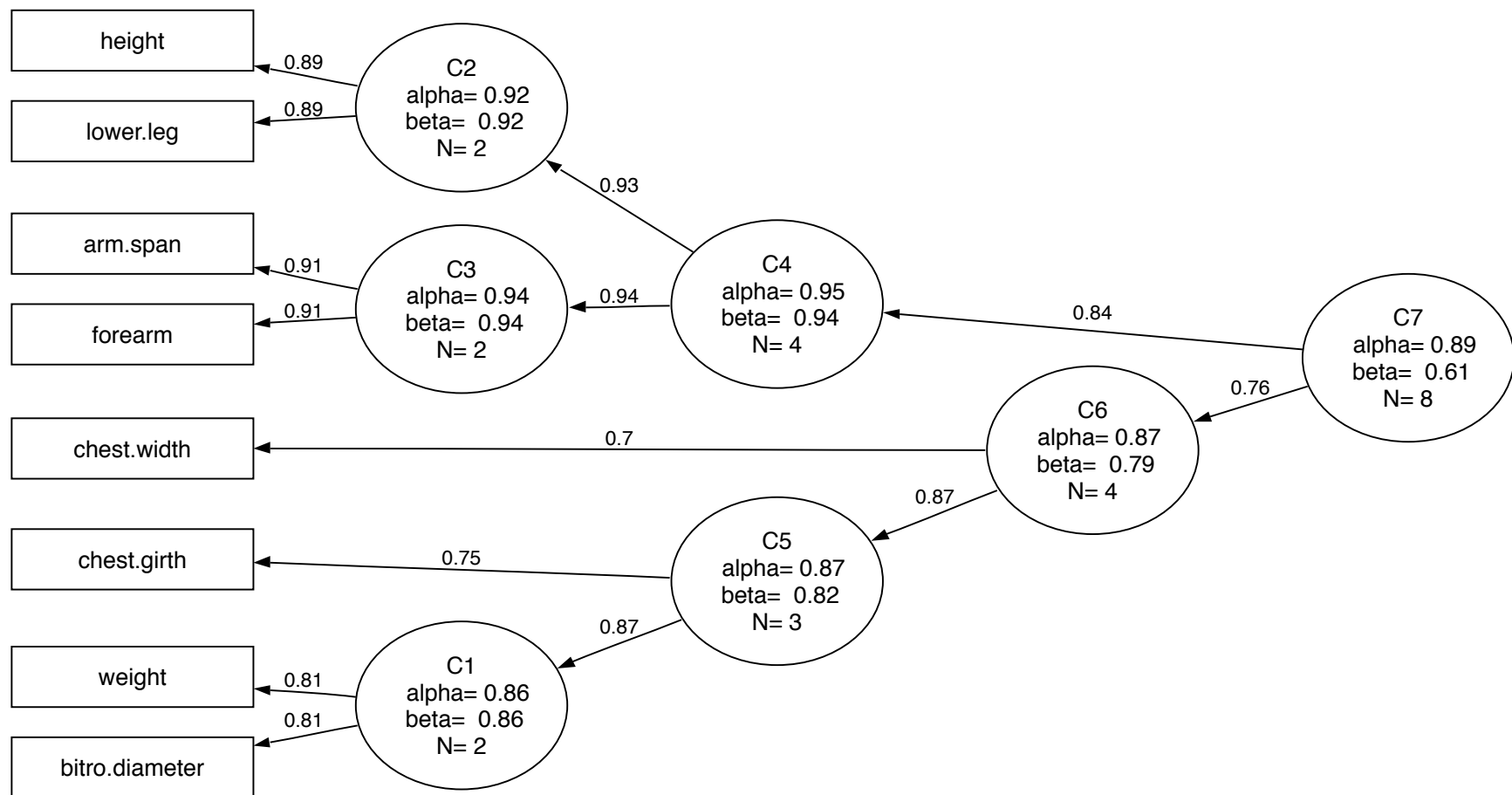
[1] 0.79

Equivalent to VSS complexity I

\$p.fit\$factorfit

[1] 0.86

One cluster solution



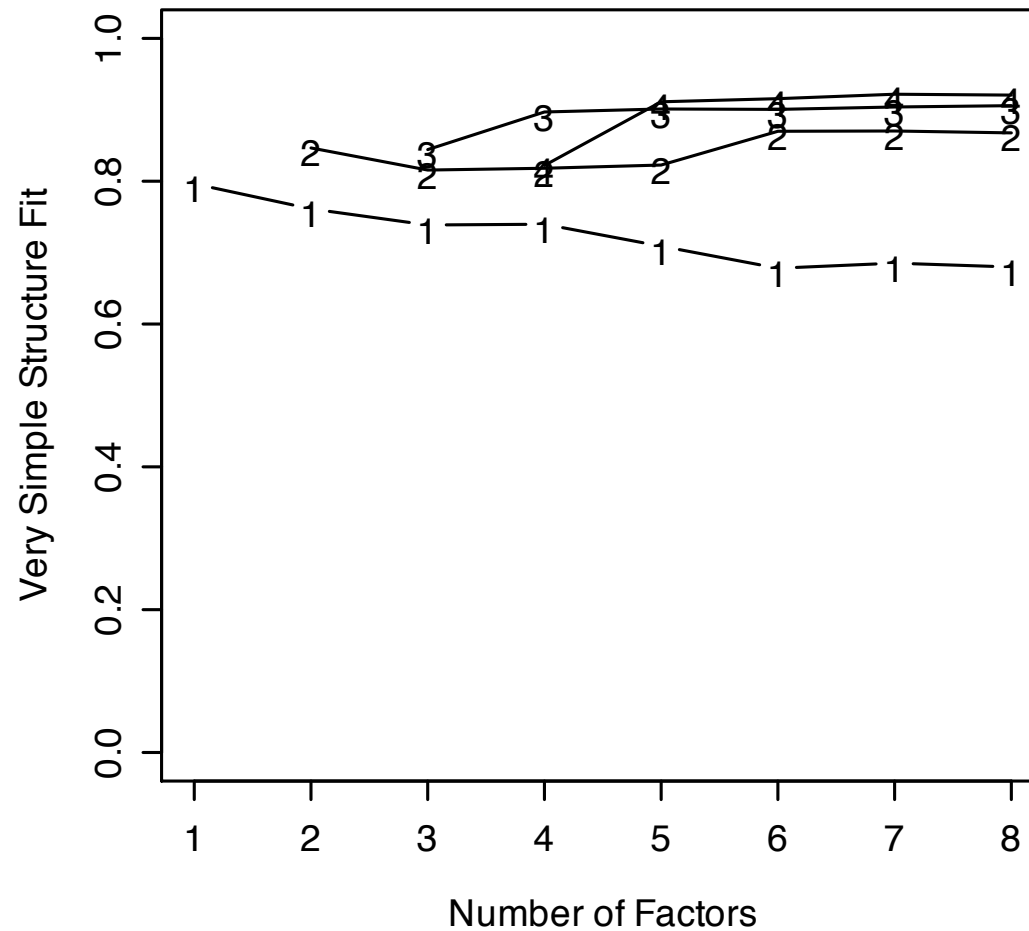
ICLUST

Yet another example

- A correlation matrix of 24 psychological tests given to 145 seventh and eight-grade children in a Chicago suburb by Holzinger and Swineford.
- Harman, H. H. (1976) *Modern Factor Analysis*, Third Edition Revised, University of Chicago Press, Table 7.4
- Factanal (maximum likelihood) suggests 5 factors are required to have non-significant residuals
- But is this overkill?

Holzinger Harmon 24

Very Simple Structure



Oblique

	PA1	PA2	PA3	PA4	
VisualPerception	0.041	0.0472	-0.6837	0.054	
Cubes	0.049	-0.0229	-0.4526	0.020	
PaperFormBoard	0.090	-0.1445	-0.5438	0.051	
Flags	0.176	-0.0371	-0.5145	-0.022	
GeneralInformation	0.759	0.1196	-0.0129	-0.024	
ParagraphComprehension	0.797	-0.0550	-0.0043	0.092	
SentenceCompletion	0.869	0.0465	-0.0071	-0.107	
WordClassification	0.557	0.1134	-0.2153	-0.009	
WordMeaning	0.859	-0.0772	0.0168	0.076	
Addition	0.076	0.8556	0.1605	0.036	
Code	0.069	0.4964	-0.0035	0.295	
CountingDots	-0.106	0.6992	-0.2155	-0.025	
StraightCurvedCapitals	0.073	0.4635	-0.4163	-0.058	
WordRecognition	0.141	-0.0026	0.1120	0.576	
NumberRecognition	0.036	-0.0049	0.0056	0.541	
FigureRecognition	-0.054	-0.0818	-0.3344	0.527	
ObjectNumber	0.039	0.1470	0.0849	0.589	
NumberFigure	-0.124	0.2477	-0.2486	0.426	[1,] 1.00 0.30 -0.41 0.40
FigureWord	0.066	0.0906	-0.1691	0.318	[2,] 0.30 1.00 -0.27 0.32
Deduction	0.327	-0.0404	-0.3119	0.207	[3,] -0.41 -0.27 1.00 -0.38
NumericalPuzzles	0.070	0.3374	-0.3635	0.106	[4,] 0.40 0.32 -0.38 1.00
ProblemReasoning	0.313	-0.0059	-0.3041	0.205	
SeriesCompletion	0.297	0.0853	-0.4425	0.102	
ArithmeticProblems	0.281	0.4194	-0.0755	0.172	

Schmid

	g factor	PA1	PA2	PA3	PA4	h2	u2
VisualPerception	0.50	0.031	0.0417	0.5429	0.042	0.47	0.53
Cubes	0.31	0.037	0.0202	0.3593	0.015	0.21	0.79
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Flags	0.40	0.134	0.0327	0.4085	0.017	0.30	0.70
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ObjectNumber	0.41	0.030	0.1297	0.0674	0.457	0.38	0.62
NumberFigure	0.46	0.094	0.2185	0.1974	0.330	0.32	0.68
FigureWord	0.39	0.050	0.0799	0.1342	0.247	0.14	0.86
Deduction	0.51	0.248	0.0357	0.2476	0.160	0.25	0.75
NumericalPuzzles	0.49	0.053	0.2977	0.2886	0.082	0.26	0.74
ProblemReasoning	0.51	0.237	0.0052	0.2415	0.159	0.23	0.77
SeriesCompletion	0.57	0.225	0.0753	0.3514	0.079	0.30	0.70
ArithmeticProblems	0.53	0.213	0.3700	0.0600	0.134	0.29	0.71

\$omega
[1] 0.64

\$alpha
[1] 0.91

Threats to interpreting correlation and the benefits of structure

- Correlations can be attenuated due to differences in skew (and error)

- Bi polar versus unipolar scales (e.g., of affect)

- How happy do you feel?

– Not at all A little Somewhat Very

- How sad do you feel?

– Not at all A little Somewhat Very

- How do you feel?

very sad sad happy very happy

Unipolar scales allow data to speak for themselves

Simulated Example of unipolar scales and the problem of skew

- Consider X and Y as random normal variables
- Let $X_+ = X$ if $X > 0$, 0 elsewhere
- Let $X_- = X$ if $X < 0$, 0 elsewhere
- Reversed $(X_+) = -X_+$
- Similarly for Y
- Examine the correlational structure
- Note that although X and $-X$ correlate -1.0 , X_+ and X_- correlate only $-.43$ and that X_+ correlates with X_+Y_+ $.66$

Determining Structure: zeros and skew

	X+	X-	Y+	Y-	X+Y+	X-Y-	X+Y-	X-Y+
X+	1.00							
X-	-0.47	1.00						
Y+	0.03	-0.01	1.00					
Y-	0.00	-0.03	-0.46	1.00				
X+Y+	0.65	-0.39	0.66	-0.39	1.00			
X-Y-	-0.40	0.63	-0.40	0.63	-0.46	1.00		
X+Y-	0.63	-0.40	-0.39	0.66	0.00	0.02	1.00	
X-Y+	-0.39	0.64	0.63	-0.40	0.00	0.00	-0.47	1.00

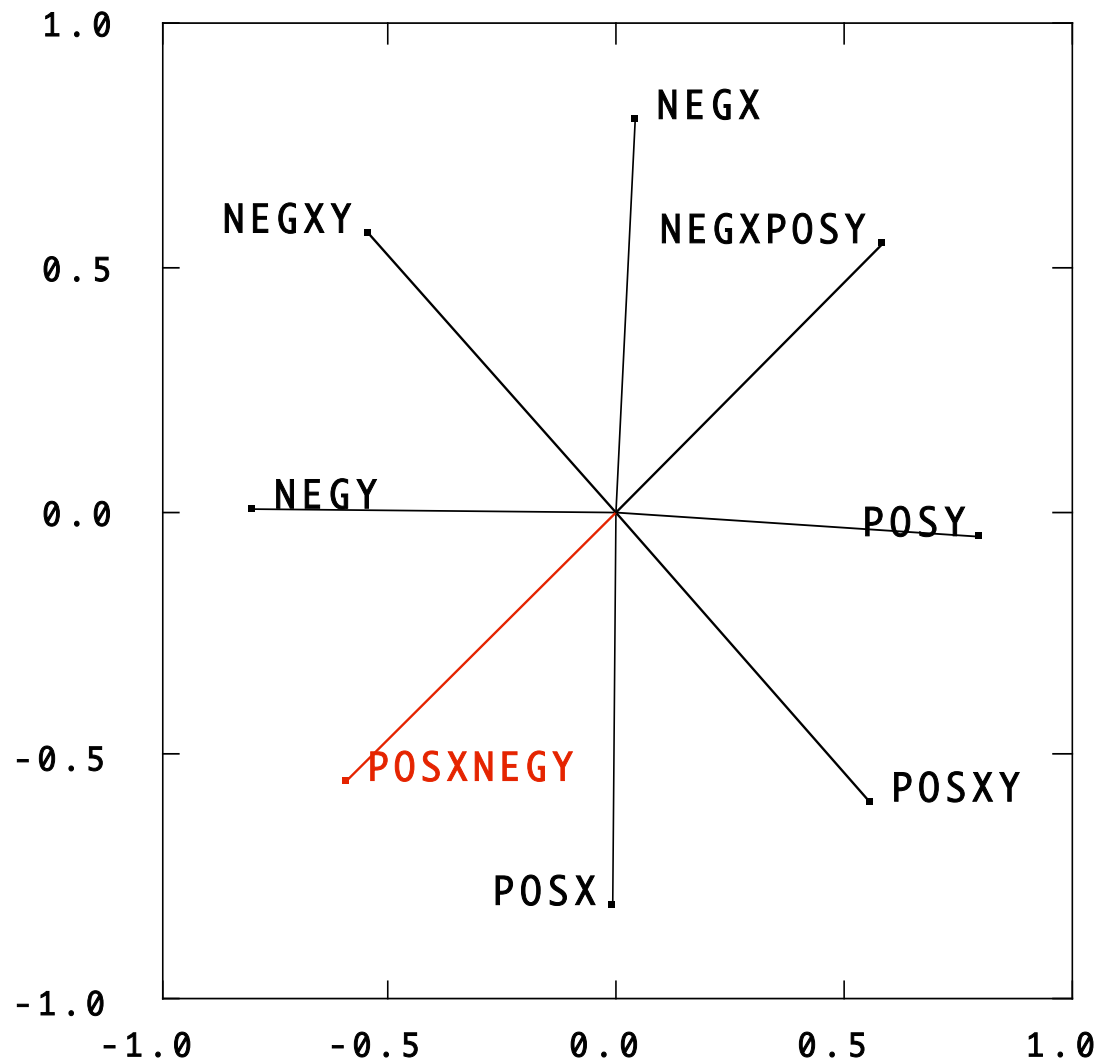
Skew and zeros: determining structure

	Y+	X+Y+	X+	X+Y-	r(Y-)	r(X-Y-)	r(X-)	X-Y+
Y	1.00							
X+Y+	0.66	1.00						
X+	0.03	0.65	1.00					
X+Y-	-0.39	0.00	0.63	1.00				
r(Y-)	0.46	0.39	0.00	-0.66	1.00			
r(X-Y-)	0.40	0.46	0.40	-0.02	0.63	1.00		
r(X-)	0.01	0.39	0.47	0.40	-0.03	0.63	1.00	
X-Y+	0.63	0.00	-0.39	-0.47	0.40	0.00	-0.64	1.00

Factor analysis shows structure

	1	2	
POSX	-0.01	-0.81	271
NEGX	0.04	0.80	87
POSY	0.80	-0.05	356
NEGY	-0.80	0.00	180
POSXY	0.56	-0.59	314
NEGXY	-0.55	0.57	136
POSXN	-0.59	-0.55	227
NEGXP	0.58	0.55	43

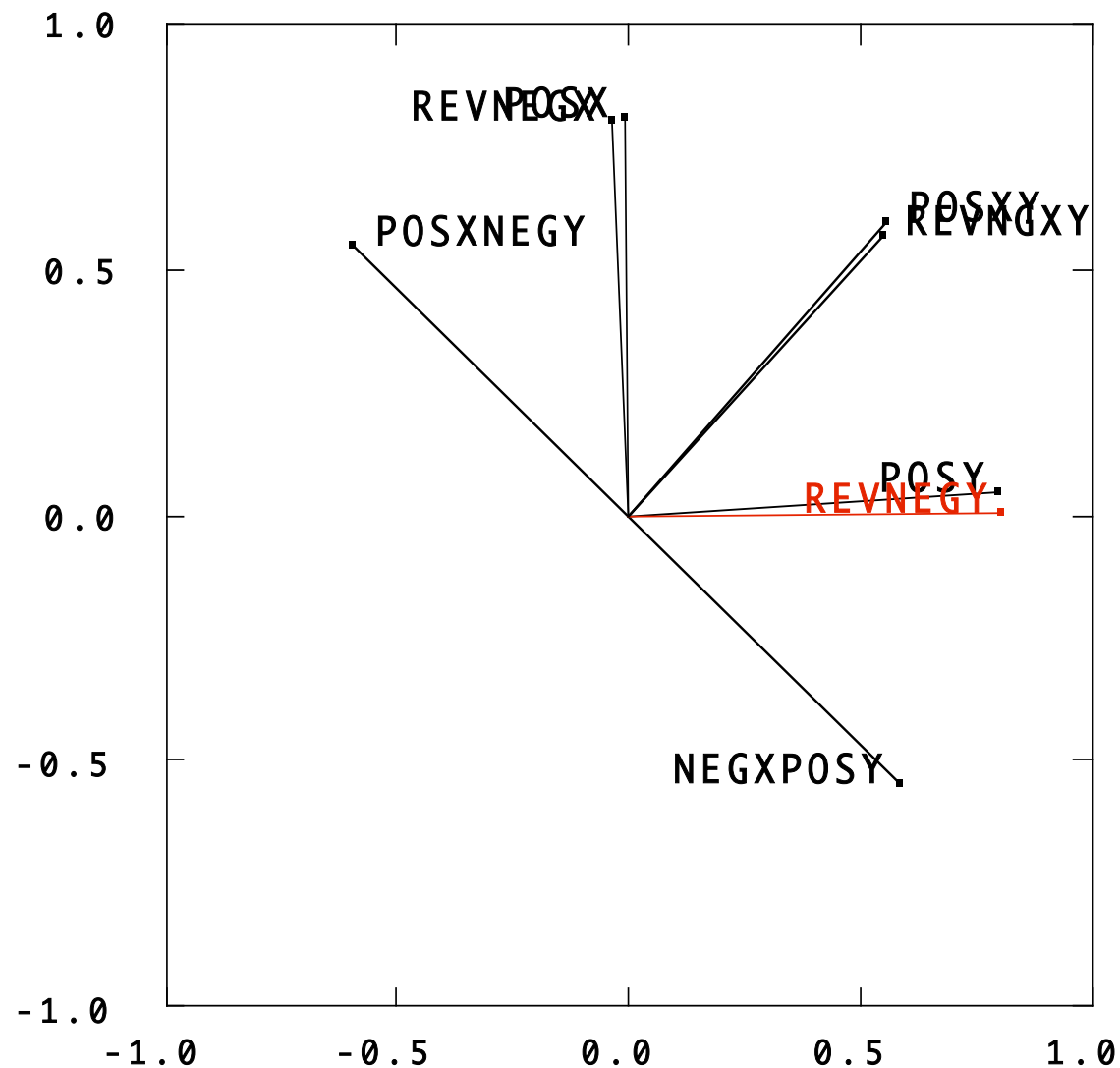
Structural representation



Factor analysis shows structure

	1	2 angle	
POSX	-0.01	0.81	89
POSY	0.80	0.05	4
POSXY	0.56	0.59	46
POSXN	-0.59	0.55	137
NEGXP	0.58	-0.55	313
REVNE	-0.04	0.80	87
REVNE	0.80	0.00	90
REVNG	0.55	0.57	46

Factor analysis shows structure



Hyperplanes and Zeros:

Defining variables by what they are not

- Tendency to interpret variables in terms of their patterns of high correlations
- But large correlations may be attenuated by skew or error
- Correlation of .7 is an angle of 45 degrees => lots of room for misinterpretation!
- Zero correlations reflect what a variable is not
 - Zeros provide definition by discriminant validity

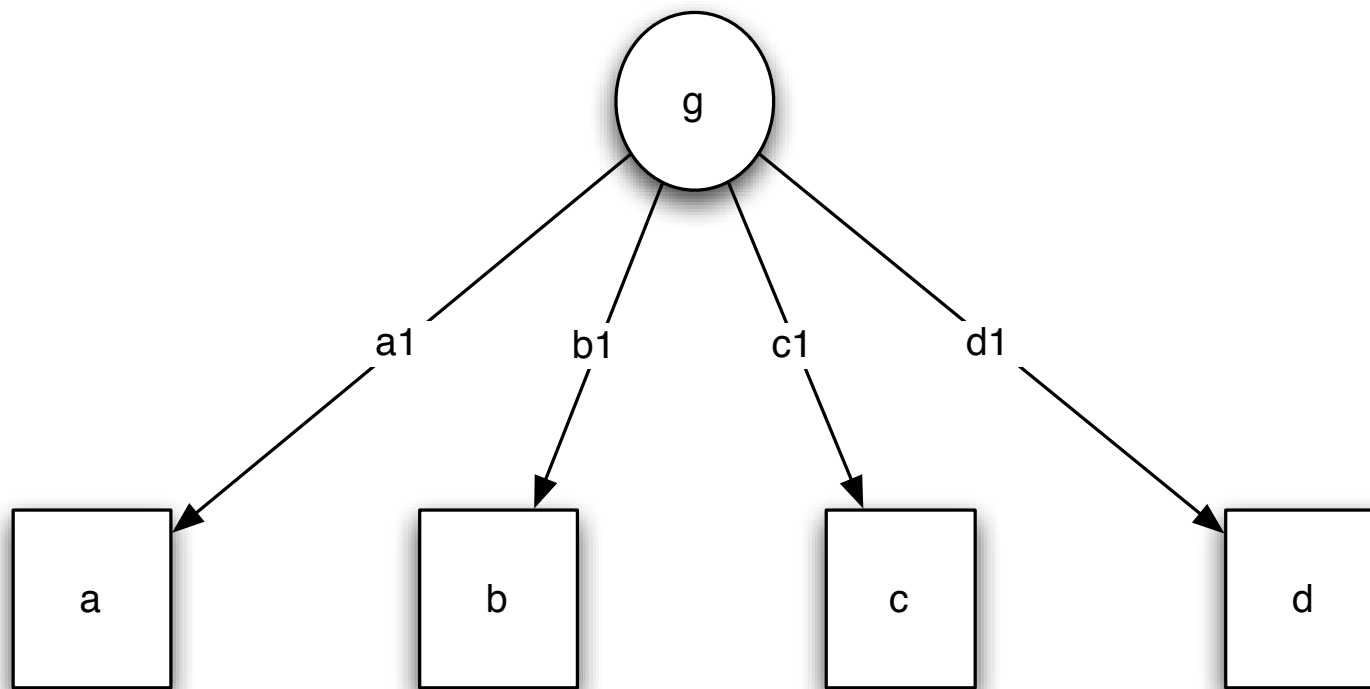
Structural Equation Models

- Estimating factor model of reliability (the measurement model)
- Estimates of validity model corrected for attenuation (the structural model)
- Maximum likelihood solutions of covariance matrices

correlation matrix

	A	B	C	D
A	1.00	0.30	0.20	0.10
B	0.30	1.0	0.20	0.20
C	0.20	0.20	1.00	0.30
D	0.10	0.20	0.30	1.00

Hypothetical structure

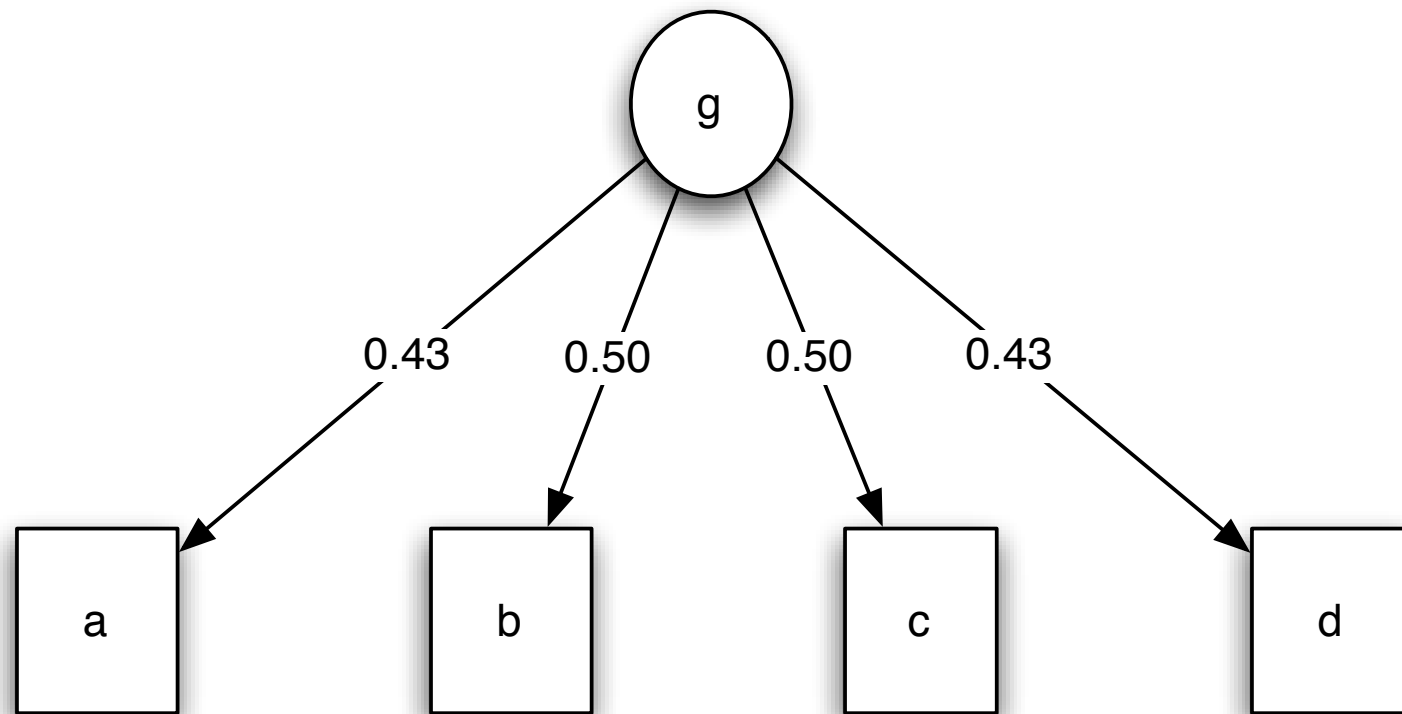


R for sem

```
obs.var2.12 = c('a', 'b', 'c', 'd')
R.prob2.12 = matrix(c(
  1.00 , .30, .20, .10,
  .30, 1.00, .20, .20,
  .20, .20, 1.00, .30,
  .10, .20, .30, 1.00),
  ncol=4,byrow=TRUE)

model2.12a=matrix(c(
  'g -> a',      'a1', NA,
  'g -> b' ,     'b1', NA,
  'g -> c' ,     'c1', NA,
  'g -> d',      'd1', NA,
  'a <-> a',      'e1', NA,
  'b <-> b',      'e2', NA,
  'c <-> c',      'e3', NA,
  'd <-> d',      'e4', NA,
  'g <-> g',      NA, 1),
  ncol=3, byrow=TRUE)
sem2.12a= sem(model2.12a,R.prob2.12,120, obs.var2.12)
summary(sem2.12a,digits=3)
```


SEM of Congeneric



Summary of SEM

Parameter Estimates

	Estimate	Std Error	z value	Pr(> z)		
a1	0.431	0.142	3.03	2.48e-03	a <---	g
b1	0.503	0.153	3.29	1.00e-03	b <---	g
c1	0.503	0.153	3.29	1.00e-03	c <---	g
d1	0.431	0.142	3.03	2.48e-03	d <---	g
e1	0.815	0.143	5.69	1.30e-08	a <-->	a
e2	0.747	0.160	4.67	2.96e-06	b <-->	b
e3	0.747	0.160	4.67	2.96e-06	c <-->	c
e4	0.815	0.143	5.69	1.30e-08	d <-->	d

Iterations = 9

Model Chisquare = 4.64 Df = 2 Pr(>Chisq) = 0.0985

Chisquare (null model) = 32.7 Df = 6

Goodness-of-fit index = 0.98

Adjusted goodness-of-fit index = 0.902

RMSEA index = 0.105 90% CI: (NA, 0.234)

Bentler-Bonnett NFI = 0.858

Tucker-Lewis NNFI = 0.704

Bentler CFI = 0.901

SRMR = 0.0496

BIC = -4.94

Normalized Residuals

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-0.91600	-0.27100	-0.08770	-0.00535	0.22300	0.89100

Compare with factanal

Uniquenesses:

```
[1] 0.815 0.747 0.747 0.815
```

Loadings:

```
      Factor1      factanal(covmat= R.prob2.12,factors=1,  
[1,] 0.431                               n.obs=120)
```

```
[2,] 0.503
```

```
[3,] 0.503
```

```
[4,] 0.431
```

Factor1

```
SS loadings      0.876
```

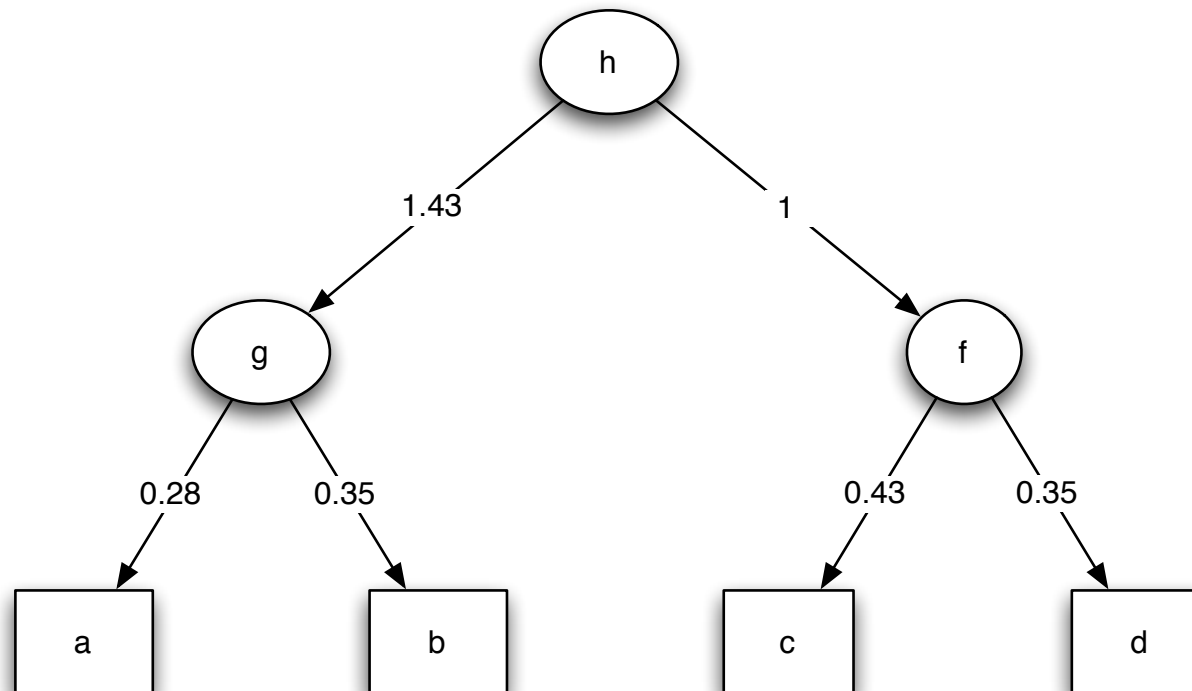
```
Proportion Var   0.219
```

Test of the hypothesis that 1 factor is
sufficient.

The chi square statistic is 4.52 on 2 degrees
of freedom.

The p-value is 0.104

an alternative model



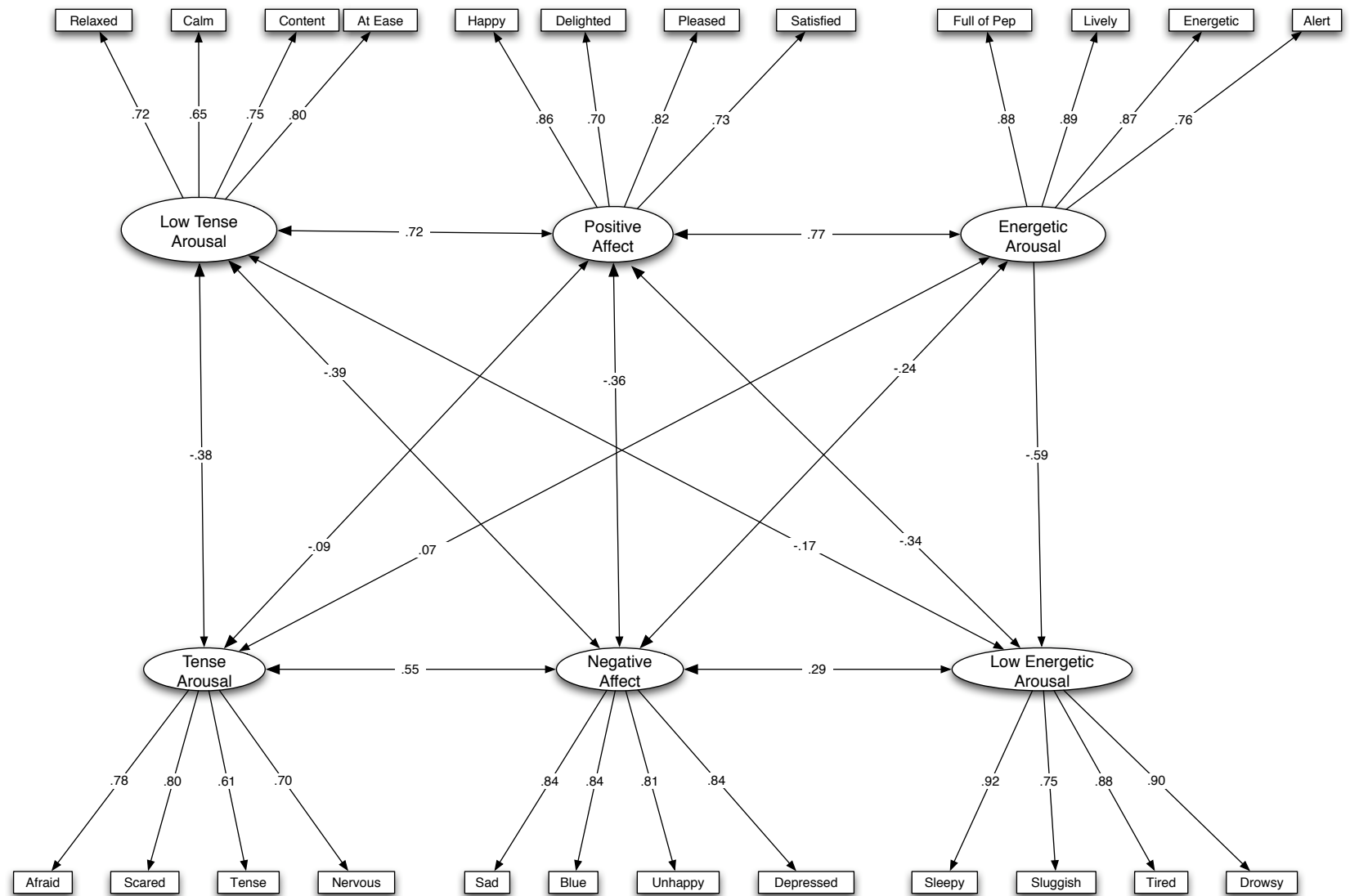
Structure of Affect

I. Is happy the opposite of sad?

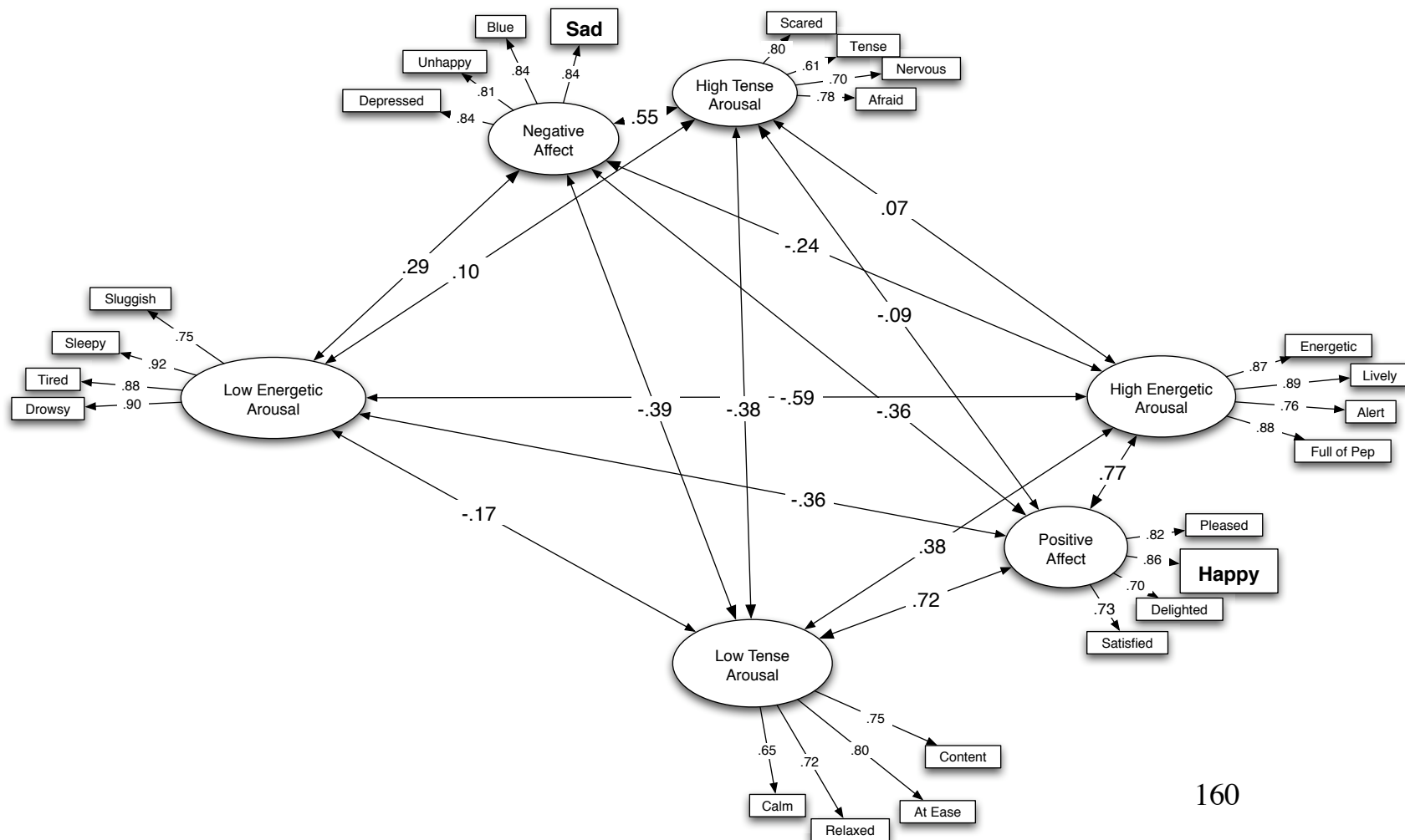
II. Is Positive Affect = - Negative Affect

III. What are the dimensions of Affect

IV. 75 affect words collected over multiple studies for > 3800 subjects



Structure of Affect



Comparing regression to latent modeling

- I. Regression weights change as a function of number of correlated predictors
- II. Path weights do not change
- III. Consider the following 6 predictors of 1 criterion

6 predictors - 1 criterion

	R						
	V1	V2	V3	V4	V5	V6	Y
V1	1.00	0.36	0.36	0.36	0.36	0.36	0.48
V2	0.36	1.00	0.36	0.36	0.36	0.36	0.48
V3	0.36	0.36	1.00	0.36	0.36	0.36	0.48
V4	0.36	0.36	0.36	1.00	0.36	0.36	0.48
V5	0.36	0.36	0.36	0.36	1.00	0.36	0.48
V6	0.36	0.36	0.36	0.36	0.36	1.00	0.48
Y	0.48	0.48	0.48	0.48	0.48	0.48	1.00

A one factor model

> f

V PAI

1 1 0.6

2 2 0.6

3 3 0.6

4 4 0.6

5 5 0.6

6 6 0.6

7 7 0.8

PAI

SS loadings 2.8

Proportion Var 0.4

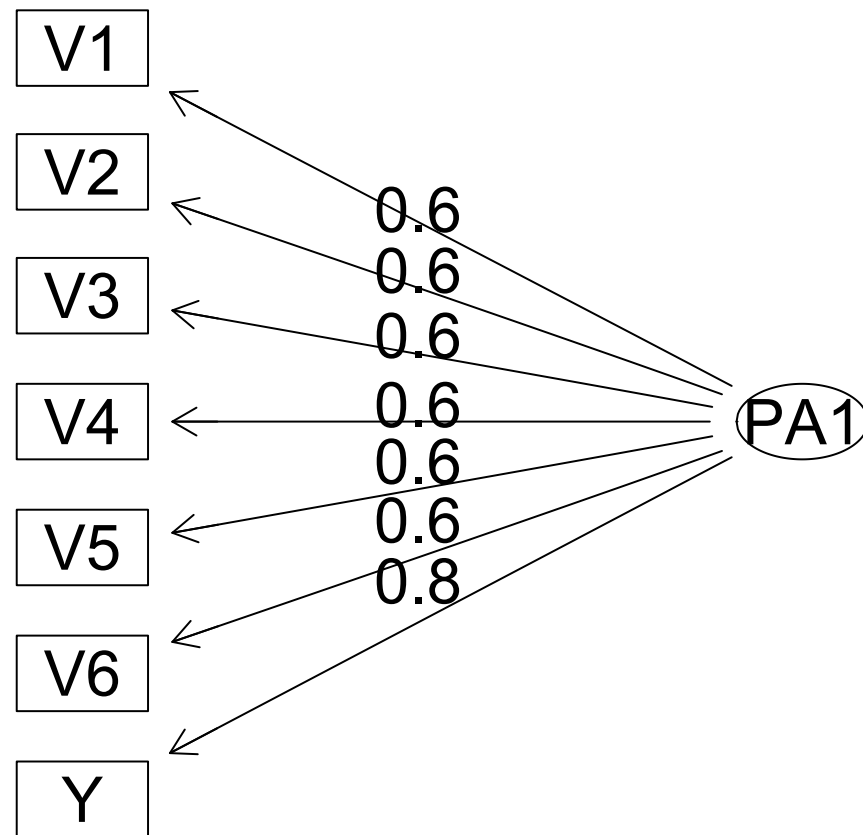
Test of the hypothesis that 1 factor is sufficient.

The degrees of freedom for the model is 14 and the fit was 0

Congeneric factor model

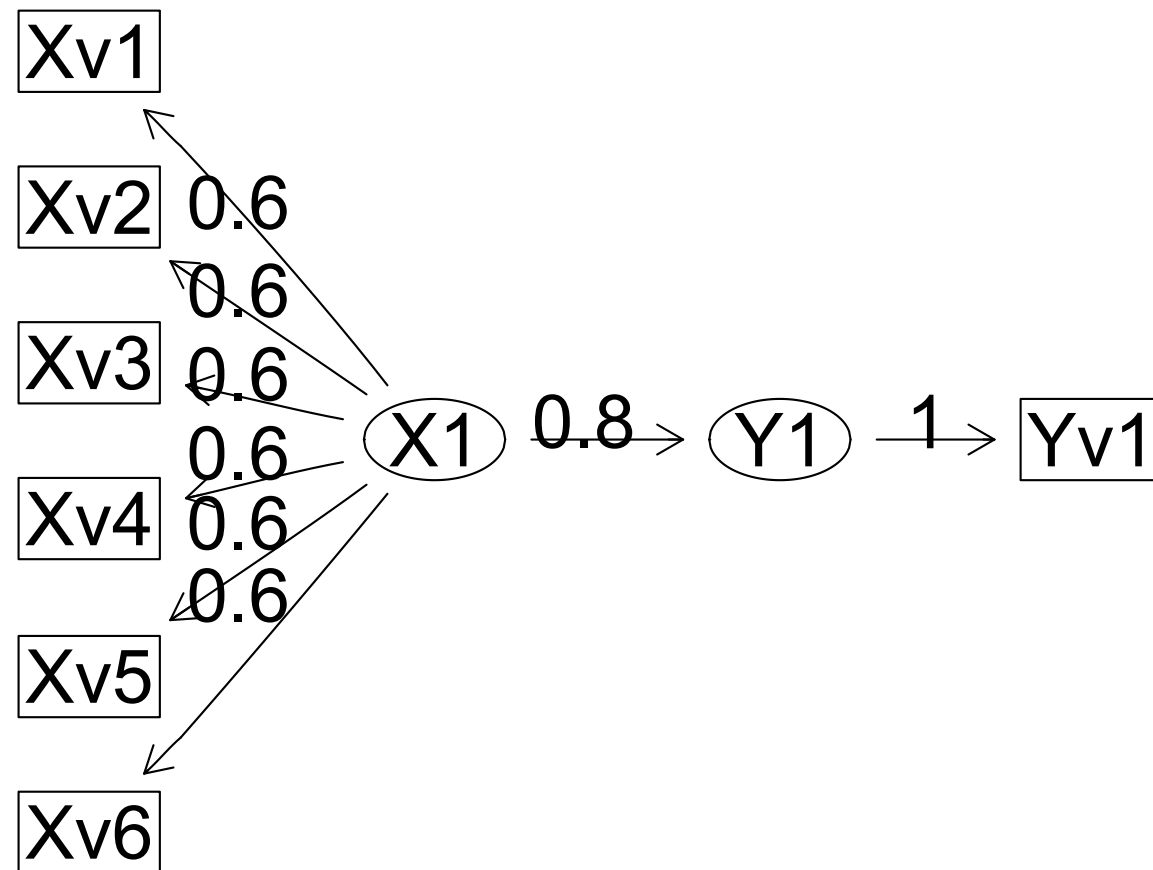
Structural model

Congeneric
factor model
accounts for
all the
observed
correlations



Structural model

All
correlations
are modeled

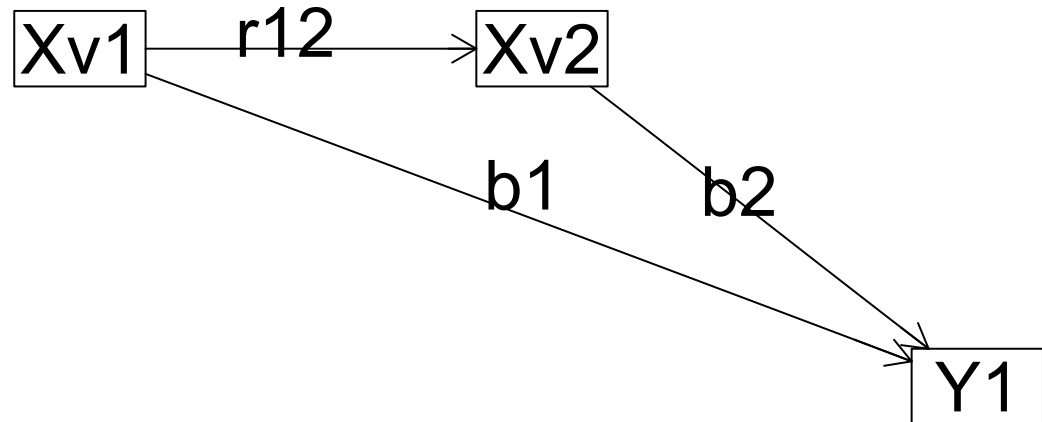


Regression model has direct and indirect paths

Regression model

$$r_{x1y} = b_1 + r_{12}b_2$$

$$r_{x2y} = b_2 + r_{12}b_1$$

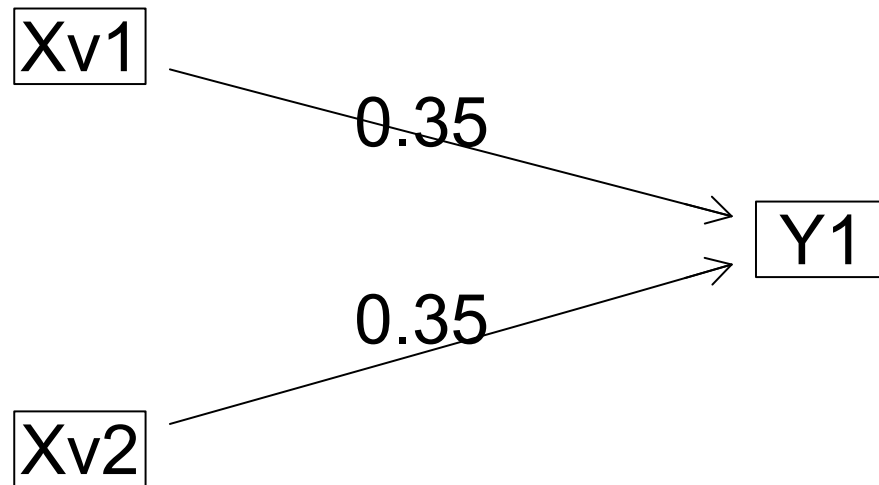


But, consider regression

$$R = .58$$

Regression model

Correlations
of .36
between
predictors
are not
shown



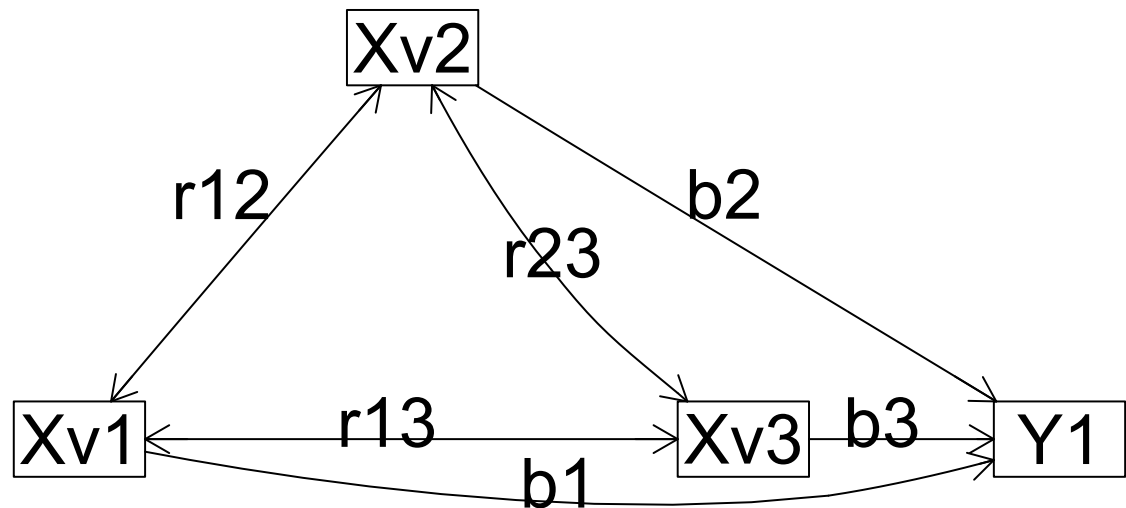
Regression weights

$$r_{x1y} = b_1 + r_{12}b_2 + r_{13}b_3$$

$$r_{x2y} = b_2 + r_{23}b_3$$

$$r_{x3y} = b_3 + r_{13}b_1 + r_{23}b_2$$

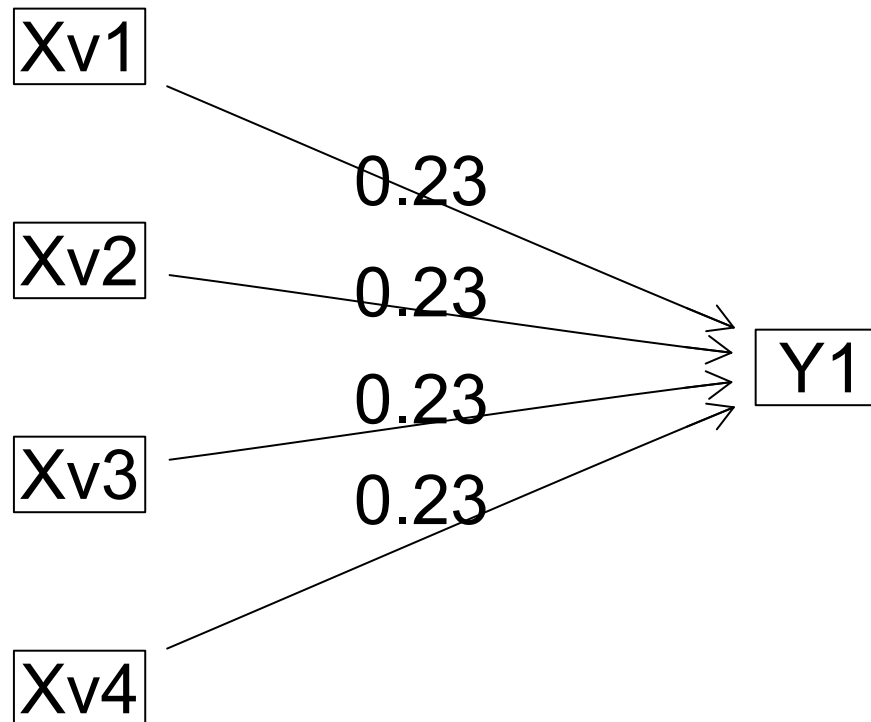
Regression model



Four predictors $R = .67$

Regression model

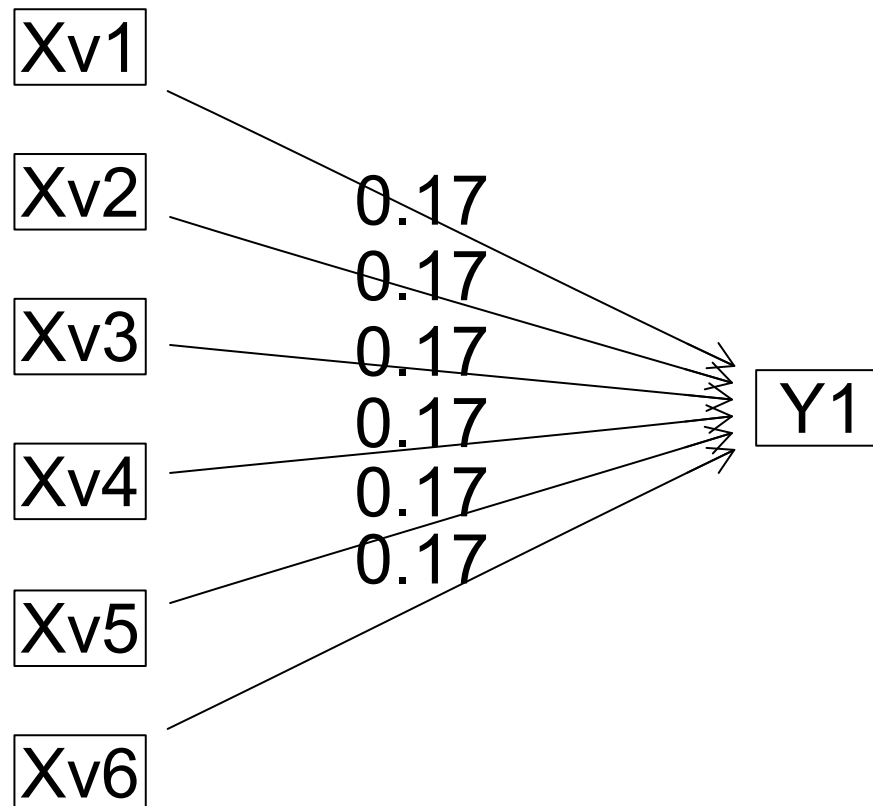
Correlations
of .36
between
predictors
are not
shown



6 predictors: $R = .7$

Regression model

Correlations
of .36
between
predictors
are not
shown



Regression vs. structural model

- Regression weights decrease as number of predictors increase
- Structural weights do not change
- If N predictors are equally correlated with each other (a_{ij}) and with criterion (r_{iy}), then $r_{iy} = b_{iy} + (N-1)a_{ij} b_{iy}$ which implies
 - $b_{iy} = r_{ij}/(1+(N-1)a_{ij})$ and $R^2 = \text{sum}(b_{iy} r_{iy}) = N b_{iy} r_{iy}$
 - $R^2 = N r_{ij}^2/(1+(N-1)a_{ij})$ which at limit $\Rightarrow$
 - $R = r_{ij}/\text{sqrt}(a_{ij})$ which is just the disattenuated r