Psychology 454: Latent Variable Modeling

lavaan and LISREL – further comments

Department of Psychology
Northwestern University
Evanston, Illinois USA

February, 2011

Outline

Prior comparisons with lavaan and LISREL suggested differences in χ^2 estimates

- Consider the case of the Bollen development data set
  - 3 measures of Industrialization
  - 4 measures of democracy in 1960
  - 4 measures of democracy in 1965
- Variety of constraints on model to achieve good fit
  - Correlated specifics
  - Constrained factor loadings

Lisrel example

```
model <- '
  # latent variable definitions
     ind60 =~ x1 + x2 + x3
     dem60 =~ y1 + y2 + y3 + y4
     dem65 =~ y5 + equal("dem60=~y2")*y6
                 + equal("dem60=~y3")*y7
                 + equal("dem60=~y4")*y8

  # regressions
    dem60 ~ ind60
    dem65 ~ ind60 + dem60
  # residual correlations
    y1 ~~ y5
    y2 ~~ y6
    y3 ~~ y7
    y4 ~~ y8'
fit <- sem(model, data=PoliticalDemocracy,
   likelihood='wishart')
summary(fit, fit.measures=TRUE)
```

```
Lavaan (0.4-6) converged normally after 88 iterations
  Number of observations                            75
  Estimator                                         ML
  Minimum Function Chi-square                   53.156
  Degrees of freedom                                40
  P-value                                        0.080
Chi-square test baseline model:
  Minimum Function Chi-square                  720.912
  Degrees of freedom                                55
  P-value                                        0.000
Full model versus baseline model:
  Comparative Fit Index (CFI)                    0.980
```

```
Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1561.202
  Loglikelihood unrestricted model (H1)      -1534.265

  Number of free parameters                         26
  Akaike (AIC)                                3174.405
  Bayesian (BIC)                              3234.659
  Sample-size adjusted Bayesian (BIC)         3152.714

Root Mean Square Error of Approximation:

  RMSEA                                          0.066
  90 Percent Confidence Interval          0.000  0.110
  P-value RMSEA <= 0.05                          0.277

Standardized Root Mean Square Residual:

  SRMR                                           0.061
```

With parameters

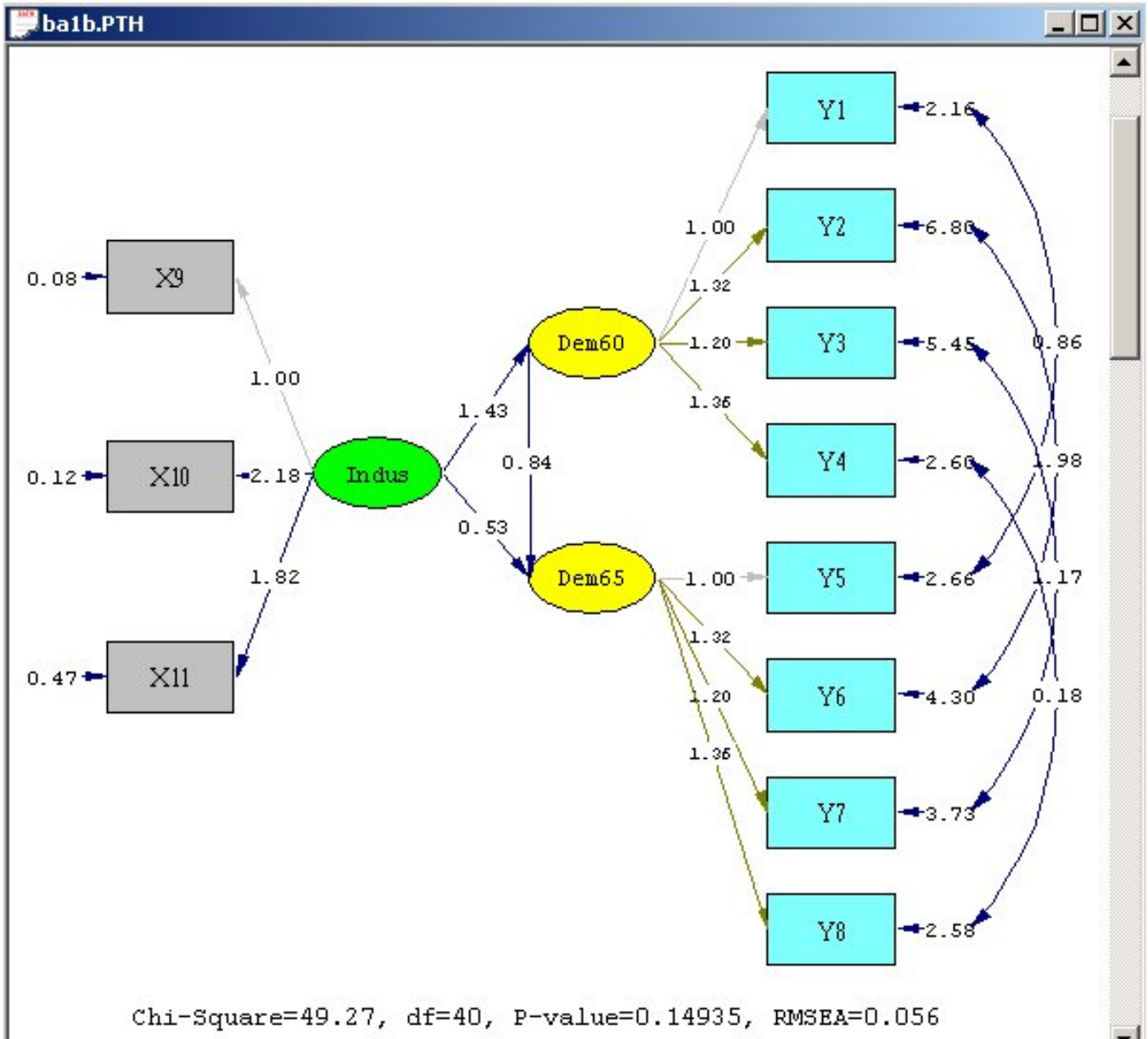
```
                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  ind60 =~
    x1                1.000
    x2                2.180    0.140   15.620    0.000
    x3                1.819    0.153   11.888    0.000
  dem60 =~
    y1                1.000
    y2                1.318    0.149    8.869    0.000
    y3                1.203    0.133    9.034    0.000
    y4                1.363    0.128   10.686    0.000
  dem65 =~
    y5                1.000
    y6                1.318
    y7                1.203
    y8                1.363

Regressions:
  dem60 ~
    ind60             1.427    0.381    3.747    0.000
  dem65 ~
    ind60             0.525    0.215    2.445    0.014
    dem60             0.845    0.071   11.850    0.000

Covariances:
  y1 ~~
    y5                0.862    0.370    2.327    0.020
  y2 ~~
    y6                1.978    0.786    2.516    0.012
  y3 ~~
    y7                1.171    0.638    1.836    0.066
  y4 ~~
    y8                0.179    0.478    0.374    0.708

Variances:
    x1                0.083    0.020    4.147    0.000
    x2                0.122    0.071    1.702    0.089
    x3                0.473    0.092    5.138    0.000
    y1                2.157    0.454    4.747    0.000
    y2                6.799    1.265    5.374    0.000
    y3                5.452    1.021    5.340    0.000
    y4                2.601    0.669    3.890    0.000
    y5                2.664    0.515    5.172    0.000
    y6                4.301    0.841    5.117    0.000
    y7                3.729    0.724    5.151    0.000
    y8                2.576    0.620    4.158    0.000
    ind60             0.455    0.088    5.138    0.000
    dem60             3.659    0.845    4.329    0.000
    dem65             0.367    0.199    1.851    0.064
```

LISREL output



LISREL output – courtesy of Yves Rosseel

DATE: 2/15/2011
TIME: 19:22

L I S R E L 8.72

BY

Karl G. Joreskog & Dag Sorbom

This program is published exclusively by
Scientific Software International, Inc.
7383 N. Lincoln Avenue, Suite 100
Lincolnwood, IL 60712, U.S.A.
Phone: (800)247-6113, (847)675-0720, Fax: (847)675-2140

Website: www.ssicentral.com

LISREL commands for Bollen Democracy problem

```
DA NI=11 NO=0 MA=CM XM=-999999
LA
y1 y2 y3 y4 y5 y6 y7 y8 x1 x2 x3
RA FI=democindus.txt
MO NY=11 NE=3 PS=DI,FR TE=FU,FI BE=FU,FI
LK
dem60 dem65 ind60
FR LY(2,1) LY(3,1) LY(4,1)
!FR LY(6,2) LY(7,2) LY(8,2)
FR LY(10,3) LY(11,3)
EQ LY(2,1) LY(6,2)
EQ LY(3,1) LY(7,2)
EQ LY(4,1) LY(8,2)
VA 1 LY(1,1) LY(5,2) LY(9,3)
FR TE(1,1) TE(2,2) TE(3,3) TE(4,4) TE(5,5) TE(6,6)
FR TE(7,7) TE(8,8) TE(9,9) TE(10,10) TE(11,11)
!FR TE(5,1) TE(4,2) TE(6,2) TE(7,3) TE(8,4) TE(8,6)
FR TE(5,1) TE(6,2) TE(7,3) TE(8,4)
FR BE(1,3) BE(2,1) BE(2,3)
OU ME=ML ND=3
```

More LISREL output

Covariance Matrix

	y1	y2	y3	y4	y5	y6
y1	6.879					
y2	6.251	15.580				
y3	5.839	5.839	10.764			
y4	6.089	9.509	6.688	11.219		
y5	5.064	5.603	4.939	5.702	6.826	
y6	5.746	9.386	4.727	7.442	4.977	11.375
y7	5.812	7.535	7.006	7.488	5.821	6.748
y8	5.671	7.758	5.639	8.013	5.339	8.247
x1	0.734	0.619	0.787	1.150	1.082	0.853
x2	1.273	1.491	1.552	2.241	2.064	1.805
x3	0.911	1.170	1.039	1.838	1.583	1.572

Covariance Matrix

	y7	y8	x1	x2	x3
y7	10.799				
y8	7.592	10.534			
x1	0.937	1.103	0.537		
x2	1.996	2.234	0.990	2.282	
x3	1.626	1.692	0.823	1.806	1.976

More LISREL output

```
[This syntax is autogenerated]
Parameter Specifications
        LAMBDA-Y
              ETA 1      ETA 2      ETA 3
            --------   --------   --------
      y1           0          0          0
      y2           1          0          0
      y3           2          0          0
      y4           3          0          0
      y5           0          0          0
      y6           0          1          0
      y7           0          2          0
      y8           0          3          0
      x1           0          0          0
      x2           0          0          4
      x3           0          0          5

        BETA
              ETA 1      ETA 2      ETA 3
            --------   --------   --------
   ETA 1           0          0          6
   ETA 2           7          0          8
   ETA 3           0          0          0

        PSI
              ETA 1      ETA 2      ETA 3
            --------   --------   --------
                9         10         11
```

The LISREL results

```
Number of Iterations =  9

LISREL Estimates (Maximum Likelihood)

        LAMBDA-Y

              ETA 1      ETA 2      ETA 3
            --------   --------   --------
      y1       1.000      - -        - -

      y2       1.318      - -        - -
             (0.149)
              8.869

      y3       1.203      - -        - -
             (0.133)
              9.034

      y4       1.363      - -        - -
             (0.128)
              10.686

      y5        - -       1.000      - -

      y6        - -       1.318      - -
                        (0.149)
                         8.869

      y7        - -       1.203      - -
                        (0.133)

        x1        - -        - -       1.000

        x2        - -        - -       2.180
                                     (0.140)
                                      15.620

        x3        - -        - -       1.819
                                     (0.153)
                                      11.888

          BETA

                    ETA 1      ETA 2      ETA 3
                  --------   --------   --------
      ETA 1        - -        - -       1.427
                                       (0.381)
                                        3.747

      ETA 2       0.845       - -       0.525
                (0.071)                (0.215)
                 11.850                 2.445

      ETA 3        - -        - -        - -
```

The regression model

Covariance Matrix of ETA

	ETA 1	ETA 2	ETA 3
ETA 1	4.584		
ETA 2	4.214	4.342	
ETA 3	0.648	0.787	0.455

PSI

Note: This matrix is diagonal.

ETA 1	ETA 2	ETA 3
3.659	0.367	0.455
(0.845)	(0.199)	(0.088)
4.329	1.851	5.138

Squared Multiple Correlations for Structural Equations

ETA 1	ETA 2	ETA 3
0.202	0.915	- -

Fits of models to variables

Squared Multiple Correlations for Y - Variables

y1	y2	y3	y4	y5	y6
0.680	0.539	0.549	0.766	0.620	0.637

Squared Multiple Correlations for Y - Variables

y7	y8	x1	x2	x3
0.627	0.758	0.846	0.947	0.761

Finally: the goodness of fit statistics of the model

```
                      Goodness of Fit Statistics

                  Degrees of Freedom = 40
        Minimum Fit Function Chi-Square = 53.156 (P = 0.0796)
Normal Theory Weighted Least Squares Chi-Square = 49.272 (P = 0.149)
           Estimated Non-centrality Parameter (NCP) = 9.272
        90 Percent Confidence Interval for NCP = (0.0 ; 31.247)

                  Minimum Fit Function Value = 0.718
          Population Discrepancy Function Value (F0) = 0.125
         90 Percent Confidence Interval for F0 = (0.0 ; 0.422)
        Root Mean Square Error of Approximation (RMSEA) = 0.0560
       90 Percent Confidence Interval for RMSEA = (0.0 ; 0.103)
         P-Value for Test of Close Fit (RMSEA < 0.05) = 0.402

          Expected Cross-Validation Index (ECVI) = 1.369
     90 Percent Confidence Interval for ECVI = (1.243 ; 1.665)
              ECVI for Saturated Model = 1.784
            ECVI for Independence Model = 17.339
```

Even more goodness of fit statistics

```
Chi-Square for Independence Model with 55 Degrees of Freedom = 1261.052
                        Independence AIC = 1283.052
                            Model AIC = 101.272
                          Saturated AIC = 132.000
                        Independence CAIC = 1319.545
                            Model CAIC = 187.527
                          Saturated CAIC = 350.954

                       Normed Fit Index (NFI) = 0.958
                    Non-Normed Fit Index (NNFI) = 0.985
                 Parsimony Normed Fit Index (PNFI) = 0.697
                    Comparative Fit Index (CFI) = 0.989
                    Incremental Fit Index (IFI) = 0.989
                      Relative Fit Index (RFI) = 0.942
                          Critical N (CN) = 89.667

                  Root Mean Square Residual (RMR) = 0.526
                        Standardized RMR = 0.0595
                    Goodness of Fit Index (GFI) = 0.892
            Adjusted Goodness of Fit Index (AGFI) = 0.822
            Parsimony Goodness of Fit Index (PGFI) = 0.541
```

Comments from Yves Rosseel (Feb 14, 2011)

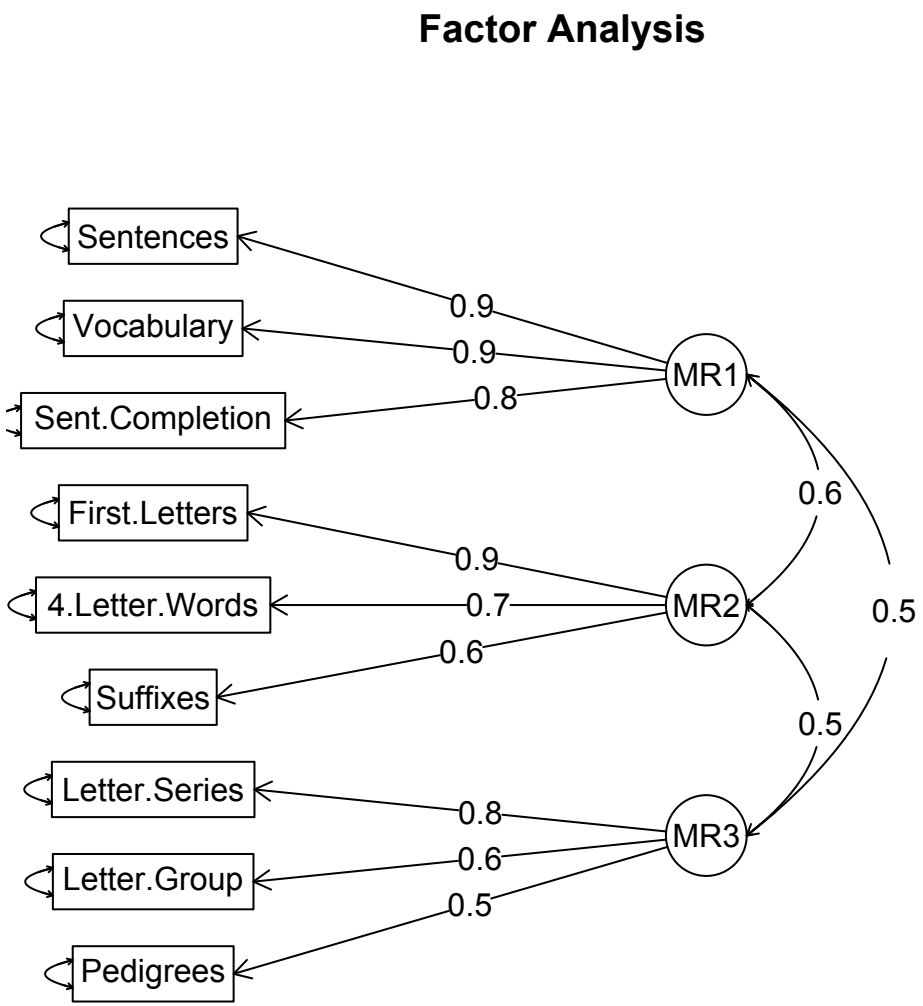
- LISREL reports (in the path diagram) not the classic 'minimum fit function chi-square', but the 'Normal Theory Weighted Least Squares Chi-Square' (which is based on GLS but with the model-implied Sigma as the weight matrix). The RMSEA is directly related to the Chi-square, so a different choice here will give a different value for RMSEA as well.
- Attached is the LISREL output for this example. As you can see, the 'minimum function' chi-square matches the lavaan one exactly.
- In the near future, I will add a mimic="LISREL" option, that takes this into account.

Structural diagrams in psych package

For the output from `fa`, `omega`, or `iclust` just diagram(result) e.g.,

```
data(bifactor)
f3 <- fa(Thurstone,3)
diagram(f3)
diagram(f3,errors=TRUE)  #add error arrows
```

diagram of EFA with errors shown



lavaan.diagram

Not yet implemented in psych for lavaan 4.-6 but shown here

```
"lavaan.diagram" <-
function(fit,title,...) {
if (is.null(fit@Model@GLIST$beta)) {model <- "cfa"} else {model <- "sem"}
if(missing(title)) {if(model =="cfa") {title="Confirmatory structure"} else {
                                        title = "Structural model"}
                          }
        fx=fit@Model@GLIST$lambda
        colnames(fx) <- fit@Model@dimNames$lambda[[2]]
         Phi <-  fit@Model@GLIST$psi
                  Rx <- fit@Model@GLIST$theta
                  v.labels <- fit@Model@dimNames$lambda[[1]]
  if(model=="cfa") {
structure.diagram(fx=fx,Phi=Phi,Rx=Rx,labels=v.labels,main=title,...)}
else {structure.diagram(fx=fx,Phi=fit@Model@GLIST$beta,Rx=Rx,
                   labels=v.labels,main=title,...) }
}
```