

Psychology 454: Latent Variable Modeling

Putting it all together:
Taking latent variables seriously

Department of Psychology
Northwestern University
Evanston, Illinois USA



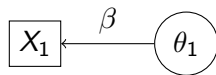
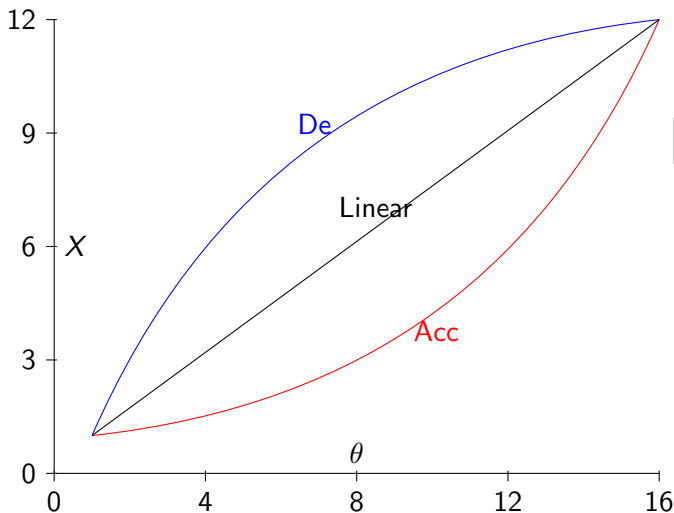
March 9, 2011

- 1 Data = Model + Residual
 - Types of data and types of scales
 - Correlation and regression
 - Multivariate Regression and Partial Correlation
 - Wright's rules: Applying path models to regression
- 2 Measurement models
 - Reliability models (Including Item Response Theory models)
 - Exploratory models
 - Confirmatory models
- 3 Structural models
 - Path diagram approach to structure using regression
 - Measurement + structure
 - Multiple Indicators, Multiple Causes
- 4 More complicated designs
 - STARTS models as a general framework
 - simplex cases as special change models
 - Applications in genetics

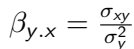
Data = Model + Residual

- ① What are the data?
 - Comparisons of rank order ($O_i < S_j, O_i < O_j, S_i < S_j$)
 - Comparisons of proximity ($|O_i - S_j| < \delta, |O_i - S_j| < |O_k - S_l|$)
- ② How are they modeled?
 - As observed variables
 - Correlation of OVs
 - Regression of OVs
 - Principal Components
 - As latent variables
 - Correlation of LVs with OVs
 - Correlation of LVs
 - Regression of LVs
- ③ How large are the residuals?

Mapping Observed Variables to Latent Variables

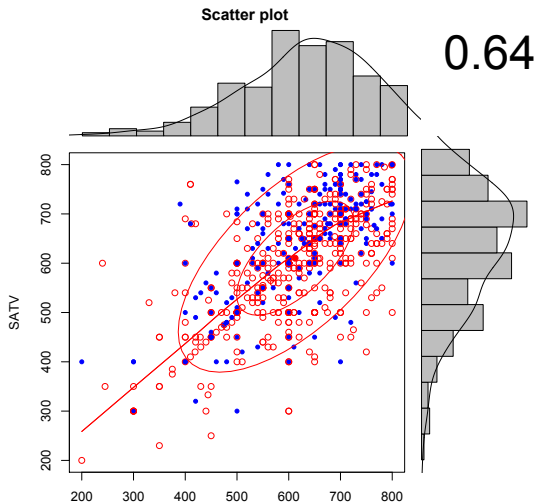


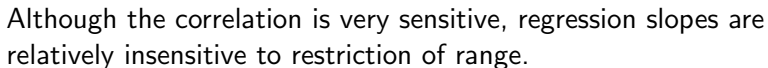
- Modeling the data
- Types of scales
 - Ordinal
 - Interval
 - Ratio

ϵ 



The correlation as a linear model



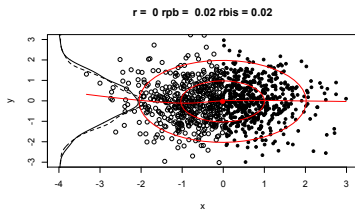
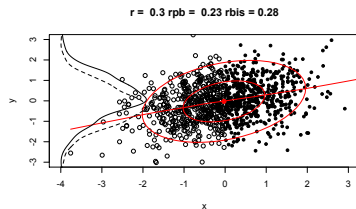
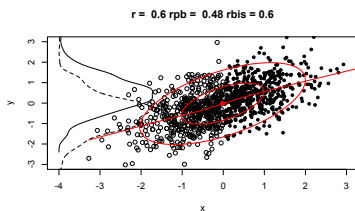
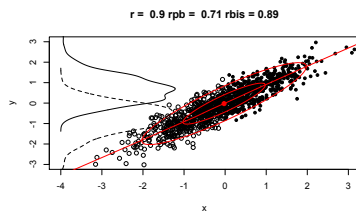


Alternative versions of the correlation coefficient

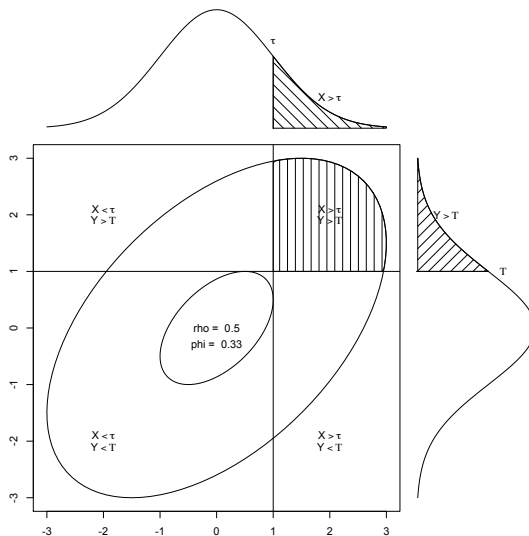
Table: A number of correlations are Pearson r in different forms, or with particular assumptions. If $r = \frac{\sum x_i y_i}{\sqrt{\sum x_i^2 \sum y_i^2}}$, then depending upon the type of data being analyzed, a variety of correlations are found.

Coefficient	symbol	X	Y	Assumptions
Pearson	r	continuous	continuous	
Spearman	ρ	ranks	ranks	
Point bi-serial	r_{pb}	dichotomous	continuous	
Phi	ϕ	dichotomous	dichotomous	
Bi-serial	r_{bis}	dichotomous	continuous	normality
Tetrachoric	r_{tet}	dichotomous	dichotomous	bivariate normality
Polychoric	r_{pc}	categorical	categorical	bivariate normality

The biserial correlation estimates the latent correlation

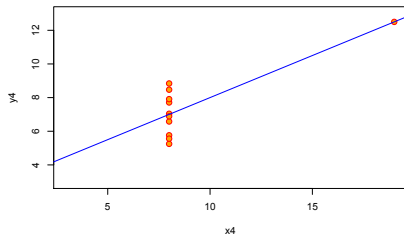
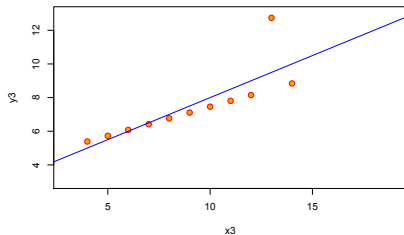
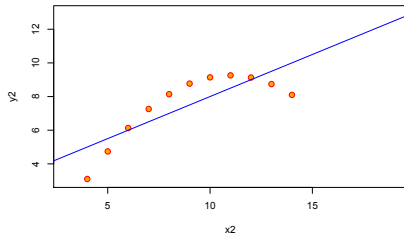
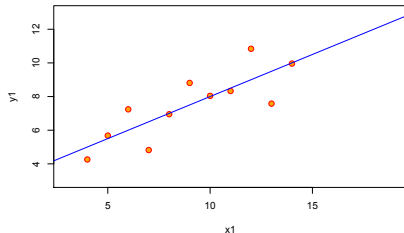


The tetrachoric correlation estimates the latent correlation



Cautions about correlations: Anscombe data set

Anscombe's 4 Regression data sets

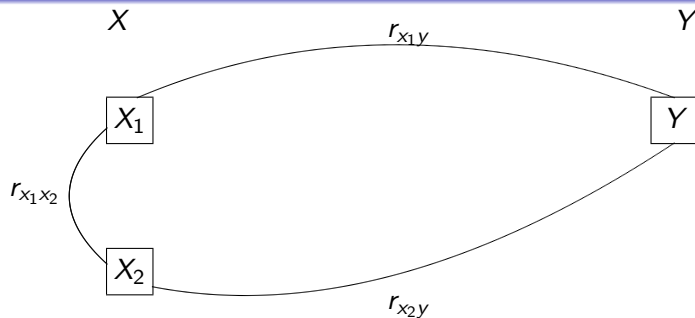


The ubiquitous correlation coefficient

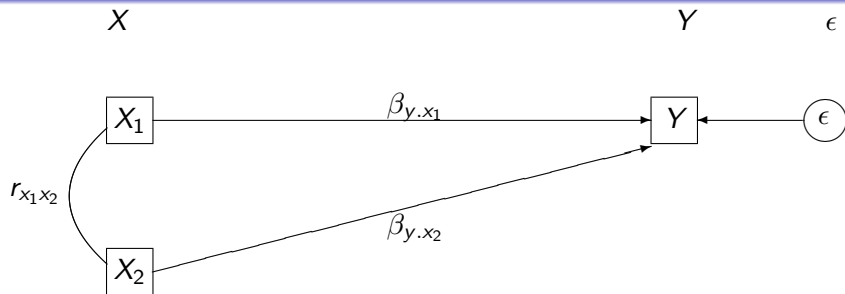
Table: Alternative Estimates of effect size. Using the correlation as a scale free estimate of effect size allows for combining experimental and correlational data in a metric that is directly interpretable as the effect of a standardized unit change in x leads to r change in standardized y .

Statistic	Estimate	r equivalent	as a function of r
Pearson correlation	$r_{xy} = \frac{C_{xy}}{\sigma_x \sigma_y}$	r_{xy}	
Regression	$b_{y.x} = \frac{C_{xy}}{\sigma_x^2}$	$r = b_{y.x} \frac{\sigma_y}{\sigma_x}$	$b_{y.x} = r \frac{\sigma_x}{\sigma_y}$
Cohen's d	$d = \frac{X_1 - X_2}{\sigma_x}$	$r = \frac{d}{\sqrt{d^2 + 4}}$	$d = \frac{2r}{\sqrt{1 - r^2}}$
Hedge's g	$g = \frac{X_1 - X_2}{s_x}$	$r = \frac{g}{\sqrt{g^2 + 4(df/N)}}$	$g = \frac{2r\sqrt{df/N}}{\sqrt{1 - r^2}}$
t - test	$t = \frac{d\sqrt{df}}{2}$	$r = \sqrt{t^2 / (t^2 + df)}$	$t = \sqrt{\frac{r^2 df}{1 - r^2}}$
F-test	$F = \frac{d^2 df}{4}$	$r = \sqrt{F / (F + df)}$	$F = \frac{r^2 df}{1 - r^2}$
Chi Square		$r = \sqrt{\chi^2 / n}$	$\chi^2 = r^2 n$
Odds ratio	$d = \frac{\ln(OR)}{1.81}$	$r = \frac{\ln(OR)}{1.81\sqrt{(\ln(OR)/1.81)^2 + 4}}$	$\ln(OR) = \frac{3.62r}{\sqrt{1 - r^2}}$
$r_{equivalent}$	r with probability p	$r = r_{equivalent}$	

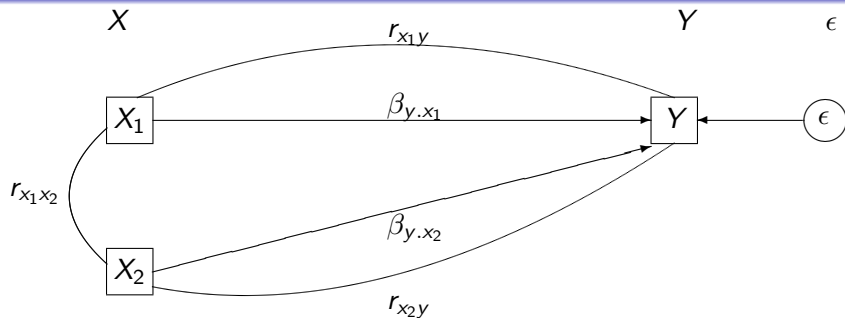
Multiple correlations



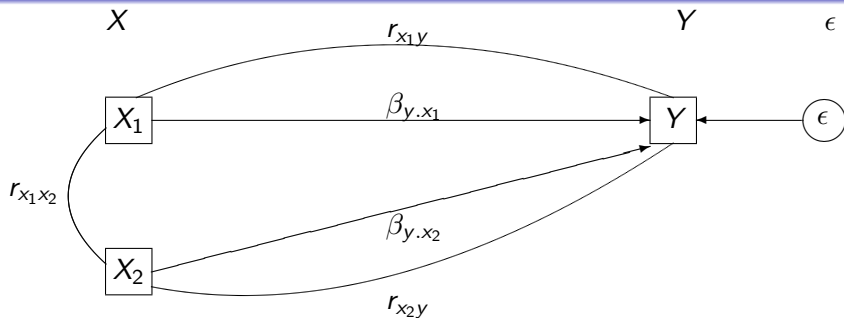
Multiple Regression



Multiple Regression: decomposing correlations



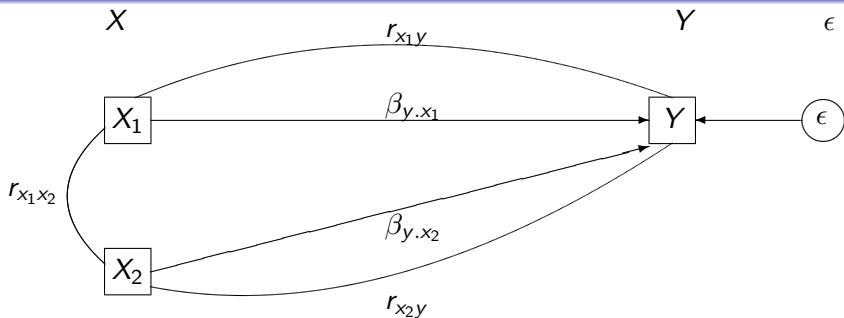
Multiple Regression: decomposing correlations



$$r_{x_1y} = \underbrace{\beta_{y.x_1}}_{\text{direct}} + \underbrace{r_{x_1x_2}\beta_{y.x_2}}_{\text{indirect}}$$

$$r_{x_2y} = \underbrace{\beta_{y.x_2}}_{\text{direct}} + \underbrace{r_{x_1x_2}\beta_{y.x_1}}_{\text{indirect}}$$

Multiple Regression: decomposing correlations



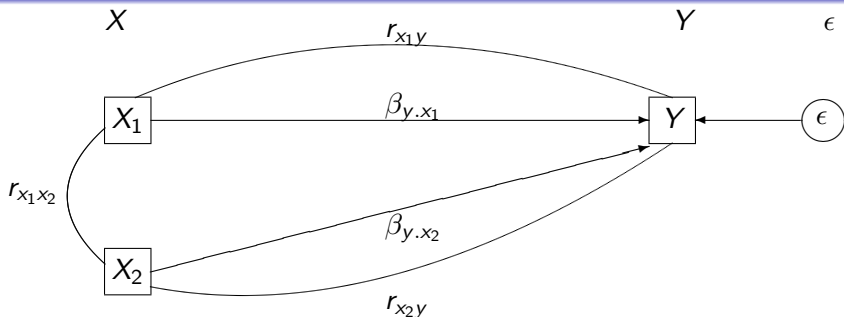
$$r_{x_1y} = \underbrace{\beta_{y.x_1}}_{\text{direct}} + \underbrace{r_{x_1x_2}\beta_{y.x_2}}_{\text{indirect}}$$

$$\beta_{y.x_1} = \frac{r_{x_1y} - r_{x_1x_2}r_{x_2y}}{1 - r_{x_1x_2}^2}$$

$$r_{x_2y} = \underbrace{\beta_{y.x_2}}_{\text{direct}} + \underbrace{r_{x_1x_2}\beta_{y.x_1}}_{\text{indirect}}$$

$$\beta_{y.x_2} = \frac{r_{x_2y} - r_{x_1x_2}r_{x_1y}}{1 - r_{x_1x_2}^2}$$

Multiple Regression: decomposing correlations



$$r_{x1y} = \underbrace{\beta_{y.x1}}_{\text{direct}} + \underbrace{r_{x1x2}\beta_{y.x2}}_{\text{indirect}}$$

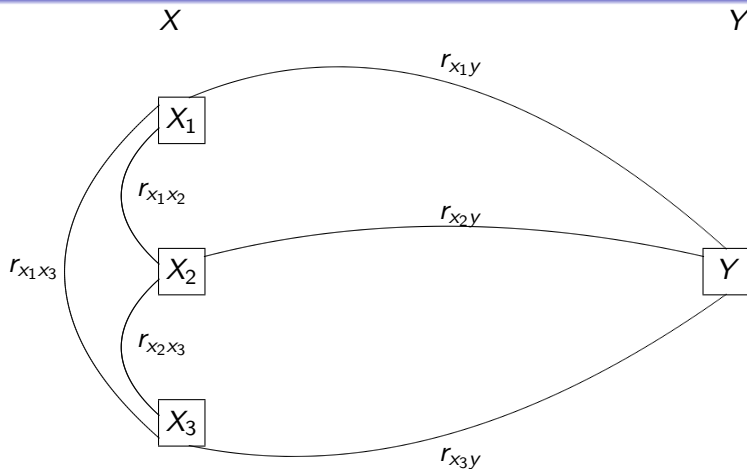
$$\beta_{y.x1} = \frac{r_{x1y} - r_{x1x2}r_{x2y}}{1 - r_{x1x2}^2}$$

$$r_{x2y} = \underbrace{\beta_{y.x2}}_{\text{direct}} + \underbrace{r_{x1x2}\beta_{y.x1}}_{\text{indirect}}$$

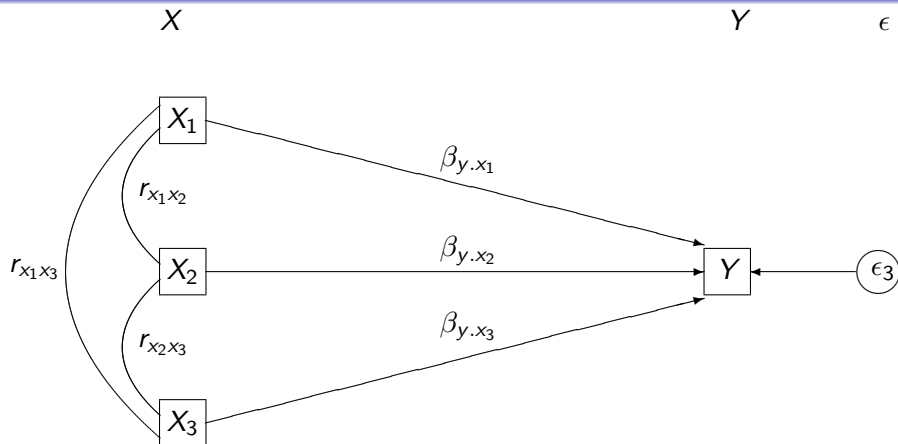
$$\beta_{y.x2} = \frac{r_{x2y} - r_{x1x2}r_{x1y}}{1 - r_{x1x2}^2}$$

$$R^2 = r_{x1y}\beta_{y.x1} + r_{x2y}\beta_{y.x2}$$

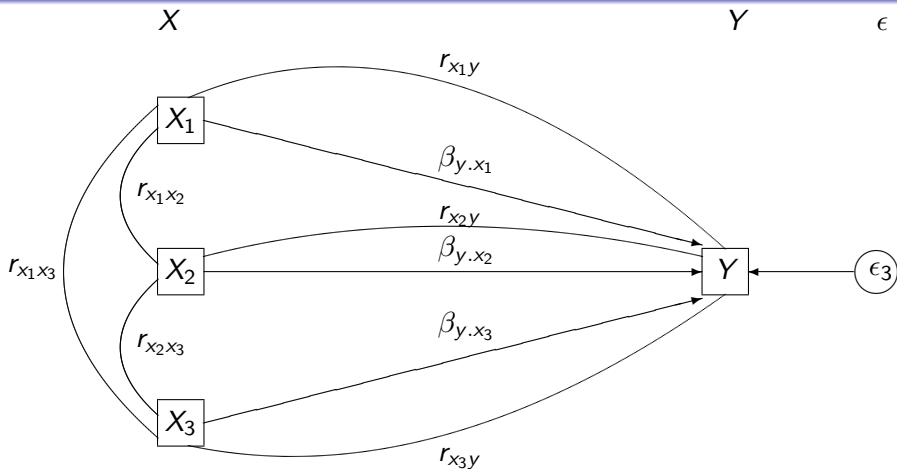
What happens with 3 predictors? The correlations



What happens with 3 predictors? β weights



What happens with 3 predictors?



$$r_{X_1 Y} = \underbrace{\beta_{Y.X_1}}_{\text{direct}} + \underbrace{r_{X_1 X_2} \beta_{Y.X_2} + r_{X_1 X_3} \beta_{Y.X_3}}_{\text{indirect}} \quad r_{X_2 Y} = \dots \quad r_{X_3 Y} = \dots$$

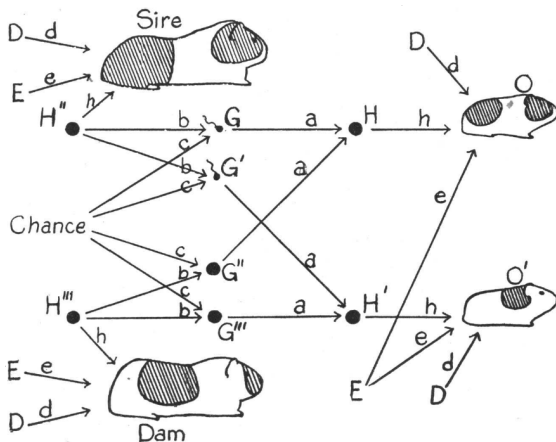
The math gets tedious

Multiple regression and matrix algebra

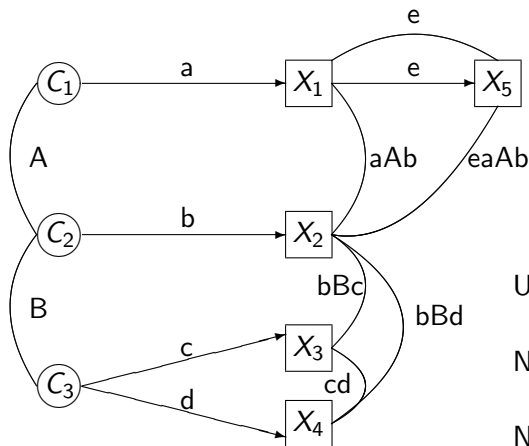
- Multiple regression requires solving multiple, simultaneous equations to estimate the direct and indirect effects.
 - Each equation is expressed as a $r_{x_i y}$ in terms of direct and indirect effects.
 - Direct effect is $\beta_{y \cdot x_i}$
 - Indirect effect is $\sum_{j \neq i} \beta_{y \cdot x_j} r_{x_j y}$
- How to solve these equations?
- Tediously, or just use **matrix algebra**

Wright's rules: Applying path models to regression

Wright's Path model of inheritance in the Guinea Pig (Wright, 1921)



The basic rules of path analysis—think genetics



Up ... and over and down ...

No down and up

No double overs

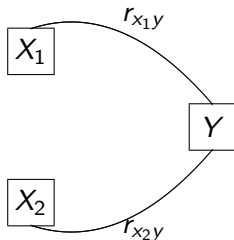
Up ... and down ...

Parents cause children

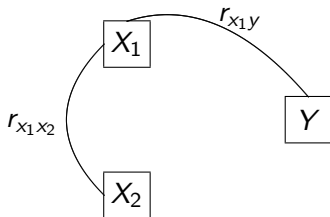
children do not cause parents

3 special cases of regression

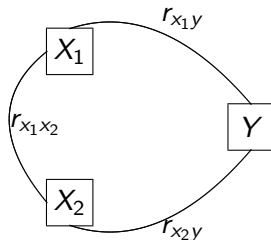
Orthogonal predictors



Suppressive predictors

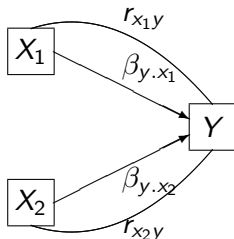


Correlated predictors

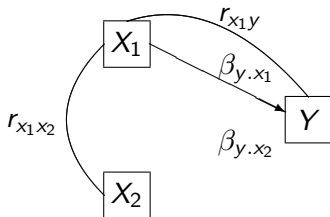


3 special cases of regression

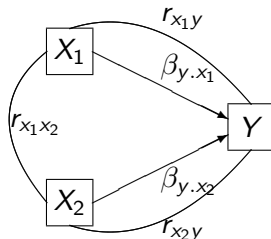
Orthogonal predictors



Suppressive predictors



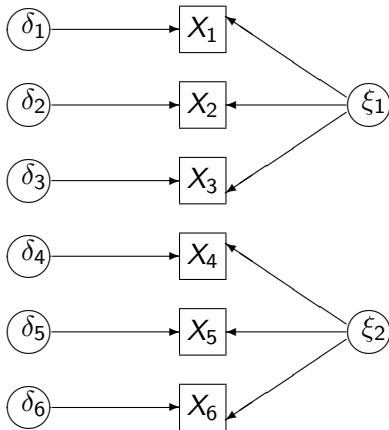
Correlated predictors



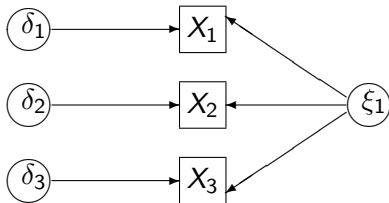
$$\beta_{y.x_1} = \frac{r_{x_1 y} - r_{x_1 x_2} r_{x_2 y}}{1 - r_{x_1 x_2}^2}$$

$$\beta_{y.x_2} = \frac{r_{x_2 y} - r_{x_1 x_2} r_{x_1 y}}{1 - r_{x_1 x_2}^2}$$

$$R^2 = r_{x_1 y} \beta_{y.x_1} + r_{x_2 y} \beta_{y.x_2}$$

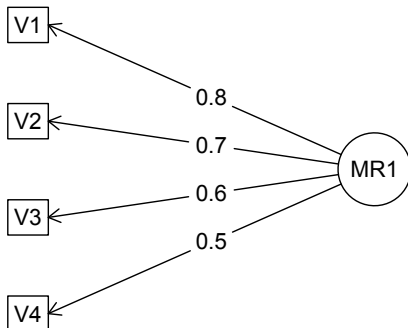
$$\delta \qquad X \qquad \xi$$


Congeneric Reliability

 δ X ξ 

Factor analysis to show a congeneric structure

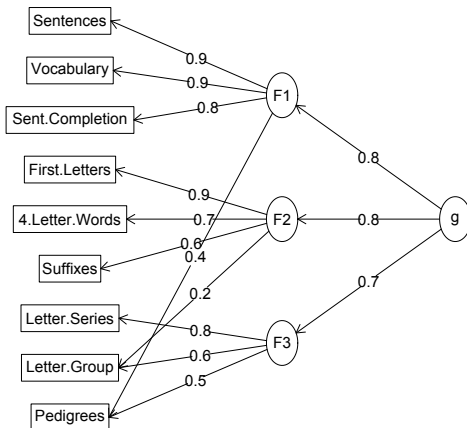
Factor Analysis



Hierarchical reliability models

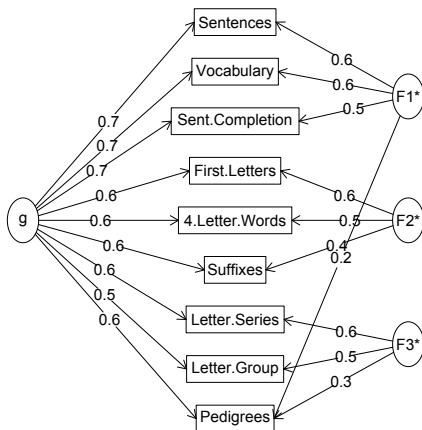
Tests can have lower level and higher order reliability

Hierarchical (multilevel) Structure



Schmid Leiman transformation estimates general factor saturation

Omega



Multiple estimates of reliability

- Generic: $\frac{\sigma_t^2}{\sigma_x^2} = \frac{\sigma_t^2}{\sigma_t^2 + \sigma_e^2} = 1 - \frac{\sigma_e^2}{\sigma_x^2}$
 - $\sigma_t^2 = \sigma_g^2 + \Sigma(\sigma_{g_i}^2) + \Sigma(\sigma_{s_j}^2)$
 - How to estimate σ_e^2 ?
 - α uses $1 - \bar{r}$ and is $\alpha = \frac{\sigma_x^2 - \Sigma(\sigma_j^2)}{\sigma_x^2} * \frac{n}{n-1}$
- $\omega_h = \frac{\sigma_g^2}{\sigma_x^2}$
- $\omega_t = \frac{\sigma_g^2 + \Sigma(\sigma_{g_i}^2)}{\sigma_x^2}$
- $\beta = \frac{2r_w}{1+r_w}$ where r_w is the correlation between the two worst split halves.
- $\beta \approx \omega_h \leq \alpha \leq \omega_t$

Guttman (1945); McDonald (1999); Revelle & Zinbarg (2009); Zinbarg, Revelle, Yovel & Li (2005)

Item Response Theory

- Item Response Theory attempts to model the response to an item in terms of two latent variables
 - Subject ability (trait value) θ
 - Item difficulty (item location) δ
 - Item discrimination (item sensitivity) α
 - $p(R|\theta, \delta) = \frac{1}{1 + e^{\alpha(\delta - \theta)}}$
- Item Response Theory may be either exploratory or confirmatory
 - Rasch Model is a confirmatory model because it constrains all factor loadings to be equal.
 - Many multidimensional IRT models constrain cross loadings to 0.
- Factor analysis of Tetrachoric/Polychoric correlations based upon normal theory yields estimates that are equivalent of IRT parameters.
 - $\delta = \frac{\tau}{\sqrt{1 - \lambda^2}}$
 - $\alpha = \frac{\lambda}{\sqrt{1 - \lambda^2}}$

Factor analysis and IRT analysis of 14 iq items

Call: fa(r = iq.tf)

Standardized loadings based
upon correlation matrix

	MR1	h2	u2
iq1	0.51	2.7e-01	0.73
iq8	0.17	2.8e-02	0.97
iq10	0.21	4.5e-02	0.96
iq15	0.08	6.3e-03	0.99
iq20	0.22	4.6e-02	0.95
iq44	0.30	9.0e-02	0.91
iq47	0.41	1.7e-01	0.83
iq2	0.07	4.3e-03	1.00
iq11	0.38	1.4e-01	0.86
iq16	0.49	2.4e-01	0.76
iq32	0.39	1.6e-01	0.84
iq37	0.00	4.9e-06	1.00
iq43	0.38	1.4e-01	0.86
iq49	0.05	2.2e-03	1.00

	MR1
SS loadings	1.34
Proportion Var	0.10

	MR1	h2	u2
iq1	0.71	5.0e-01	0.50
iq8	0.25	6.4e-02	0.94
iq10	0.29	8.4e-02	0.92
iq15	0.12	1.6e-02	0.98
iq20	0.35	1.2e-01	0.88
iq44	0.38	1.4e-01	0.86
iq47	0.51	2.6e-01	0.74
iq2	0.11	1.1e-02	0.99
iq11	0.59	3.5e-01	0.65
iq16	0.64	4.1e-01	0.59
iq32	0.52	2.7e-01	0.73
iq37	0.00	1.3e-05	1.00
iq43	0.49	2.4e-01	0.76
iq49	0.07	4.6e-03	1.00

	MR1
SS loadings	2.48
Proportion Var	0.18

Item Response Analysis using
Factor Analysis =

Call: irt.fa(x = iq.tf)

Item discrimination and location
for factor MR1

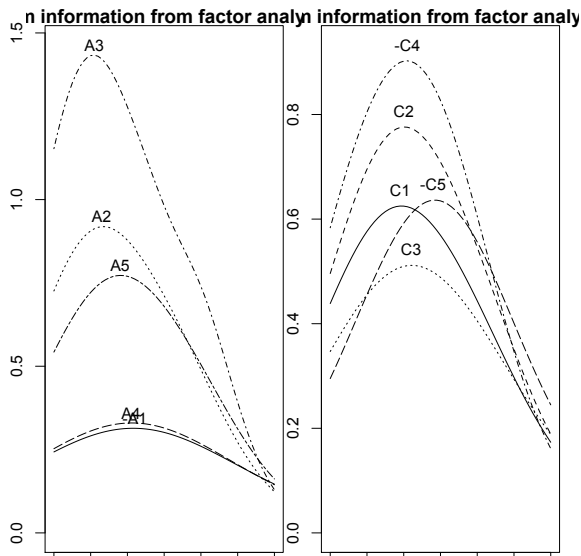
	discrimination	location
iq1	1.00	-1.35
iq8	0.26	-0.95
iq10	0.30	0.23
iq15	0.13	-0.45
iq20	0.37	-1.24
iq44	0.41	-0.46
iq47	0.60	-0.34
iq2	0.11	0.42
iq11	0.73	-1.58
iq16	0.84	-1.01
iq32	0.61	-0.19
iq37	0.00	0.60
iq43	0.57	-1.03
iq49	0.07	0.42



1. *Journal of the American Medical Association*, 2000; 284: 2689-2695.

- The generalization to ordered categorical items is relatively straightforward (Samejima, 1969).
- Much easier to see this in terms of factor of polychoric correlations with conversion to IRT parameters.
 - Example is from two scales of the Big Five.
 - Calculating polychorics is tedious
- `irt.fa` will find either tetrachoric or polychoric correlation, depending upon the data.
- Graphic output includes Item Information Curves, Item Characteristic Curves, and Test Information Curves.

Item information curves for factors 1 and 2 of BFI



100

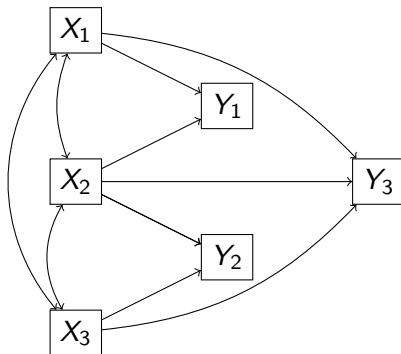
- Factors model the correlations: $R \approx FF' + U^2$
 - Factors are latent (unobserved) entities that are *estimated*
 - $\hat{F}_s = XW = XR^{-1}F$
 - Factor scores are indeterminate and need to be estimated, but the structure is determinant, given the model.
- Components model the observed scores
 - $C_s = XC$
 - This will model the error in X as well as X.
- The question of what is optimal number of factors/components to extract remains an open question. Good alternatives include:
 - Parallel analysis
 - Scree Test
 - Very Simple Structure
 - Minimum Average Partial

Downloaded from <http://ajph.org/> on November 10, 2015

- 1 Exploratory models allow all cross loadings to be estimated.
 - Rotation to simple structure makes for more interpretable solution, but the cross loadings still are not forced to zero.
 - By using more parameters than confirmatory analysis, EFA will necessarily provide better fits, but at the cost of more complexity.
 - Alternative rotations will produce different appearing solutions, but all fit equally well.
- 2 Confirmatory analysis specifies some parameters to be 0.
 - Usually not allowing many if any cross loadings.
 - This will lead to lower levels of fit, but also more parsimonious models.
- 3 The supposed advantage of confirmatory models is to allow model testing.
 - This is particularly useful when examining structural similarity across groups and across ages.
 - Knowing that the measures are the same across groups is important when making comparisons.

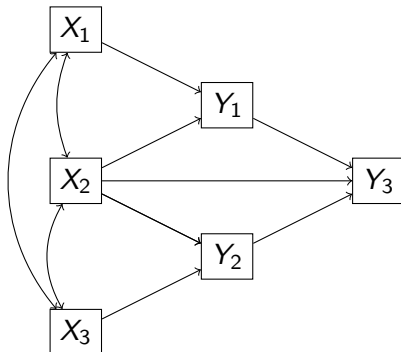
More complicated regression - 3 dependent variables

```
# ex3.11
Data <- read.table("ex3.11.dat")
names(Data) <- c("y1", "y2", "y3",
  "x1", "x2", "x3")
model.ex3.11 <- ' y1 + y2 + y3 ~ x1 + x2 + x3 '
fit <- sem(model.ex3.11, data=Data)
summary(fit, standardized=TRUE)
```



More complicated regression - a path model

```
# ex3.11
Data <- read.table("ex3.11.dat")
names(Data) <- c("y1", "y2", "y3",
  "x1", "x2", "x3")
model.ex3.11 <- ' y1 + y2 ~ x1 + x2 + x3
y3 ~ y1 + y2 + x2 '
fit <- sem(model.ex3.11, data=Data)
summary(fit, standardized=TRUE, fit.measures=TRUE)
```



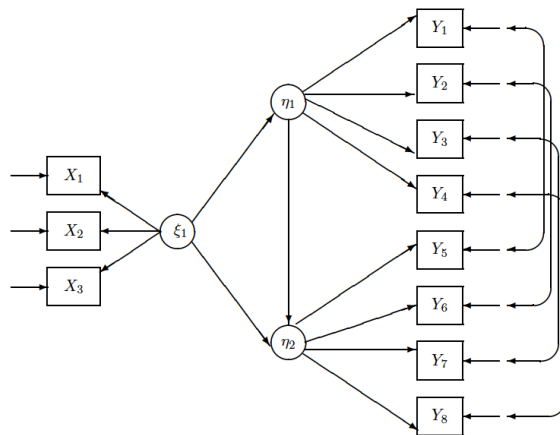
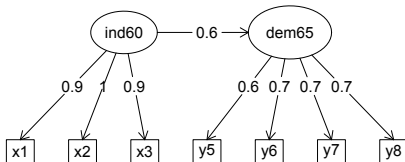


Figure 2: Panel Model of Democracy and Industrialization

The most simple model: Industrialization leads to Democracy

```
lavaan.diagram(sem.fit,lr=FALSE,e.size=.2)
```

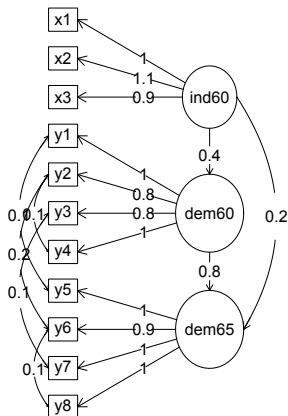
Structural model



The very modified model – another way

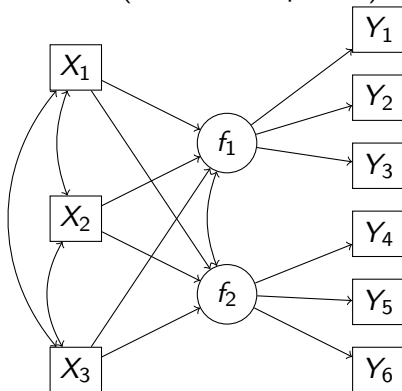
```
lavaan.diagram(fit3,cut=0)
```

Structural model



A MIMIC model

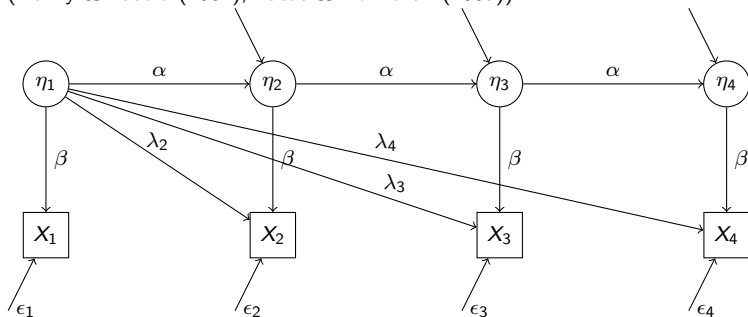
Consider the case of 6 variables with two latent factors that have 3 covariates (MPlus Example 5.8)



Stable Traits, Auto Regressive Trait, State Components

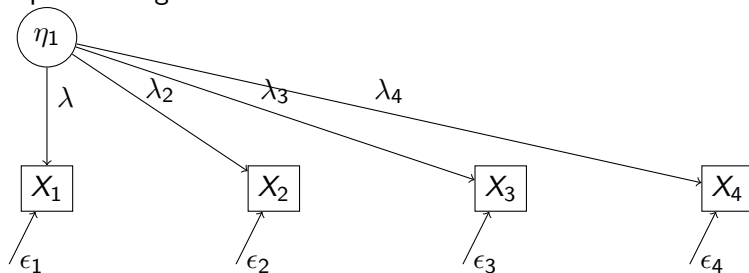
- 1 Test retest reliabilities reflect three different components and we need multiple measures to estimate them.
- 2 Stable Traits should lead to same score over time (λ)
- 3 Auto regressive traits suggest slow drop off in correlation over time (α)
- 4 State leads to internal consistency ($\frac{\beta^2}{\beta^2 + \epsilon^2}$) but not stability

(Kenny & Zautra (2001); Lucas & Donnellan (2007))



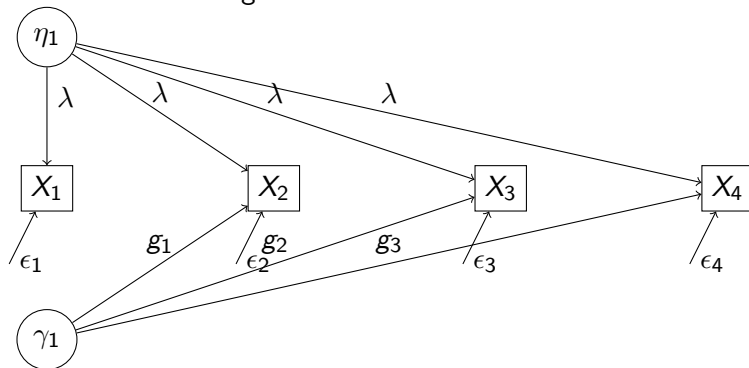
Stability across time

If $\alpha = 0$ and $\lambda_i = k$ then this is just congeneric measurement with equal loadings.



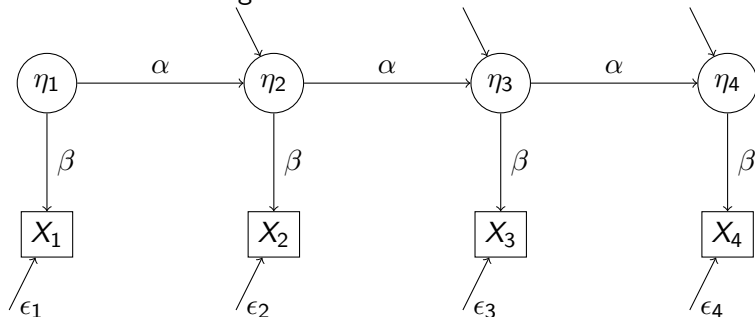
Growth across time

If $\alpha = 0$ and $\lambda = k$ then this is just congeneric measurement, but we can also model growth.



Simplex change over time

Predicts diminishing correlations across time.



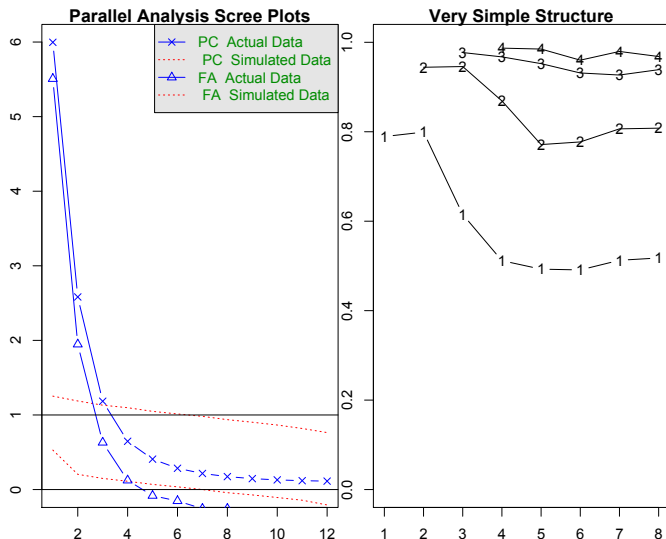
A growth set with simplex structure

```
> s12 <- sim.simplex(nvar =12, r=.8,mu=NULL, n=0)
> round(s12,2)
```

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12
V1	1.00	0.80	0.64	0.51	0.41	0.33	0.26	0.21	0.17	0.13	0.11	0.09
V2	0.80	1.00	0.80	0.64	0.51	0.41	0.33	0.26	0.21	0.17	0.13	0.11
V3	0.64	0.80	1.00	0.80	0.64	0.51	0.41	0.33	0.26	0.21	0.17	0.13
V4	0.51	0.64	0.80	1.00	0.80	0.64	0.51	0.41	0.33	0.26	0.21	0.17
V5	0.41	0.51	0.64	0.80	1.00	0.80	0.64	0.51	0.41	0.33	0.26	0.21
V6	0.33	0.41	0.51	0.64	0.80	1.00	0.80	0.64	0.51	0.41	0.33	0.26
V7	0.26	0.33	0.41	0.51	0.64	0.80	1.00	0.80	0.64	0.51	0.41	0.33
V8	0.21	0.26	0.33	0.41	0.51	0.64	0.80	1.00	0.80	0.64	0.51	0.41
V9	0.17	0.21	0.26	0.33	0.41	0.51	0.64	0.80	1.00	0.80	0.64	0.51
V10	0.13	0.17	0.21	0.26	0.33	0.41	0.51	0.64	0.80	1.00	0.80	0.64
V11	0.11	0.13	0.17	0.21	0.26	0.33	0.41	0.51	0.64	0.80	1.00	0.80
V12	0.09	0.11	0.13	0.17	0.21	0.26	0.33	0.41	0.51	0.64	0.80	1.00

simplex cases as special change models

How many factors in a simplex?



simplex cases as special change models

3 or 4 factor solution?

Factor Analysis using method = minres

Call: fa(r = s12, nfactors = 3, n.obs = 500)

Standardized loadings based upon correlation matrix

	MR1	MR2	MR3	h2	u2
V1	0.05	-0.49	0.57	0.60	0.40
V2	0.11	-0.57	0.62	0.80	0.20
V3	0.29	-0.56	0.52	0.83	0.17
V4	0.54	-0.45	0.29	0.74	0.26
V5	0.77	-0.29	0.06	0.74	0.26
V6	0.94	-0.10	-0.10	0.81	0.19
V7	0.94	0.10	-0.10	0.81	0.19
V8	0.77	0.29	0.06	0.74	0.26
V9	0.54	0.45	0.29	0.74	0.26
V10	0.29	0.56	0.52	0.83	0.17
V11	0.11	0.57	0.62	0.80	0.20
V12	0.05	0.49	0.57	0.60	0.40

	MR1	MR2	MR3
SS loadings	4.14	2.33	2.54
Proportion Var	0.35	0.19	0.21
Cumulative Var	0.35	0.54	0.75

With factor correlations of

	MR1	MR2	MR3
MR1	1.00	0	0.54
MR2	0.00	1	0.00
MR3	0.54	0	1.00

Standardized loadings based upon correlation matrix

	MR3	MR2	MR1	MR4	h2	u2
V1	-0.02	0.86	-0.07	0.04	0.68	0.321
V2	-0.01	0.96	0.00	0.03	0.92	0.082
V3	0.04	0.64	0.37	-0.05	0.78	0.218
V4	0.07	0.30	0.71	-0.07	0.79	0.214
V5	0.06	0.02	0.88	0.04	0.86	0.137
V6	-0.01	-0.04	0.69	0.36	0.79	0.209
V7	-0.04	-0.01	0.36	0.69	0.79	0.209
V8	0.02	0.06	0.04	0.88	0.86	0.137
V9	0.30	0.07	-0.07	0.71	0.79	0.214
V10	0.64	0.04	-0.05	0.37	0.78	0.218
V11	0.96	-0.01	0.03	0.00	0.92	0.082
V12	0.86	-0.02	0.04	-0.07	0.68	0.321

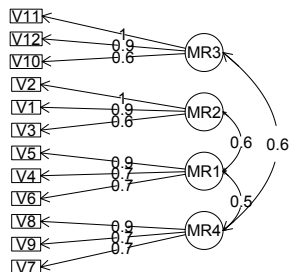
	MR3	MR2	MR1	MR4
SS loadings	2.38	2.38	2.44	2.44
Proportion Var	0.20	0.20	0.20	0.20
Cumulative Var	0.20	0.40	0.60	0.80

With factor correlations of

	MR3	MR2	MR1	MR4
MR3	1.00	0.14	0.20	0.55
MR2	0.14	1.00	0.55	0.20
MR1	0.20	0.55	1.00	0.48
MR4	0.55	0.20	0.48	1.00

3 and 4 factor solutions

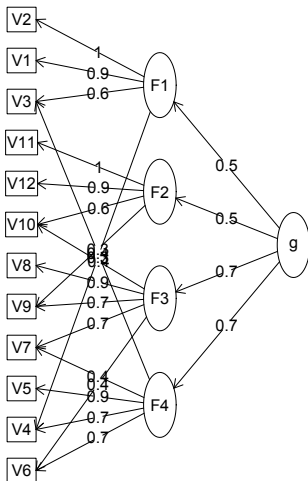
Factor Analysis



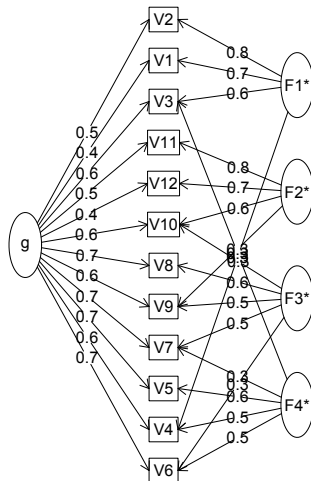
simplex cases as special change models

omega solutions

Hierarchical (multilevel) Structure



Omega

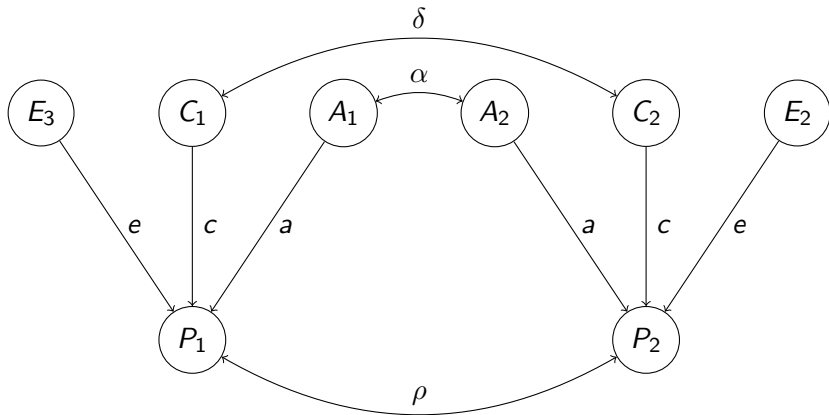


Clustering a simplex

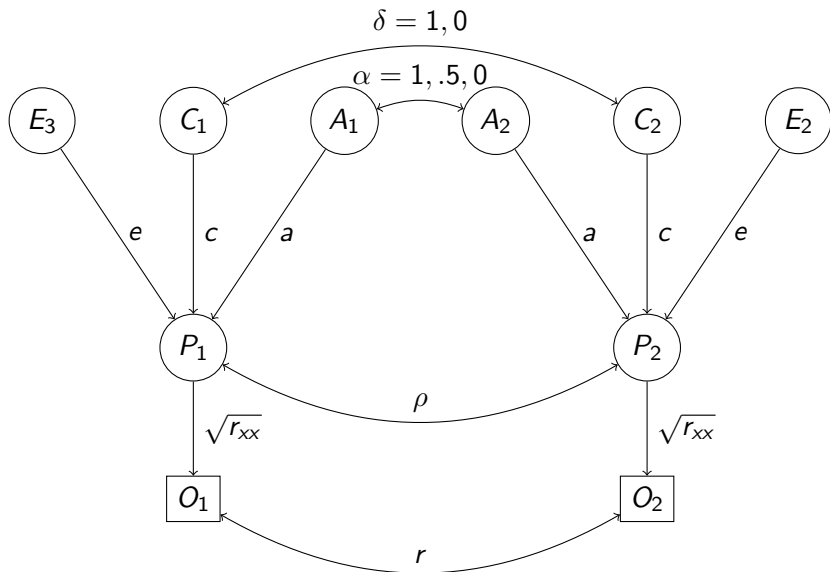
Figure 1 is a path diagram of a structural equation model. The model includes 11 latent variables (C1-C11) and 12 observed variables (V1-V12). Latent variables C1-C11 are represented by circles and contain their factor loadings (α) and error variances (β). Observed variables V1-V12 are represented by rectangles. Standardized path coefficients are shown on the arrows. The model shows a hierarchical structure where C1-C11 are measured by V1-V12, and C1-C11 are also measured by V1-V12. The model is a second-order CFA model where C1-C11 are first-order factors and C11 is a second-order factor. The model is a second-order CFA model where C1-C11 are first-order factors and C11 is a second-order factor. The model is a second-order CFA model where C1-C11 are first-order factors and C11 is a second-order factor.

ACE models: Additive genetic, Common environment, unique Environment

- Sources of variance in the common behavioral genetic models
 - Additive genetic variance
 - Shared family environmental variance
 - Unique environmental variance
- Modeled by the association between various family combinations
 - Monozygotic twins raised apart (sharing just genetic variance)
 - Monozygotic twins raised together (sharing genetic + environmental variance)
 - Dizygotic twins raised apart (sharing .5 genetic variance)
 - Dizygotic twins raised together (sharing .5 genetic + environmental variance)
 - Adopteds together (sharing just environmental variance)
 - unrelateds apart (sharing nothing)
- Estimate $\sigma_g^2, \sigma_c^2, \sigma_e^2$ and define $h^2 = \frac{\sigma_g^2}{\sigma_g^2 + \sigma_c^2 + \sigma_e^2}$

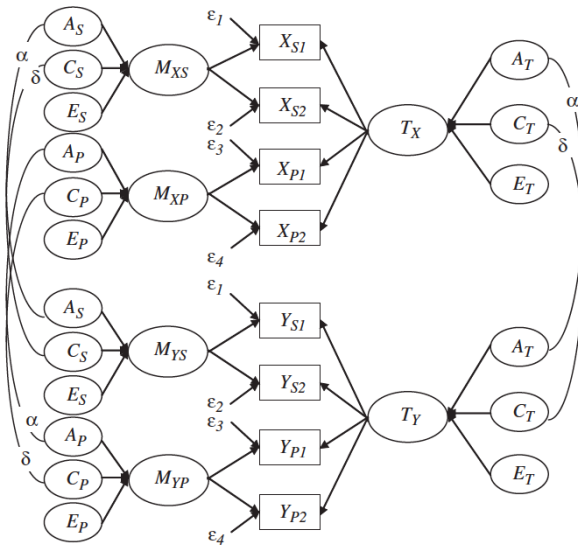


$\alpha = 1, .5, 0$ and $\delta = 1, 0$ depending upon family configuration.

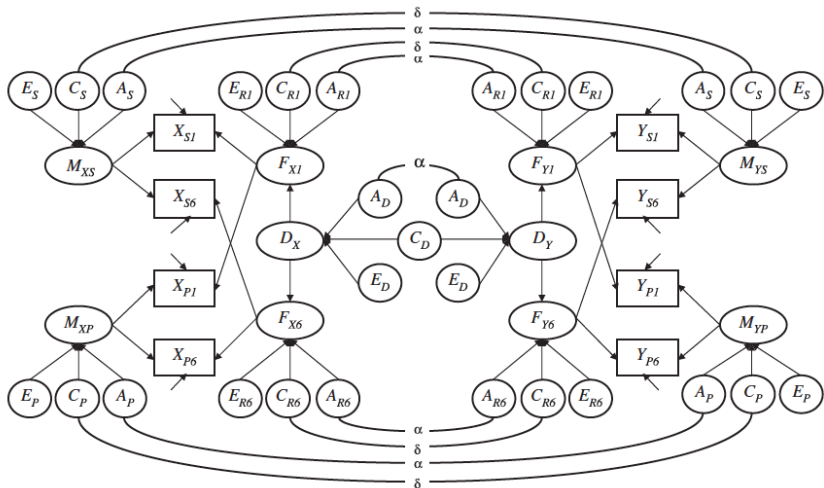
ACE model: with error correction. Model the observed correlations.

100

Multi method (self and peer) genetics analysis



Multi method (self and peer), multi-facets, genetics analysis



1

- The power of analysis is no greater than the quality of the data
 - $\text{Data} = \text{Model} + \text{Residual}$
 - $\text{Model} = \text{Data} - \text{Residual}$
 - Model requires good data
- Multiple models will fit the data
 - Models should be compared to each other, not just the Null model
 - Is a model better than an alternative model?
 - How do you know?
 - What other models fit equally well?
- Be creative while being cautious and critical

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Guttman, L. (1945). A basis for analyzing test-retest reliability. *Psychometrika*, 10(4), 255–282.

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