Psychometric Theory
Scale Construction and Factor Analysis
Homework

Scale construction - I

- You are considering developing a scale to measure trait X. You sample 12 items from the domain of "X-ness" in scale A and find that their average variance is .25 and that their average covariance is .10.
- What is the variance of this 12 item test?
- What is the alpha reliability of this test?
- What is the average interitem correlation?

Scale Construction-2

- Another set of 10 items (B), sampled from a similar domain has an average variance of .3 and an average covariance of .05.
- What is the variance of this set of items.
- What is the alpha for this set of items.
- What is the average correlation for this set?

Scale Construction-3

- The average covariance of between the items of set A and set B is .03.
- What is the average correlation between items in sets A and B?
- What is the covariance of sets A and B?
- What is the correlation between set A and B

Scale Construction-4

- Given the two subscales found previously, what is the variance of the composite scale made up of subscale A and subscale B?
- What is the alpha reliability of this composite?
- What is the general factor saturation of this composite (how much of the total test variance is a general factor)?

Scale construction-5

- What is the correlation of an average item in subtest A with the total score on scale A?
- What is the correlation of an average item in subtest B with the total score on scale B?
- What is the correlation of an average item in subtest A with the total score on the composite of subtests A and B?

Scale construction-6

- For a 5 item scale with item variances of 1 and item covariances of 0, what is the correlation of an item with the total scale?
- For a 10 item scale with item variances of 1 and item covariances of 0, what is the correlation of an item with the total scale?

Factor Analysis - 1

- The item loadings on the first factor of the correlation matrix from subset A are all .632.
- What is the communality for each item?
- What is the eigen value of this factor?

Factor Analysis -2

- Consider the following factor loading matrix
- What are the correlations between V_1, V_2, V_3, and V_5?
- What the communalities and eigen values for this matrix
- Draw the structure of this factor matrix

	F_1	F_2	F_3	h^2
V_1	.9	.0	.0	
V_2	.8	.0	.0	
V_3	.7	.0	.0	
V_4	.0	.7	.4	
V_5	.3	.6	.0	
V_6	.0	.5	.0	
V_7	.0	.0	.5	
V_8	.0	.0	.4	
V_9	.0	0	.3	
eigen value				

Factor Analysis -3

- Draw the structure associated with this factor matrix
- Find the correlations between the variables.

	F_1	F_2	F_3
V_1	.8	.0	.0
V_2	.7	.0	.0
V_3	.6	.0	.0
V_4	.0	.7	.0
V_5	.0	.6	.0
V_6	.0	.5	.0
V_7	.0	.0	.5
V_8	.0	.0	.4
V_9	.0	0	.3

Factor Analysis -4

- Consider the following factor loading matrix with a second level factor of with loadings of .9, .8 and .7
- What are the correlations between V1, V2, V3, and V5?
- Draw the structure.

	F_1	F_2	F_3
V_1	.8	.0	.0
V_2	.7	.0	.0
V_3	.6	.0	.0
V_4	.0	.7	.0
V_5	.0	.6	.0
V_6	.0	.5	.0
V_7	.0	.0	.5
V_8	.0	.0	.4
V_9	.0	0	.3

Psychometric Theory
Homework 3

Scale construction-1

- You are considering developing a scale to measure trait X. You sample 12 items from the domain of "X-ness" and find that their average variance is .25 and that their average covariance is .10.
- What is the variance of this 12 item test?
  - $V_t = k*V_a + k*(k-1)*C_a =$
  - $12*.25 + 12*11*.1 = V_t = 16.2$
- What is the alpha reliability of this test?
  - $\text{Alpha} = \{(V_t-\sum V_a)/V_t\}*\{k/(k-1)\}$
  - $\text{alpha} = (16.2-12*.25)/16.2*(12/11) = .89$
- What is the average interitem correlation?
  - $r_{xy} = c_{xy}/\text{sqrt}(V_x*V_y) = .1/\text{sqrt}(.25*.25) = .4$

Scale Construction-2

- Another set of 10 items (B), sampled from a similar domain has an average variance of .3 and an average covariance of .05.
- What is the variance of this set of items.
  - $V_t = k*V_a + k*(k-1)*C_a =$
  - $10*.30 + 10*9*.05 = V_t = 7.5$
- What is the alpha for this set of items.
  - $\text{Alpha} = \{(V_t-\sum V_a)/V_t\}*\{k/(k-1)\}$
  - $\text{alpha} = (7.5-10*.30)/7.5*(10/9) = .67$
- What is the average correlation for this set?
  - $r_{xy} = c_{xy}/\text{sqrt}(V_x*V_y) = .05/\text{sqrt}(.30*.30) = .167$

Scale Construction-3

- The average covariance of between the items of set A and set B is .03.
- What is the average correlation between items in sets A and B?
  - $r_{xy} = c_{xy}/\text{sqrt}(V_x * V_y) = .03/\text{sqrt}(.25*.3) = .1095$
- What is the covariance of sets A and B?
  - $C_{xy} = k_x * k_y * c_{xy} \Rightarrow 12 * 10 * .03 = 3.6$
- What is the correlation between set A and B
  - $r_{xy} = c_{xy}/\text{sqrt}(V_x * V_y) \Rightarrow r_{ab} = 3.6/\text{sqrt}(16.2*7.5) = .327$

Scale Construction-4

- Given the two subscales found previously, what is the variance of the composite scale made up of subscale A and subscale B?
  - $V_{(a+b)} = V_a + V_b + 2C_{ab} = 16.2 + 7.5 + 2* 3.6 = 30.9$
- What is the alpha reliability of this composite?
  - Alpha = $\{(V_t-\sum V_a)/V_t\}*\{k/(k-1)\}$
  - alpha = (30.9-(12*.25 + 10*.3))/30.9*(22/21) = .84
- What is the general factor saturation of this composite (how much of the total test variance is a general factor)?

General factor of two scales

$V_T =$

	A	B
A	V_a	C_{ab}
B	C_{ab}	V_b

=

	A	B
A	$k_a v_a + k_a*(k_a-1)c_a$	$k_a * k_b * c_{ab}$
B	$k_a * k_b * c_{ab}$	$k_b v_b + k_b*(k_b-1)c_b$

$= 16.2 + 7.5 + 2* 3.6 = 30.9$

$V_g =$

	A	B
A	$k_a * k_a * c_{ab}$	$k_a * k_b * c_{ab}$
B	$k_a * k_b * c_{ab}$	$k_b * k_b * c_{ab}$

=

	A	B
A	12*12 *.03	12*10 *.03
B	12*10 *.03	10*10 *.03

= 14.52

$V_g/V_t =$ 14.52/30.9= .47

Scale construction-5

- What is the correlation of an average item in subtest A with the total score on scale A?
  - $r_{xy} = c_{xy}/\text{sqrt}(V_x * V_y)$
  - $C_{xy} = v_a + (k\text{-}1) * c_a = .25 + 11 * .1 = 1.35$
  - $V_x = v_a = .25 \quad V_y = V_A = 16.2 \quad r_{aA} = 1.35/\text{sqrt}(16.2 * .25) = .67$
- What is the correlation of an average item in subtest B with the total score on scale B?
  - $C_{xy} = v_a + (k\text{-}1) * c_a = .30 + 9 * .05 = .75$
  - $V_x = v_a = .30 \quad V_y = V_A = 7.5 \quad r_{aA} = .75/\text{sqrt}(7.5 * .3) = .50$

Scale Construction -5c

- What is the correlation of an average item in subtest A with the total score on the composite of subtests A and B?
- $r_{xy} = c_{xy}/\text{sqrt}(V_x * V_y)$
- $C_{xy} = .25 + 11 * .1 + 10 * .03 = 1.65$
- $V_x = v_a = .25 \quad V_y = V_{AB} = 30.9$
  - $V_{(a+b)} = V_a + V_b + 2C_{ab} = 16.2 + 7.5 + 2 * 3.6 = 30.9$
- $r_{aAB} = 1.65/\text{sqrt}(30.9 * .25) = .59$

Scale construction-6

- For a 5 item scale with item variances of 1 and item covariances of 0, what is the correlation of an item with the total scale?
  - $r_{xy} = c_{xy}/\text{sqrt}(V_x * V_y)$
  - $c_{xt} = v_a + (k-1)*c_a = 1 + 4*0 = 1$
  - $V_x = 1 \quad V_t = k*V_a + k*(k-1)*C_a = 5$
  - $r_{xt} = 1/\text{sqrt}(1*5) = .44$
- For a 10 item scale with item variances of 1 and item covariances of 0, what is the correlation of an item with the total scale?
  - $r_{xt} = 1/\text{sqrt}(1*10) = .312$

Factor Analysis - 1

- The item loadings on the first factor of the correlation matrix from subset A are all .632.
- What is the communality for each item?
  - communality is amount of variance in an item accounted for by all the factors = $\sum f_i^2 = .632^2 = .4$
- What is the eigen value of this factor?
  - Eigen value is the amount of variance in all the items accounted for by a factor (sum over items)
  - $\sum f_{ij}^2 = 12 * .632^2 = 3.6$

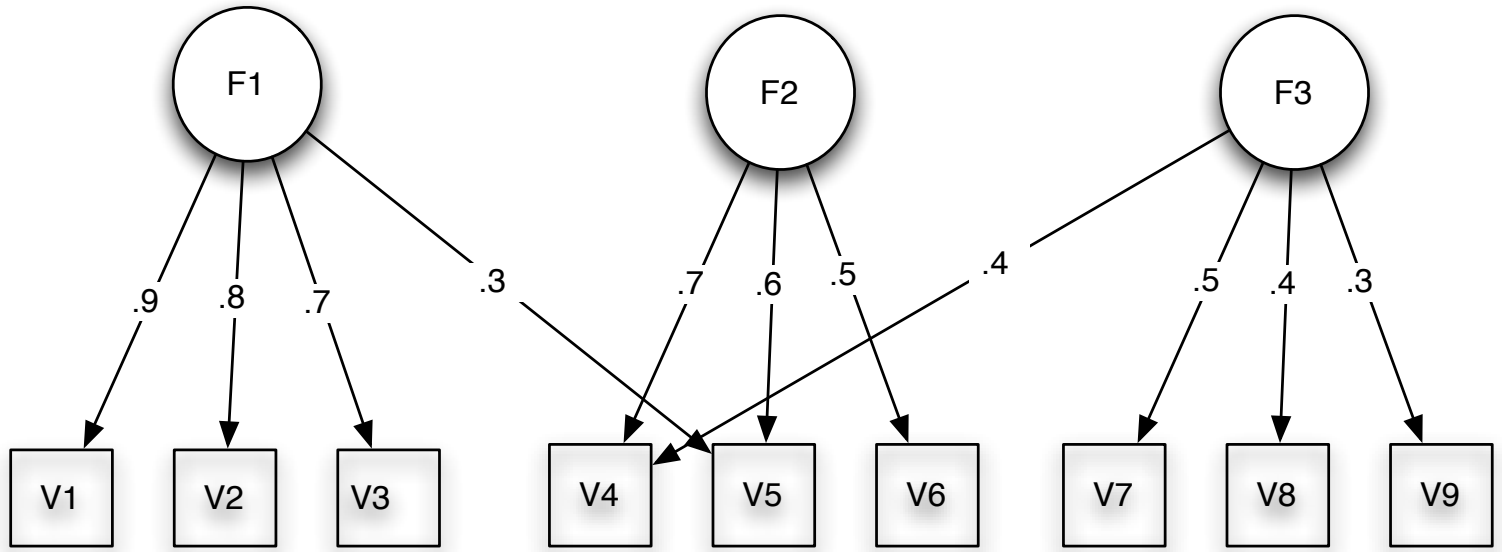
Factor Analysis -2

- Consider the following factor loading matrix
- What are the correlations between V_1, V_2, V_3, and V_5?
- What the communalities and eigen values for this matrix

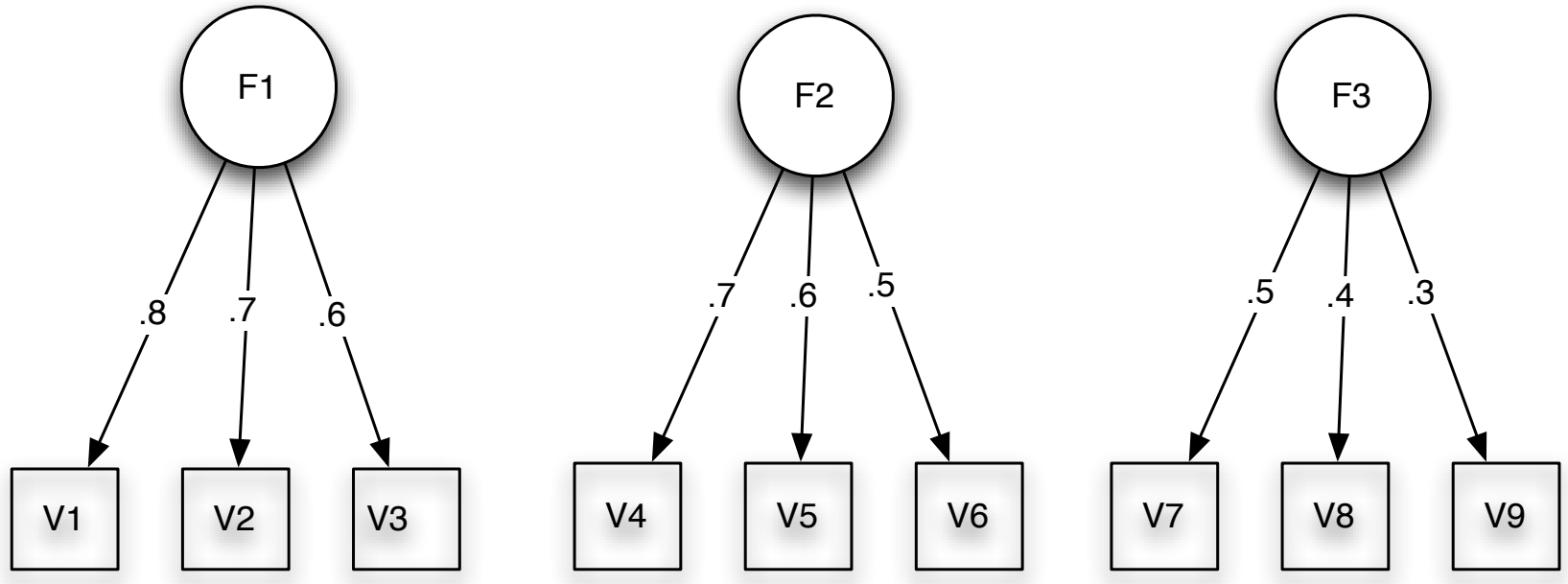
	F_1	F_2	F_3	h^2
V_1	.9	.0	.0	.81
V_2	.8	.0	.0	.64
V_3	.7	.0	.0	.49
V_4	.0	.7	.4	.65
V_5	.3	.6	.0	.45
V_6	.0	.5	.0	.25
V_7	.0	.0	.5	.25
V_8	.0	.0	.4	.16
V_9	.0	0	.3	.09
eigen value	2.03	1.10	.66	

	V1	V2	V3	V5
V1	.81			
V2	.72	.64		
V3	.63	.56	.49	
V5	.27	.24	.21	.45

Factor Structure -2



Factor Analysis 3 orthogonal factors



Correlation matrix of uncorrelated factors

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]
[1,]	1.00	0.56	0.48	0.00	0.00	0.00	0.00	0.0	0.00
[2,]	0.56	1.00	0.42	0.00	0.00	0.00	0.00	0.0	0.00
[3,]	0.48	0.42	1.00	0.00	0.00	0.00	0.00	0.0	0.00
[4,]	0.00	0.00	0.00	1.00	0.42	0.35	0.00	0.0	0.00
[5,]	0.00	0.00	0.00	0.42	1.00	0.30	0.00	0.0	0.00
[6,]	0.00	0.00	0.00	0.35	0.30	1.00	0.00	0.0	0.00
[7,]	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.3	0.24
[8,]	0.00	0.00	0.00	0.00	0.00	0.00	0.30	1.0	0.20
[9,]	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.2	1.00

Factor Analysis -4

- Consider the following factor loading matrix with a second level factor of with loadings of .9, .8 and .7
- What are the correlations between V1, V2, V3, and V5?

	F_1	F_2	F_3
V_1	.8	.0	.0
V_2	.7	.0	.0
V_3	.6	.0	.0
V_4	.0	.7	.0
V_5	.0	.6	.0
V_6	.0	.5	.0
V_7	.0	.0	.5
V_8	.0	.0	.4
V_9	.0	0	.3

	V1	V2	V3	V5
V1	.64			
V2	.56	.49		
V3	.48	.56	.36	
V5	.35	.30	.26	.36

Correlation matrix of correlated factors

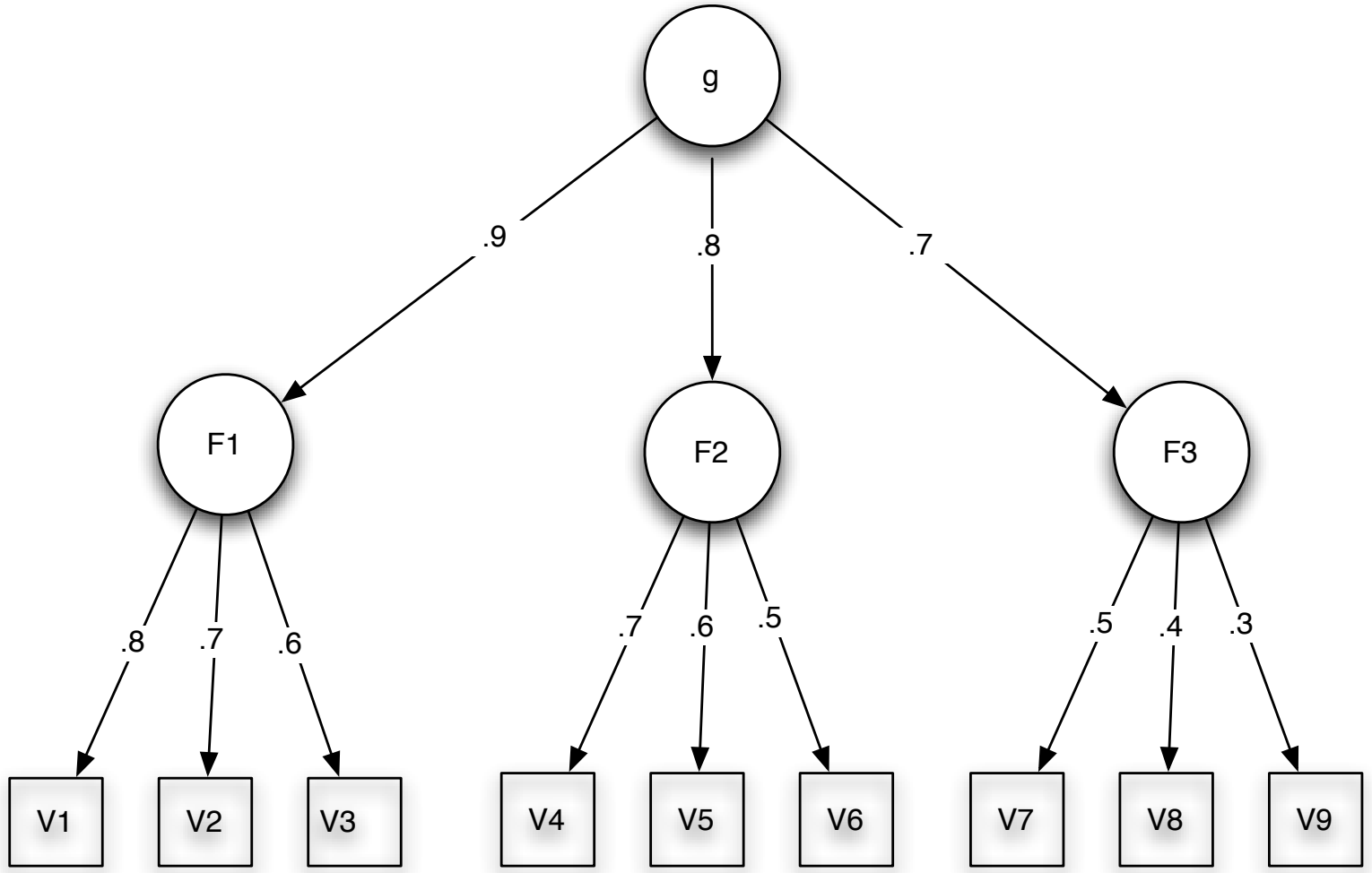
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]
[1,]	1.00	0.56	0.48	0.40	0.35	0.29	0.30	0.25	0.20
[2,]	0.56	1.00	0.42	0.35	0.30	0.25	0.26	0.22	0.18
[3,]	0.48	0.42	1.00	0.30	0.26	0.22	0.23	0.19	0.15
[4,]	0.40	0.35	0.30	1.00	0.42	0.35	0.24	0.20	0.16
[5,]	0.35	0.30	0.26	0.42	1.00	0.30	0.20	0.17	0.13
[6,]	0.29	0.25	0.22	0.35	0.30	1.00	0.17	0.14	0.11
[7,]	0.30	0.26	0.23	0.24	0.20	0.17	1.00	0.30	0.24
[8,]	0.25	0.22	0.19	0.20	0.17	0.14	0.30	1.00	0.20
[9,]	0.20	0.18	0.15	0.16	0.13	0.11	0.24	0.20	1.00

Orthogonal factors

Loadings:

	Factor1	Factor2	Factor3
[1,]	0.70	0.30	0.25
[2,]	0.61	0.26	0.22
[3,]	0.52	0.22	0.19
[4,]	0.24	0.63	0.18
[5,]	0.21	0.54	0.16
[6,]	0.17	0.45	0.13
[7,]	0.17	0.15	0.56
[8,]	0.14	0.12	0.46
[9,]	0.11	0.10	0.37

2 level factor solution



Oblique factors

Structure Matrix

	Factor1	Factor2	Factor3
[1,]	0.8	0.0	0.0
[2,]	0.7	0.0	0.0
[3,]	0.6	0.0	0.0
[4,]	0.0	0.7	0.0
[5,]	0.0	0.6	0.0
[6,]	0.0	0.5	0.0
[7,]	0.0	0.0	0.6
[8,]	0.0	0.0	0.5
[9,]	0.0	0.0	0.4

Factor correlations

	[,1]	[,2]	[,3]
[1,]	1.00	0.72	0.63
[2,]	0.72	1.00	0.56
[3,]	0.63	0.56	1.00

Schmid-Leiman orthogonalization

	g factor	Factor1	Factor2	Factor3	h2	u2
[1,]	0.72	0.35	0.00	0.00	0.64	0.36
[2,]	0.63	0.31	0.00	0.00	0.49	0.51
[3,]	0.54	0.26	0.00	0.00	0.36	0.64
[4,]	0.56	0.00	0.42	0.00	0.49	0.51
[5,]	0.48	0.00	0.36	0.00	0.36	0.64
[6,]	0.40	0.00	0.30	0.00	0.25	0.75
[7,]	0.42	0.00	0.00	0.43	0.36	0.64
[8,]	0.35	0.00	0.00	0.36	0.25	0.75
[9,]	0.28	0.00	0.00	0.29	0.16	0.84

Schmid Leiman

g factor + orthogonal primaries