Psychology 454: Latent Variable Modeling
Week 4: Latent models of real data

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Outline

Real data rather than simulated

1. Prior examples were all artificial data sets.
   - These have the advantage that we know truth.
   - They have the disadvantage that they don't give us experience with real data.
2. Some classic data sets are available in *psych*, *lavaan* and *sem*.
3. Using the `data` function to see what is available.
   - `data(package="psych")`
4. Eventually, you need to try your own data.

Some of the psych data sets

```
data(package="psych")

Bechtoldt                    Seven data sets showing a bifactor solution.
Bechtoldt.1                  Seven data sets showing a bifactor solution.
Bechtoldt.2                  Seven data sets showing a bifactor solution.
Chen (Schmid)                12 variables created by Schmid and Leiman to show the
                             Schmid-Leiman Transformation
Dwyer                        8 cognitive variables used by Dwyer for an example.
Gorsuch                      Example data set from Gorsuch (1997) for an example factor
                             extension.
Harman.5                     5 socio-economic variables from Harman (1967)
Harman.8                     Correlations of eight physical variables (from Harman, 196
Harman.Burt (Harman)         Two data sets from Harman (1967). 9 cognitive variables fro
                             Holzinger and 8 emotional variables from Burt
Harman.Holzinger (Harman)    Two data sets from Harman (1967). 9 cognitive variables fro
                             Holzinger and 8 emotional variables from Burt
Holzinger                    Seven data sets showing a bifactor solution.
Holzinger.9                  Seven data sets showing a bifactor solution.
Reise                        Seven data sets showing a bifactor solution.
Schmid                       12 variables created by Schmid and Leiman to show the
                             Schmid-Leiman Transformation
Thurstone                    Seven data sets showing a bifactor solution.
Thurstone.33                 Seven data sets showing a bifactor solution.
Tucker                       9 Cognitive variables discussed by Tucker and Lewis (1973)
West (Schmid)                12 variables created by Schmid and Leiman to show the
                             Schmid-Leiman Transformation
all.income (income)          US family income from US census 2008
bfi                          25 Personality items representing 5 factors
blot                         Bonds Logical Operations Test - BLOT
bock.table (bock)            Bock and Liberman (1970) data set of 1000 observations of
                             the LSAT
```

More of the psych data sets

```
burt                         11 emotional variables from Burt (1915)
cities                       Distances between 11 US cities
city.location (cities)       Distances between 11 US cities
cubits                       Galtons example of the relationship between height and
                             cubit or forearm length
cushny                       A data set from Cushny and Peebles (1905) on the effect of
                             three drugs on hours of sleep, used by Student (1908)
epi.bfi                      13 personality scales from the Eysenck Personality Invento
                             and Big 5 inventory
flat (affect)                Two data sets of affect and arousal scores as a function o
                             personality and movie conditions
galton                       Galton Mid parent child height data
heights                      A data.frame of the Galton (1888) height and cubit data se
income                       US family income from US census 2008
iqitems                      16 multiple choice IQ items
lsat6 (bock)                 Bock and Liberman (1970) data set of 1000 observations of
                             the LSAT
lsat7 (bock)                 Bock and Liberman (1970) data set of 1000 observations of
                             the LSAT
maps (affect)                Two data sets of affect and arousal scores as a function o
                             personality and movie conditions
msq                          75 mood items from the Motivational State Questionnaire fo
                             3896 participants
neo                          NEO correlation matrix from the NEO_PI_R manual
peas                         Galtons Peas
sat.act                      3 Measures of ability: SATV, SATQ, ACT
schmid.leiman (Schmid)       12 variables created by Schmid and Leiman to show the
                             Schmid-Leiman Transformation
veg (vegetables)             Paired comparison of preferences for 9 vegetables
withinBetween                An example of the distinction between within group and
                             between group correlations
```

Data sets in sem and lavaan

```
data(package='sem')
data(package ='lavaan')
```

```
Data sets in package 'sem':

Bollen        Bollen''s Data on Industrialization and Political Democracy
CNES        Variables from the 1997 Canadian National Election Study
Klein            Klein''s Data on the U. S. Economy'
Kmenta␣␣␣␣␣␣␣Partly␣Artificial␣Data␣on␣the␣U.␣S.␣Economy

Data␣sets␣in␣package␣'lavaan':

Demo.growth␣␣␣␣␣␣Demo␣dataset␣for␣a␣illustrating␣a␣linear␣growth␣model.
HolzingerSwineford1939␣␣␣␣␣␣Holzinger␣and␣Swineford␣Dataset␣(9␣Variables)
PoliticalDemocracy␣␣␣␣␣␣␣␣␣␣␣Industrialization␣And␣Political␣Democracy␣Dataset
```

Also can read in from clipboard, web, from hard drive and import

- To read from clipboard: `read.clipboard()`
- In the following calls, the parameters may be deleted
  - read.clipboard() #assumes headers and tab or space delimited
  - read.clipboard.csv() #assumes headers and comma delimited
  - read.clipboard.tab() #assumes headers and tab delimited (e.g., excel spreadsheets
  - read.clipboard.lower() #read in a matrix given the lower off diagonal
  - read.clipboard.upper() #read in a matrix given the upper off diagonal
  - read.clipboard.fwf() #read in data using a fixed format width (see read.fwf for instructions)
- To read from a file (Including txt, csv, sav, rds, Rda files)
  - my.data <- read.file() #this opens the system directory – navigate it to your file

More on getting your data

To read from a local file, read.file will open the OS directory, you search for the file, it will then read it. The current version will read .txt, .csv, .sav,, .rds, and .rda files.

R code

```
#specify the name and address of the remote file
datafilename <- "http://personality-project.org/r/datasets/maps.mixx.epi.bfi.data"
person.data  <- read.file(datafilename)  #read the data file
#or
# datafilename <- file.read()     # use the OS to find the file and read it

names(person.data)  #list the names of the variables

# Read a remote spss .sav file
file.name <- "http://personality-project.org/r/datasets/finkel.sav"
eli.data <- read.file(file.name)  #converts the spss file
```

Use the bifactor examples

The study of latent variable models in general and sem in particular is the combination of measurement models with structural models. The bifactor examples are useful to understand issues in reliability and estimating measurement models.

```
data(Thurstone)
data(Thurstone.33)
data(Holzinger)
data(Holzinger.9)
data(Bechtoldt)
data(Bechtoldt.1)
data(Bechtoldt.2)
data(Reise)
data(Chen)
data(West)
```

```
Two more data sets with similar structures are found in the Harman data set.

Bechtoldt.1: 17 x 17 correlation matrix of ability tests, N = 212.
Bechtoldt.2: 17 x 17 correlation matrix of ability tests, N = 213.
Holzinger: 14 x 14 correlation matrix of ability tests, N = 355
Holzinger.9: 9 x 9 correlation matrix of ability tests, N = 145
Reise: 16 x 16 correlation matrix of health satisfaction items. N = 35,000
Thurstone: 9 x 9 correlation matrix of ability tests, N = 213
Thurstone.33: Another 9 x 9 correlation matrix of ability items, N=4175
```

Simulate a bifactor model using `sim.structure`

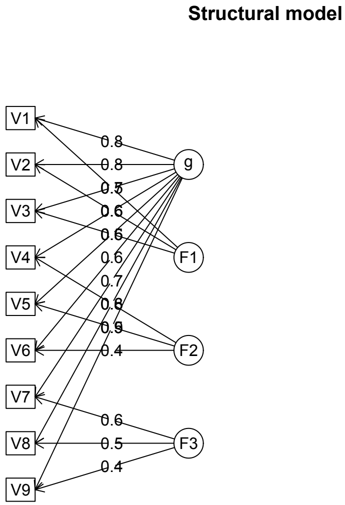
First, make up a bifactor model and then simulate it. We use the `sim.structure` function with a specified factor loadings matrix.

R code

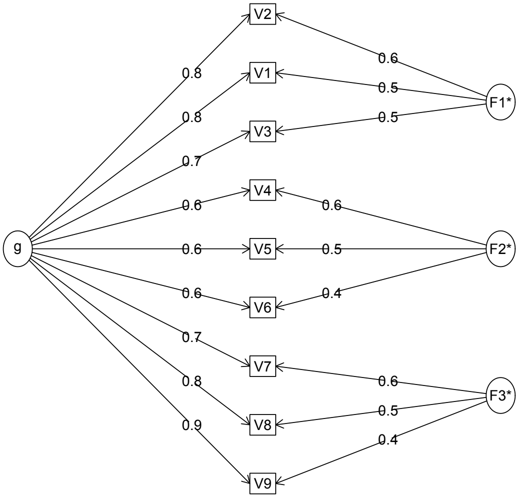
```
f <- matrix(c(.8,.75,.7,.65,.6,.65,.7,.8,.9,.5,.6,.5,
     rep(0,9),.6,.5,.4,  rep(0,9),.6,.5,.4),ncol=4)
colnames(f) <- cs(g, F1, F2, F3)
rownames(f) <- paste0("V", 1:9)
f
```

```
      g  F1  F2  F3
V1 0.80 0.5 0.0 0.0
V2 0.75 0.6 0.0 0.0
V3 0.70 0.5 0.0 0.0
V4 0.65 0.0 0.6 0.0
V5 0.60 0.0 0.5 0.0
V6 0.65 0.0 0.4 0.0
V7 0.70 0.0 0.0 0.6
V8 0.80 0.0 0.0 0.5
V9 0.90 0.0 0.0 0.4
```

The structure diagram of a bifactor model using `structure.diagram`



The structure diagram of a bifactor model using `omega.diagram`



Simulate the data

R code

```
set.seed(17)
R <- sim.structure(f,n=500)
R
```

```
Call: sim.structure(fx = f, n = 500)
 $model (Population correlation matrix)
     V1   V2   V3   V4   V5   V6   V7   V8   V9
V1 1.00 0.90 0.81 0.52 0.48 0.52 0.56 0.64 0.72
V2 0.90 1.00 0.82 0.49 0.45 0.49 0.52 0.60 0.68
V3 0.81 0.82 1.00 0.45 0.42 0.45 0.49 0.56 0.63
V4 0.52 0.49 0.45 1.00 0.69 0.66 0.45 0.52 0.59
V5 0.48 0.45 0.42 0.69 1.00 0.59 0.42 0.48 0.54
V6 0.52 0.49 0.45 0.66 0.59 1.00 0.45 0.52 0.59
V7 0.56 0.52 0.49 0.45 0.42 0.45 1.00 0.86 0.87
V8 0.64 0.60 0.56 0.52 0.48 0.52 0.86 1.00 0.92
V9 0.72 0.68 0.63 0.59 0.54 0.59 0.87 0.92 1.00
$reliability (population reliability)
[1] 0.89 0.92 0.74 0.78 0.61 0.58 0.85 0.89 0.97
$r  (Sample correlation matrix  for sample size =  500 )
     V1   V2   V3   V4   V5   V6   V7   V8   V9
V1 1.00 0.91 0.83 0.60 0.54 0.55 0.58 0.66 0.74
V2 0.91 1.00 0.84 0.57 0.50 0.51 0.54 0.61 0.68
V3 0.83 0.84 1.00 0.51 0.46 0.47 0.50 0.55 0.61
V4 0.60 0.57 0.51 1.00 0.74 0.68 0.52 0.60 0.64
V5 0.54 0.50 0.46 0.74 1.00 0.64 0.49 0.55 0.59
V6 0.55 0.51 0.47 0.68 0.64 1.00 0.54 0.60 0.65
V7 0.58 0.54 0.50 0.52 0.49 0.54 1.00 0.89 0.88
V8 0.66 0.61 0.55 0.60 0.55 0.60 0.89 1.00 0.93
V9 0.74 0.68 0.61 0.64 0.59 0.65 0.88 0.93 1.00
```

Bifactor rotation does not get the structure

R code

```
f4<- fa(R$model,4,rotate="bifactor")
f4
```

```
Factor Analysis using method =  minres
Call: fa(r = R$model, nfactors = 4, rotate = "bifactor")
Standardized loadings (pattern matrix) based upon correlation matrix
    MR1   MR3   MR2   MR4   h2    u2 com
V1 0.85  0.03  0.42  0.04 0.89 0.110 1.5
V2 0.83 -0.01  0.49 -0.04 0.92 0.077 1.6
V3 0.76  0.01  0.41  0.00 0.74 0.260 1.5
V4 0.59  0.66  0.01 -0.03 0.78 0.218 2.0
V5 0.54  0.56  0.01  0.00 0.61 0.390 2.0
V6 0.59  0.48  0.01  0.07 0.58 0.417 2.0
V7 0.84 -0.07 -0.36 -0.07 0.85 0.150 1.4
V8 0.90 -0.01 -0.29  0.02 0.89 0.110 1.2
V9 0.95  0.04 -0.22  0.12 0.97 0.030 1.1

                       MR1  MR3  MR2  MR4
SS loadings           5.38 0.98 0.84 0.03
Proportion Var        0.60 0.11 0.09 0.00
Cumulative Var        0.60 0.71 0.80 0.80
Proportion Explained  0.74 0.14 0.12 0.00
Cumulative Proportion 0.74 0.88 1.00 1.00

Mean item complexity =  1.6
Test of the hypothesis that 4 factors are sufficient.

The degrees of freedom for the null model are  36  and the objective function was  8.96
```

What about Maximum Likelihood?

R code

```
f4<- fa(R$model,4,rotate="bifactor", fm="mle")
f4
```

```
  f4<- fa(R$model,4,rotate="bifactor", fm="mle")
> f4
Factor Analysis using method =  ml
Call: fa(r = R$model, nfactors = 4, rotate = "bifactor", fm = "mle")
Standardized loadings (pattern matrix) based upon correlation matrix
    ML1   ML3   ML2   ML4   h2    u2 com
V1 0.84  0.03  0.42  0.04 0.89 0.110 1.5
V2 0.83 -0.01  0.49 -0.04 0.92 0.077 1.6
V3 0.76  0.01  0.41  0.00 0.74 0.260 1.5
V4 0.59  0.66  0.01 -0.03 0.78 0.217 2.0
V5 0.54  0.56  0.01  0.00 0.61 0.390 2.0
V6 0.59  0.48  0.01  0.07 0.58 0.418 2.0
V7 0.84 -0.07 -0.36 -0.07 0.85 0.150 1.4
V8 0.90 -0.01 -0.29  0.02 0.89 0.110 1.2
V9 0.95  0.04 -0.22  0.12 0.97 0.030 1.1

                       ML1  ML3  ML2  ML4
SS loadings           5.38 0.98 0.84 0.03
Proportion Var        0.60 0.11 0.09 0.00
Cumulative Var        0.60 0.71 0.80 0.80
Proportion Explained  0.74 0.14 0.12 0.00
Cumulative Proportion 0.74 0.88 1.00 1.00
```

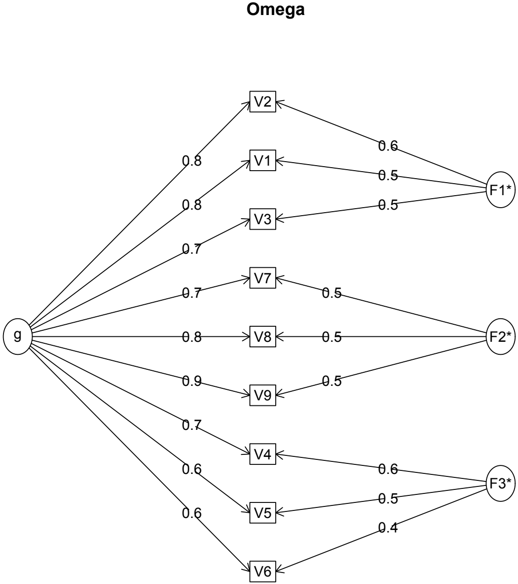
omega solution does better

R code

```
om <- omega(R$model)
om
```

```
Omega
Call: omega(m = R$model)
Alpha:                  0.93
G.6:                    0.95
Omega Hierarchical:     0.82
Omega H asymptotic:     0.85
Omega Total             0.97

Schmid Leiman Factor loadings greater than  0.2
      g   F1*   F2*   F3*   h2   u2   p2
V1 0.79  0.52              0.89 0.11 0.69
V2 0.76  0.58              0.92 0.08 0.63
V3 0.70  0.50              0.74 0.26 0.66
V4 0.66              0.58 0.77 0.23 0.56
V5 0.60              0.50 0.62 0.38 0.59
V6 0.63              0.43 0.58 0.42 0.67
V7 0.72        0.55        0.83 0.17 0.63
V8 0.79        0.52        0.90 0.10 0.70
V9 0.85        0.47        0.95 0.05 0.76
With eigenvalues of:
   g  F1*  F2*  F3*
4.76 0.87 0.79 0.78
general/max  5.49   max/min =   1.11
mean percent general =  0.65    with sd =  0.06 and cv of  0.09
Explained Common Variance of the general factor =  0.66
```

omega of the bifactor model



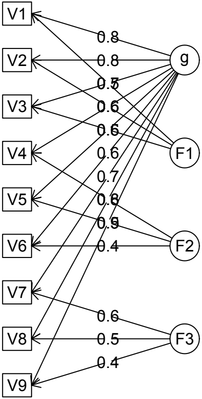
Create the lavaan code using structure.diagram

R code

```
structure.diagram(f,simple=FALSE)
 mod <- structure.diagram(f,simple=FALSE)
```

```
 mod$lavaan
 g =~  + V1 + V2 + V3 + V4 + V5 + V6 + V7 + V8 + V9
F1 =~  + V1 + V2 + V3
F2 =~  + V4 + V5 + V6
F3 =~  + V7 + V8 + V9
```

Structural model



lavaan gets the right answer (if we help)

R code

```
library(lavaan)
 bi.cfa <- cfa(model = mod$lavaan, sample.cov =R$model,
     sample.nobs=500,  orthogonal=TRUE,std.lv=TRUE)
summary(bi.cfa)
```

```
lavaan (0.5-23.1097) converged normally after  42 iterations

  Number of observations                           500

  Estimator                                         ML
  Minimum Function Test Statistic                0.000
  Degrees of freedom                                18
  P-value (Chi-square)                           1.000

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard
```

lavaan loadings are correct

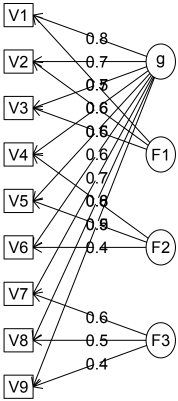
But, we had to specify that we wanted an orthogonal solution

R code

```
bi.cfa <- cfa(model=bi.mod,sample.cov=R$model,sample.nobs=500,
                    orthogonal=TRUE,std.lv=TRUE)
```

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  g =~
    V1                0.799    0.040   19.957    0.000
    V2                0.749    0.041   18.229    0.000
    V3                0.699    0.042   16.623    0.000
    V4                0.649    0.042   15.345    0.000
    V5                0.599    0.043   13.887    0.000
    V6                0.649    0.042   15.345    0.000
    V7                0.699    0.044   15.750    0.000
    V8                0.799    0.042   19.059    0.000
    V9                0.899    0.039   23.086    0.000
  F1 =~
    V1                0.499    0.034   14.709    0.000
    V2                0.599    0.033   18.408    0.000
    V3                0.499    0.037   13.386    0.000
  F2 =~
    V4                0.599    0.048   12.532    0.000
    V5                0.499    0.047   10.586    0.000
    V6                0.400    0.044    9.130    0.000
  F3 =~
    V7                0.599    0.039   15.337    0.000
    V8                0.499    0.041   12.228    0.000
    V9                0.400    0.040    9.976    0.000
```

Confirmatory structure



Bifactor models versus Schmid-Leimam solutions

1. Bifactor model was originally the Spearman (1904) model
   - A general factor for all the items
   - Unique factors for each item
2. Modern Bifactor models developed by Holzinger and Swineford (1937, 1939)
3. Schmid and Leiman (1957) showed another way of finding a general and group factors
4. Applied particularly to personality items by Reise et al. (2007); Reise (2012)
5. But Chen et al. (2006) showed the problem of proportionality constraints in Schmid-Leiman solutions.
6. Rodriguez et al. (2016b,a) elaborate on bifactor models.
7. Bifactor rotations do not always do a good job of recovering bifactors from EFA data.

Thurstone correlation matrix

```
> colnames(Thurstone) <- abbreviate(rownames(Thurstone),6)
> Thurstone
```

```
                Sntncs Vcblry Snt.Cm Frst.L 4.Lt.W Suffxs Lttr.S Pedgrs Lttr.G
Sentences        1.000  0.828  0.776  0.439  0.432  0.447  0.447  0.541  0.380
Vocabulary       0.828  1.000  0.779  0.493  0.464  0.489  0.432  0.537  0.358
Sent.Completion  0.776  0.779  1.000  0.460  0.425  0.443  0.401  0.534  0.359
First.Letters    0.439  0.493  0.460  1.000  0.674  0.590  0.381  0.350  0.424
4.Letter.Words   0.432  0.464  0.425  0.674  1.000  0.541  0.402  0.367  0.446
Suffixes         0.447  0.489  0.443  0.590  0.541  1.000  0.288  0.320  0.325
Letter.Series    0.447  0.432  0.401  0.381  0.402  0.288  1.000  0.555  0.598
Pedigrees        0.541  0.537  0.534  0.350  0.367  0.320  0.555  1.000  0.452
Letter.Group     0.380  0.358  0.359  0.424  0.446  0.325  0.598  0.452  1.000
```

The Thurstone 9 variable problem is taken from Bechtoldt (1961) who took data from Thurstone and Thurstone (1941) and split it into two samples of 212 and 213. McDonald (1999) in turn took the 17 variables and chose 9 that show a clear bifactor structure. This example is used in the sem pacakge as well as PROC CALIS in SAS. It is one of the classic demonstrations.

Better yet, use `lowerMat`

R code

```
lowerMat(Thurstone)
```

```
                Sntnc Vcblr Snt.C Frs.L 4.L.W Sffxs Ltt.S Pdgrs Ltt.G
Sentences       1.00
Vocabulary      0.83  1.00
Sent.Completion 0.78  0.78  1.00
First.Letters   0.44  0.49  0.46  1.00
4.Letter.Words  0.43  0.46  0.42  0.67  1.00
Suffixes        0.45  0.49  0.44  0.59  0.54  1.00
Letter.Series   0.45  0.43  0.40  0.38  0.40  0.29  1.00
Pedigrees       0.54  0.54  0.53  0.35  0.37  0.32  0.56  1.00
Letter.Group    0.38  0.36  0.36  0.42  0.45  0.32  0.60  0.45  1.00
```

Or, if you like APA style tables in Latex use `cor2latex`

R code

```
cor2latex(Thurstone, font.size='tiny')
```

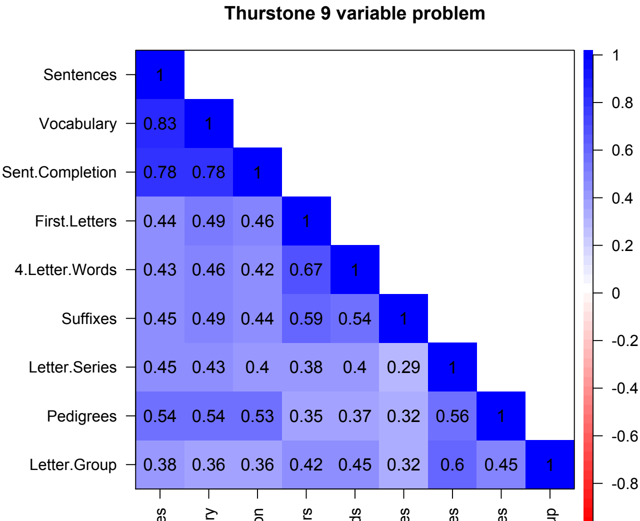
Table: cor2latex

A correlation table from the psych package in R.

Variable	Sntnc	Vcblr	Snt.C	Frs.L	4.L.W	Sffxs	Ltt.S	Pdgrs	Ltt.G
Sentences	1.00								
Vocabulary	0.83	1.00							
Sent.Completion	0.78	0.78	1.00						
First.Letters	0.44	0.49	0.46	1.00					
4.Letter.Words	0.43	0.46	0.42	0.67	1.00				
Suffixes	0.45	0.49	0.44	0.59	0.54	1.00			
Letter.Series	0.45	0.43	0.40	0.38	0.40	0.29	1.00		
Pedigrees	0.54	0.54	0.53	0.35	0.37	0.32	0.56	1.00	
Letter.Group	0.38	0.36	0.36	0.42	0.45	0.32	0.60	0.45	1.00

Thurstone 9 variable problem

```
cor.plot(Thurstone,TRUE,upper=FALSE,main="Thurstone 9 variable problem")
```



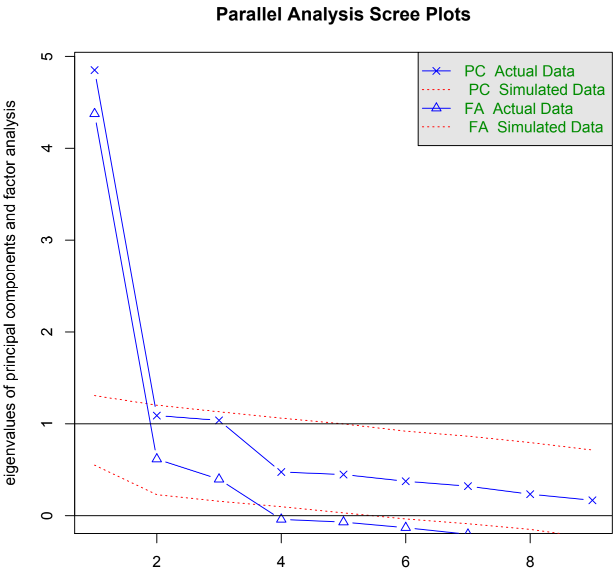
The number of factors problem

- "The number of factors problem is easy, I solve it everyday before breakfast. The problem is the right number" Kaiser as quoted by Horn and Engstrom (1979)
  1. Extract factors until χ^2 is not significant.
  2. Extract factors until change in χ^2 is not significant.
  3. Parallel analysis.
  4. Scree test
  5. Very Simple Structure (VSS)
  6. Minimum Absolute Partial (Velicer's MAP test)
  7. Minimum BIC or AIC
  8. Complexity
  9. Eigen Value > 1 (worst rule)
- An alternative is the factor redundancy or measure of effective dimensionality Del Giudice (2020).

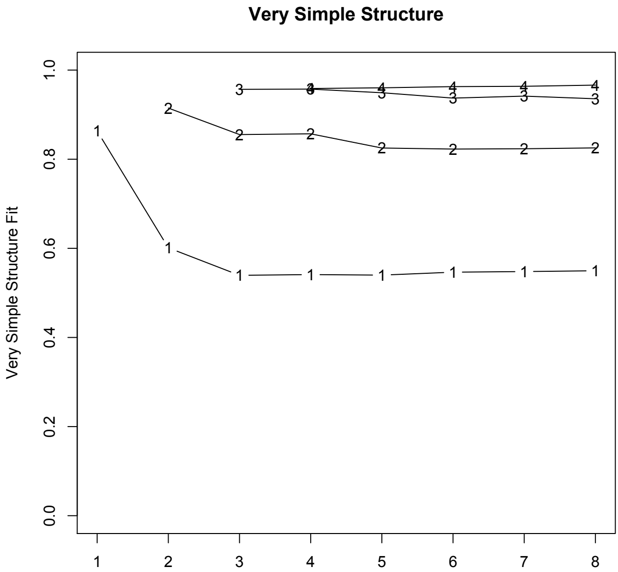
Number of factors?

```
> fa.parallel(Thurstone,n.obs=213)
> vss <- VSS(Thurstone,n.obs=213,SMC=FALSE)
> vss

Parallel analysis suggests that the number of factors =  3  and
                                   the number of components =  1

Very Simple Structure
Call: VSS(x = Thurstone, n.obs = 213, SMC = FALSE)
VSS complexity 1 achieves a maximimum of 0.86  with  1  factors
VSS complexity 2 achieves a maximimum of 0.91  with  2  factors

The Velicer MAP criterion achieves a minimum of 0.07  with  3  factors

Velicer MAP
[1] 0.07 0.07 0.07 0.11 0.20 0.31 0.59 1.00

Very Simple Structure Complexity 1
[1] 0.86 0.60 0.54 0.54 0.54 0.55 0.55 0.55

Very Simple Structure Complexity 2
[1] 0.00 0.91 0.86 0.86 0.83 0.82 0.82 0.83
```

Parallel analysis of the Thurstone data set



VSS fit function



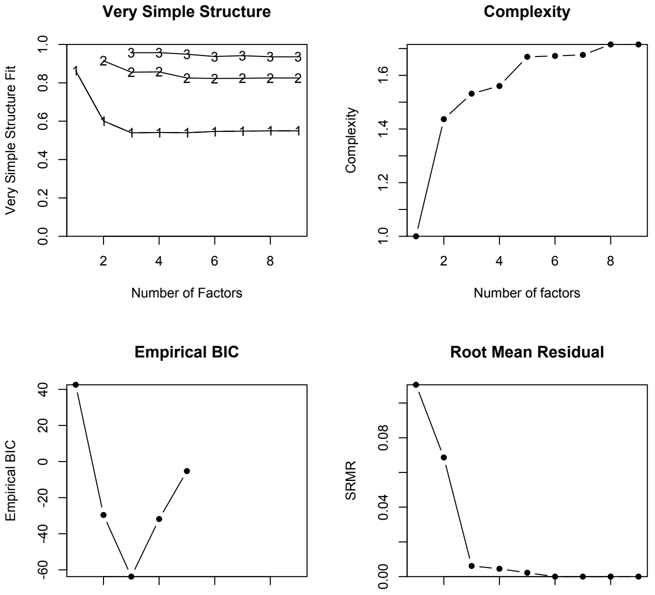
And many more

R code

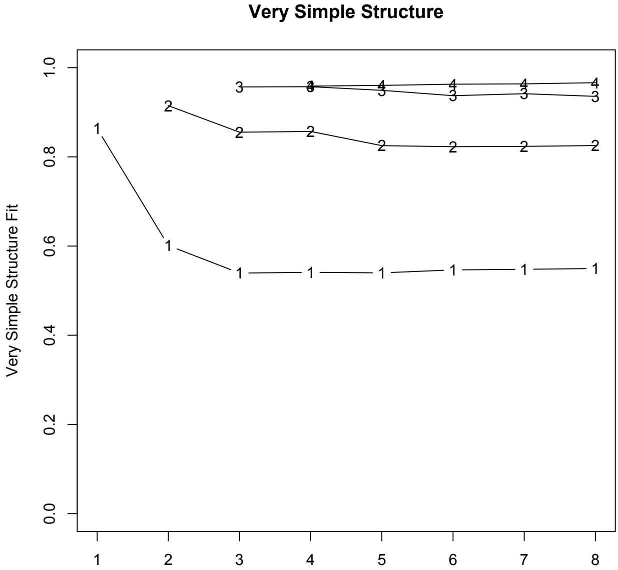
```
nfactors{Thurstone,n.obs=213)
```

```
Number of factors
Call: vss(x = x, n = n, rotate = rotate, diagonal = diagonal, fm = fm,
    n.obs = n.obs, plot = FALSE, title = title, use = use, cor = cor)
VSS complexity 1 achieves a maximimum of 0.86  with  1  factors
VSS complexity 2 achieves a maximimum of 0.91  with  2  factors
The Velicer MAP achieves a minimum of 0.07  with  3  factors
Empirical BIC achieves a minimum of  -63.76  with  3  factors
Sample Size adjusted BIC achieves a minimum of  -23.49  with  3  factors

Statistics by number of factors
  vss1 vss2   map dof   chisq     prob sqresid  fit RMSEA   BIC SABIC complex  eChisq    S
1 0.86 0.00 0.075  27 2.3e+02 2.1e-34    3.63 0.86  0.19  86.8 172.3     1.0 1.9e+02 1.1e
2 0.60 0.91 0.067  19 8.3e+01 5.6e-10    2.26 0.91  0.13 -18.8  41.4     1.4 7.2e+01 6.9e
3 0.54 0.86 0.066  12 2.8e+00 1.0e+00    1.15 0.96  0.00 -61.5 -23.5     1.5 5.8e-01 6.1e
4 0.54 0.86 0.114   6 1.5e+00 9.6e-01    1.10 0.96  0.00 -30.7 -11.7     1.6 3.1e-01 4.5e
5 0.54 0.83 0.195   1 2.4e-01 6.2e-01    1.02 0.96  0.00  -5.1  -1.9     1.7 7.5e-02 2.2e
6 0.55 0.82 0.312  -3 4.8e-07      NA    0.98 0.96    NA    NA    NA     1.7 1.9e-07 3.6e
7 0.55 0.82 0.590  -6 4.2e-09      NA    0.94 0.96    NA    NA    NA     1.7 1.5e-09 3.1e
8 0.55 0.83 1.000  -8 0.0e+00      NA    0.86 0.97    NA    NA    NA     1.7 3.3e-19 4.6e
9 0.55 0.83    NA  -9 0.0e+00      NA    0.86 0.97    NA    NA    NA     1.7 3.2e-27 4.6e
```

`nfactors`(Thurstone,n.obs=213) gives multiple solutions



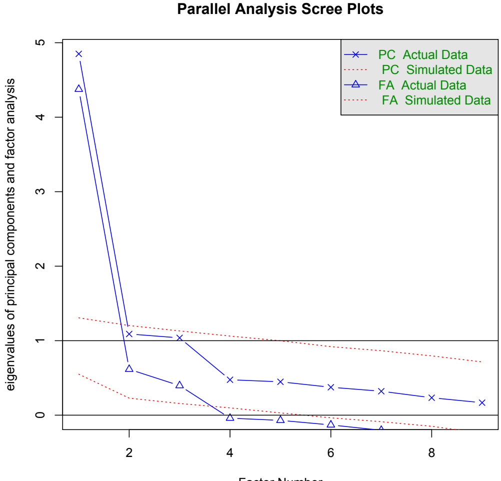
A Very Simple Structure Plot



Parallel analysis

> fa.parallel(Thurstone,n.obs=213)

Parallel analysis suggests that the number of factors = 3 and the number of components = 1



Minimal Residual FA

fa(Thurstone,n.obs=213)

```
> fa(Thurstone,n.obs=213)
Factor Analysis using method =  minres
Call: fa(r = Thurstone, n.obs = 213)
Standardized loadings based upon correlation matrix
    MR1   h2   u2
V1 0.87 0.75 0.25
V2 0.88 0.77 0.23
V3 0.83 0.70 0.30
V4 0.62 0.39 0.61
V5 0.61 0.37 0.63
V6 0.59 0.34 0.66
V7 0.57 0.32 0.68
V8 0.64 0.41 0.59
V9 0.52 0.27 0.73
                 MR1
SS loadings     4.32
Proportion Var 0.48
Test of the hypothesis that 1 factor is sufficient.
The degrees of freedom for the null model are  36  and the objective function was
5.2 with Chi Square of  1081.97
The degrees of freedom for the model are 27  and the objective function was  1.12
The root mean square of the residuals is  0.08
The df corrected root mean square of the residuals is  0.13
The number of observations was  213  with Chi Square =  231.55  with prob <  2.1e-34
Tucker Lewis Index of factoring reliability =  0.738
RMSEA index =  0.191  and the 90 % confidence intervals are  0.19 0.194
BIC =  86.79
Fit based upon off diagonal values = 0.95
Measures of factor score adequacy
                                                  MR1
```

Maximum likelihood

```
> fa(Thurstone,n.obs=213,fm="mle")
Factor Analysis using method =  ml
Call: fa(r = Thurstone, n.obs = 213, fm = "mle")
Standardized loadings based upon correlation matrix
    ML1   h2   u2
V1 0.88 0.78 0.22
V2 0.90 0.80 0.20
V3 0.85 0.72 0.28
V4 0.59 0.35 0.65
V5 0.57 0.33 0.67
V6 0.57 0.32 0.68
V7 0.54 0.29 0.71
V8 0.63 0.40 0.60
V9 0.49 0.24 0.76
                ML1
SS loadings    4.22
Proportion Var 0.47
Test of the hypothesis that 1 factor is sufficient.
The degrees of freedom for the null model are  36  and the objective function was
5.2 with Chi Square of  1081.97
The degrees of freedom for the model are 27  and the objective function was  1.1
The root mean square of the residuals is  0.08
The df corrected root mean square of the residuals is  0.14
The number of observations was  213  with Chi Square =  228.59  with prob <  8e-34
Tucker Lewis Index of factoring reliability =  0.742
RMSEA index =  0.19  and the 90 % confidence intervals are  0.189 0.192
BIC =  83.83
Fit based upon off diagonal values = 0.94
Measures of factor score adequacy
                                               ML1
Correlation of scores with factors            0.96
Multiple R square of scores with factors      0.93
Minimum correlation of possible factor scores  0.86
```

Comparing min res and mle

```
Standardized loadings based upon correlation matrix
    MR1   h2   u2
V1 0.87 0.75 0.25
V2 0.88 0.77 0.23
V3 0.83 0.70 0.30
V4 0.62 0.39 0.61
V5 0.61 0.37 0.63
V6 0.59 0.34 0.66
V7 0.57 0.32 0.68
V8 0.64 0.41 0.59
V9 0.52 0.27 0.73

                MR1
SS loadings    4.32
Proportion Var 0.48
Chi Square =  231.55
    with prob <  2.1e-34
```

```
    ML1   h2   u2
V1 0.88 0.78 0.22
V2 0.90 0.80 0.20
V3 0.85 0.72 0.28
V4 0.59 0.35 0.65
V5 0.57 0.33 0.67
V6 0.57 0.32 0.68
V7 0.54 0.29 0.71
V8 0.63 0.40 0.60
V9 0.49 0.24 0.76

                ML1
SS loadings    4.22
Proportion Var 0.47
Chi Square =  228.59
    with prob <  8e-34
```

2 maximum likelihood factors

```
> f2 <- fa(Thurstone,2,n.obs=213,fm="mle")
> f2

Factor Analysis using method =  ml
Call: fa(r = Thurstone, nfactors = 2, n.obs = 213, fm = "mle")
Standardized loadings based upon correlation matrix
    ML1   ML2   h2   u2
V1  0.94 -0.05 0.83 0.17
V2  0.89  0.03 0.82 0.18
V3  0.85  0.02 0.73 0.27
V4 -0.02  0.84 0.68 0.32
V5 -0.04  0.84 0.66 0.34
V6  0.13  0.59 0.46 0.54
V7  0.28  0.35 0.32 0.68
V8  0.50  0.17 0.39 0.61
V9  0.12  0.49 0.32 0.68
                ML1  ML2
SS loadings     2.92 2.29
Proportion Var 0.32 0.25
Cumulative Var 0.32 0.58
 With factor correlations of
    ML1  ML2
ML1 1.00 0.64
ML2 0.64 1.00
```

2 factor goodness of fits

```
Test of the hypothesis that 2 factors are sufficient.

The degrees of freedom for the null model are  36  and the objecti
          was  5.2 with Chi Square of  1081.97
The degrees of freedom for the model are 19  and the
           objective function was  0.4

The root mean square of the residuals is  0.05
The df corrected root mean square of the residuals is  0.1
The number of observations was  213  with Chi Square =  82.84
                    with prob <  6e-10

Tucker Lewis Index of factoring reliability =  0.884
RMSEA index =  0.128  and the 90 % confidence intervals are
0.127 0.131
BIC =  -19.02
Fit based upon off diagonal values = 0.98
Measures of factor score adequacy
                                                  ML1  ML2
Correlation of scores with factors               0.97 0.93
Multiple R square of scores with factors         0.93 0.86
Minimum correlation of possible factor scores    0.86 0.72
```

3 MLE factors of Thurstone data

```
f3 <- fa(Thurstone,3,n.obs=213,fm="mle")
f3
```

```
Call: fa(r = Thurstone, nfactors = 3, n.obs = 213, fm = "mle")
Standardized loadings based upon correlation matrix
     ML1   ML2   ML3   h2   u2
V1   0.91 -0.04  0.04 0.83 0.17
V2   0.89  0.06 -0.03 0.84 0.16
V3   0.83  0.04  0.00 0.73 0.27
V4   0.00  0.86  0.01 0.73 0.27
V5  -0.01  0.74  0.10 0.63 0.37
V6   0.18  0.63 -0.08 0.50 0.50
V7   0.03 -0.01  0.84 0.72 0.28
V8   0.37 -0.05  0.47 0.50 0.50
V9  -0.06  0.21  0.64 0.53 0.47
                 ML1  ML2  ML3
SS loadings     2.64 1.86 1.49
Proportion Var  0.29 0.21 0.17
Cumulative Var  0.29 0.50 0.67
 With factor correlations of
     ML1  ML2  ML3
ML1 1.00 0.59 0.54
ML2 0.59 1.00 0.52
ML3 0.54 0.52 1.00
```

MinRes solution

R code

```
f3 <- fa(Thurstone,3,n.obs=213) #defaults to minres
 f3
```

```
Factor Analysis using method =  minres
Call: fa(r = Thurstone, nfactors = 3, n.obs = 213)
Standardized loadings (pattern matrix) based upon correlation matrix
                   MR1   MR2   MR3   h2   u2 com
Sentences         0.90 -0.03  0.04 0.82 0.18 1.0
Vocabulary        0.89  0.06 -0.03 0.84 0.16 1.0
Sent.Completion   0.84  0.03  0.00 0.74 0.26 1.0
First.Letters     0.00  0.85  0.00 0.73 0.27 1.0
Four.Letter.Words -0.02  0.75  0.10 0.63 0.37 1.0
Suffixes          0.18  0.63 -0.08 0.50 0.50 1.2
Letter.Series     0.03 -0.01  0.84 0.73 0.27 1.0
Pedigrees         0.38 -0.05  0.46 0.51 0.49 2.0
Letter.Group     -0.06  0.21  0.63 0.52 0.48 1.2

                      MR1  MR2  MR3
SS loadings          2.65 1.87 1.49
Proportion Var       0.29 0.21 0.17
Cumulative Var       0.29 0.50 0.67
Proportion Explained 0.44 0.31 0.25
Cumulative Proportion 0.44 0.75 1.00

 With factor correlations of
     MR1  MR2  MR3
MR1 1.00 0.59 0.53
MR2 0.59 1.00 0.52
MR3 0.53 0.52 1.00
```

with MLE fit statistics

```
Test of the hypothesis that 3 factors are sufficient.

The degrees of freedom for the null model are  36  and the
        objective function was  5.2 with Chi Square of  1081.97
The degrees of freedom for the model are 12  and the
        objective function was  0.01

The root mean square of the residuals is  0
The df corrected root mean square of the residuals is  0.01
The number of observations was  213  with
           Chi Square =  2.82  with prob <  1

Tucker Lewis Index of factoring reliability =  1.027
RMSEA index =  0  and the 90 % confidence intervals are  0 0.023
BIC =  -61.51
Fit based upon off diagonal values = 1
Measures of factor score adequacy
                                                 ML1  ML2  ML3
Correlation of scores with factors              0.96 0.92 0.90
Multiple R square of scores with factors        0.93 0.85 0.81
Minimum correlation of possible factor scores   0.86 0.71 0.63
```

MinRes fits are almost identical

```
Mean item complexity =  1.2
Test of the hypothesis that 3 factors are sufficient.

The degrees of freedom for the null model are  36  and
             the objective function was  5.2 with Chi Square of  1081.97
The degrees of freedom for the model are 12  and the objective function was  0.01

The root mean square of the residuals (RMSR) is  0.01
The df corrected root mean square of the residuals is  0.01

The harmonic number of observations is  213 with the empirica
             l chi square  0.52  with prob <  1
The total number of observations was  213  with Likelihood Chi Square =  2.98  with prob

Tucker Lewis Index of factoring reliability =  1.026
RMSEA index =  0  and the 90 % confidence intervals are  0 0
BIC =  -61.36
Fit based upon off diagonal values = 1
Measures of factor score adequacy
                                                  MR1  MR2  MR3
Correlation of (regression) scores with factors   0.96 0.92 0.90
Multiple R square of scores with factors          0.93 0.85 0.82
Minimum correlation of possible factor scores     0.86 0.71 0.63
>
```

Orthogonal Rotations and oblique Transformations

- Should the latent variables be allowed to correlate?
  - Factors as extracted are orthogonal.
  - Factors as interpreted typically are allowed to be oblique
- Consider five cases
  - unrotated: factors as extracted (rotate="none")
  - rotated using VARIMAX (or Quartimax)
  - transformed using PROCRUSTES
  - transformed using oblimin
  - transformed using geominQ

Unrotated versus Varimax Rotation

```
> fa(Thurstone,3,n.obs=213,rotate="none")

Factor Analysis using method =
minres
Call: fa(r = Thurstone, nfactors = 3, n.obs = 213, rotate = "none")
Standardized loadings based upon correlation matrix
    MR1   MR2   MR3   h2   u2
V1 0.87 -0.27  0.02 0.82 0.18
V2 0.88 -0.24 -0.06 0.84 0.16
V3 0.83 -0.22 -0.03 0.73 0.27
V4 0.66  0.45 -0.32 0.73 0.27
V5 0.63  0.43 -0.21 0.63 0.37
V6 0.60  0.24 -0.29 0.50 0.50
V7 0.60  0.32  0.50 0.72 0.28
V8 0.65  0.05  0.29 0.50 0.50
V9 0.54  0.38  0.30 0.53 0.47

                 MR1  MR2  MR3
SS loadings     4.47 0.86 0.67
Proportion Var  0.50 0.10 0.07
Cumulative Var  0.50 0.59 0.67
```

```
> fa(Thurstone,3,n.obs=213,rotate="Var

    MR1  MR2  MR3  h2   u2
V1 0.86 0.20 0.22 0.82 0.18
V2 0.85 0.27 0.18 0.84 0.16
V3 0.80 0.24 0.19 0.73 0.27
V4 0.29 0.78 0.20 0.73 0.27
V5 0.27 0.70 0.26 0.63 0.37
V6 0.36 0.60 0.10 0.50 0.50
V7 0.28 0.18 0.78 0.72 0.28
V8 0.48 0.15 0.50 0.50 0.50
V9 0.20 0.32 0.62 0.53 0.47

                 MR1  MR2  MR3
SS loadings     2.73 1.78 1.48
Proportion Var  0.30 0.20 0.16
Cumulative Var  0.30 0.50 0.67
```

Varimax versus Quartimax

```
> fa(Thurstone,3,n.obs=213,rotate="varimax")
```

	MR1	MR2	MR3	h2	u2
V1	0.86	0.20	0.22	0.82	0.18
V2	0.85	0.27	0.18	0.84	0.16
V3	0.80	0.24	0.19	0.73	0.27
V4	0.29	0.78	0.20	0.73	0.27
V5	0.27	0.70	0.26	0.63	0.37
V6	0.36	0.60	0.10	0.50	0.50
V7	0.28	0.18	0.78	0.72	0.28
V8	0.48	0.15	0.50	0.50	0.50
V9	0.20	0.32	0.62	0.53	0.47

	MR1	MR2	MR3
SS loadings	2.73	1.78	1.48
Proportion Var	0.30	0.20	0.16
Cumulative Var	0.30	0.50	0.67

```
> fa(Thurstone,3,n.obs=213,rotate="qua
```

	MR1	MR2	MR3	h2	u2
V1	0.91	0.02	0.05	0.82	0.18
V2	0.91	0.09	0.01	0.84	0.16
V3	0.85	0.07	0.03	0.73	0.27
V4	0.47	0.71	0.11	0.73	0.27
V5	0.45	0.63	0.18	0.63	0.37
V6	0.48	0.51	0.01	0.50	0.50
V7	0.45	0.12	0.71	0.72	0.28
V8	0.58	0.05	0.40	0.50	0.50
V9	0.37	0.27	0.56	0.53	0.47

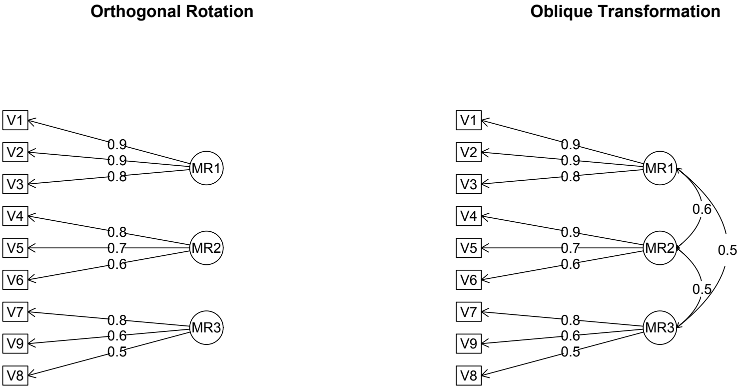
	MR1	MR2	MR3
SS loadings	3.70	1.27	1.03
Proportion Var	0.41	0.14	0.11
Cumulative Var	0.41	0.55	0.67

Varimax versus Quartimin

```
> fa(Thurstone,3,n.obs=213,
          rotate="Varimax")

    MR1  MR2  MR3  h2   u2
V1 0.86 0.20 0.22 0.82 0.18
V2 0.85 0.27 0.18 0.84 0.16
V3 0.80 0.24 0.19 0.73 0.27
V4 0.29 0.78 0.20 0.73 0.27
V5 0.27 0.70 0.26 0.63 0.37
V6 0.36 0.60 0.10 0.50 0.50
V7 0.28 0.18 0.78 0.72 0.28
V8 0.48 0.15 0.50 0.50 0.50
V9 0.20 0.32 0.62 0.53 0.47

                     MR1  MR2  MR3
SS loadings         2.73 1.78 1.48
 Proportion Var 0.30 0.20 0.16
 Cumulative Var 0.30 0.50 0.67
```

```
> fa(Thurstone,3,n.obs=213,rotate="qua

     MR1   MR2   MR3   h2   u2
V1  0.91 -0.04  0.04 0.82 0.18
V2  0.89  0.06 -0.03 0.84 0.16
V3  0.83  0.04  0.00 0.73 0.27
V4  0.00  0.86  0.00 0.73 0.27
V5 -0.01  0.74  0.10 0.63 0.37
V6  0.18  0.63 -0.08 0.50 0.50
V7  0.03 -0.01  0.84 0.72 0.28
V8  0.37 -0.05  0.47 0.50 0.50
V9 -0.06  0.21  0.64 0.53 0.47

                     MR1  MR2  MR3
SS loadings         2.64 1.86 1.50
 Proportion Var 0.29 0.21 0.17
 Cumulative Var 0.29 0.50 0.67
  With factor correlations of
     MR1  MR2  MR3
MR1 1.00 0.59 0.54
MR2 0.59 1.00 0.52
MR3 0.54 0.52 1.00
```

Visualizing the difference: Varimax versus quartimin



Specifying the parameters in sem using the RAM notation

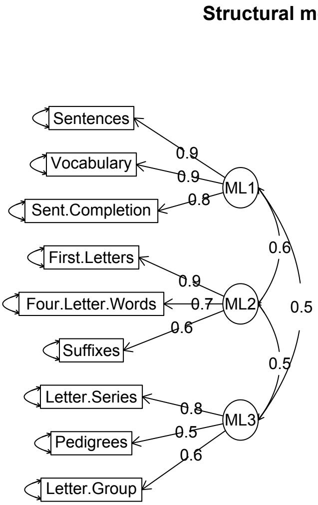
- Need to specify each path
- Need to specify the error variance paths as well
  - A short cut can be done by using *psych*
  - See the vignette ‘`psych for sem`’' for more details
- Apply this to the Thurstone problem

create the sem commands by using psych

```
f3 <- fa(Thurstone,3,fm='mle')
mod3 <- structure.diagram(f3,cut=.45,errors=TRUE)
mod3
```

```
      Path          Parameter  Value
 [1,] "ML1->V1"     "F1V1"     NA
 [2,] "ML1->V2"     "F1V2"     NA
 [3,] "ML1->V3"     "F1V3"     NA
 [4,] "ML2->V4"     "F2V4"     NA
 [5,] "ML2->V5"     "F2V5"     NA
 [6,] "ML2->V6"     "F2V6"     NA
 [7,] "ML3->V7"     "F3V7"     NA
 [8,] "ML3->V8"     "F3V8"     NA
 [9,] "ML3->V9"     "F3V9"     NA
[10,] "V1<->V1"     "x1e"      NA
[11,] "V2<->V2"     "x2e"      NA
...
[18,] "V9<->V9"     "x9e"      NA
[19,] "ML2<->ML1"   "rF2F1"    NA
[20,] "ML3<->ML1"   "rF3F1"    NA
[21,] "ML3<->ML2"   "rF3F2"    NA
[22,] "ML1<->ML1"   NA         "1"
[23,] "ML2<->ML2"   NA         "1"
[24,] "ML3<->ML3"   NA         "1"
```

Thurstone structural model from structure.diagram



The lavaan code is a bit easier to read

R code

```
f3 <- fa(Thurstone,3,fm='mle')
mod3 <- structure.diagram(f3,cut=.45,errors=TRUE)
mod3$lavaan
```

```
 mod3$lavaan
 ML1 =~  + Sentences + Vocabulary + Sent.Completion
 ML2 =~  + First.Letters + Four.Letter.Words + Suffixes
[ML3 =~  + Letter.Series + Pedigrees + Letter.Group
 ML3 ~~ ML2
```

But is this really “confirmatory”” How did I decide that ML2 and ML3 should correlate?

Running sem

```
> rownames(Thurstone) <- colnames(Thurstone) #to get the names to match the model
 sem3 <- sem(mod3$sem, Thurstone, N=213)
summary(sem3, digits=2)


 Model Chisquare =  38   Df =  24 Pr(>Chisq) = 0.033
 Chisquare (null model) =  1102   Df =  36
 Goodness-of-fit index =  0.96
 Adjusted goodness-of-fit index =  0.92
 RMSEA index =  0.053   90% CI: (0.015, 0.083)
 Bentler-Bonnett NFI =  0.97
 Tucker-Lewis NNFI =  0.98
 Bentler CFI =  0.99
 SRMR =  0.044
 BIC =  -90

 Normalized Residuals
   Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
 -0.97    -0.42     0.00     0.04     0.09     1.63
```

With parameter estimates

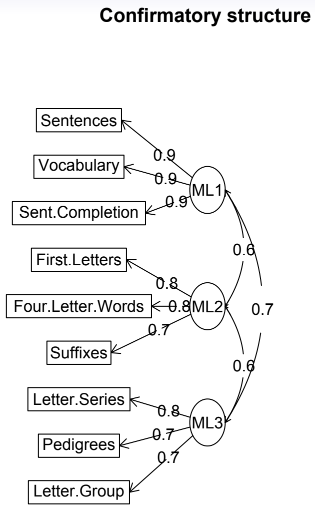
```
 Parameter Estimates
      Estimate  Std Error  z value  Pr(>|z|)
F1V1  0.90      0.054      16.7     0.0e+00   V1 <--- ML1
F1V2  0.91      0.054      17.0     0.0e+00   V2 <--- ML1
F1V3  0.86      0.056      15.3     0.0e+00   V3 <--- ML1
F2V4  0.84      0.061      13.8     0.0e+00   V4 <--- ML2
F2V5  0.80      0.062      12.9     0.0e+00   V5 <--- ML2
F2V6  0.70      0.064      10.9     0.0e+00   V6 <--- ML2
F3V7  0.78      0.065      12.0     0.0e+00   V7 <--- ML3
F3V8  0.72      0.067      10.7     0.0e+00   V8 <--- ML3
F3V9  0.70      0.067      10.5     0.0e+00   V9 <--- ML3
x1e   0.18      0.028       6.4     1.7e-10   V1 <--> V1
x2e   0.16      0.028       5.9     3.0e-09   V2 <--> V2
x3e   0.27      0.033       8.0     1.6e-15   V3 <--> V3
x4e   0.30      0.051       5.9     2.7e-09   V4 <--> V4
x5e   0.36      0.052       7.0     3.4e-12   V5 <--> V5
x6e   0.51      0.060       8.4     0.0e+00   V6 <--> V6
x7e   0.39      0.062       6.3     2.3e-10   V7 <--> V7
x8e   0.48      0.065       7.4     1.8e-13   V8 <--> V8
x9e   0.51      0.065       7.7     9.5e-15   V9 <--> V9
rF2F1 0.64      0.051      12.6     0.0e+00   ML1 <--> ML2
rF3F1 0.67      0.054      12.5     0.0e+00   ML1 <--> ML3
rF3F2 0.64      0.059      10.7     0.0e+00   ML2 <--> ML3
```

The lavaan results

R code

```
fit3 <- cfa(mod3$lavaan,sample.cov=Thurstone,sample.nobs=213,std.lv
summary(fit3)
```

```
 Number of observations                          213

  Estimator                                       ML
  Minimum Function Test Statistic             38.376
  Degrees of freedom                              24
  P-value (Chi-square)                         0.032

 Parameter Estimates:

  Information                               Expected
  Standard Errors                           Standard

 Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  ML1 =~
    Sentences         0.903    0.054   16.727    0.000
    Vocabulary        0.912    0.054   17.005    0.000
    Sent.Completin    0.854    0.056   15.317    0.000
  ML2 =~
    First.Letters     0.834    0.060   13.783    0.000
    Four.Lttr.Wrds    0.795    0.061   12.937    0.000
    Suffixes          0.701    0.064   10.960    0.000
  ML3 =~
    Letter.Series     0.779    0.064   12.173    0.000
    Pedigrees         0.718    0.065   10.998    0.000
    Letter.Group      0.702    0.066   10.679    0.000
```


Confirmatory structure
Sentences
Vocabulary
Sent.Completion
First.Letters
Four.Letter.Words
Suffixes
Letter.Series
Pedigrees
Letter.Group
ML1
ML2
ML3
0.9
0.9
0.9
0.8
0.8
0.7
0.8
0.7
0.7
0.6
0.7
0.6

Exploratory bifactor model

R code

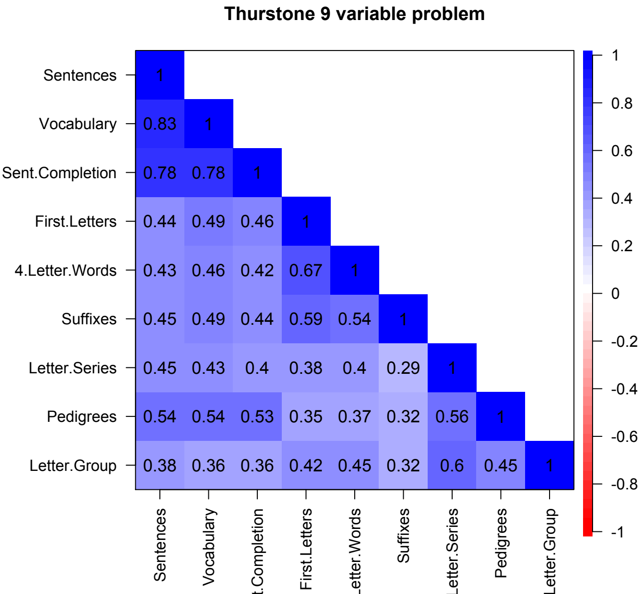
```
omt <- omega(Thurstone)
omega.diagram(omt,sl=FALSE)
```

```
Omega
Call: omega(m = Thurstone)
Alpha:                 0.89
G.6:                   0.91
Omega Hierarchical:    0.74
Omega H asymptotic:    0.79
Omega Total            0.93

Schmid Leiman Factor loadings greater than  0.2
      g   F1*  F2*  F3*  h2   u2   p2
V1 0.71  0.57            0.82 0.18 0.61
V2 0.73  0.55            0.84 0.16 0.63
V3 0.68  0.52            0.73 0.27 0.63
V4 0.65       0.56       0.73 0.27 0.57
V5 0.62       0.49       0.63 0.37 0.61
V6 0.56       0.41       0.50 0.50 0.63
V7 0.59            0.61 0.72 0.28 0.48
V8 0.58  0.23      0.34 0.50 0.50 0.66
V9 0.54            0.46 0.53 0.47 0.56

With eigenvalues of:
   g  F1*  F2*  F3*
3.58 0.96 0.74 0.71
```

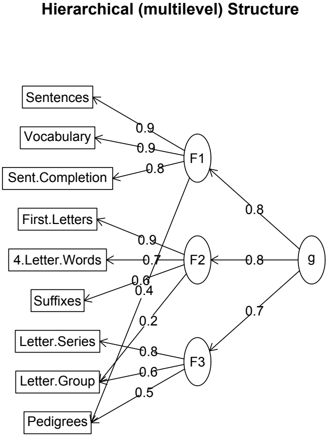
With fit statistics

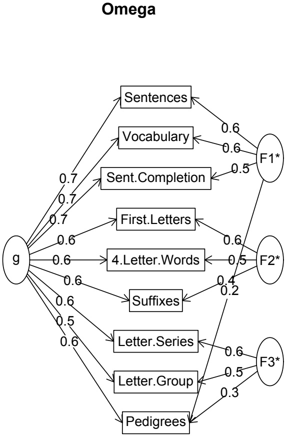
```
With eigenvalues of:
   g  F1*  F2*  F3*
3.58 0.96 0.74 0.71
general/max  3.71   max/min =   1.35
mean percent general =  0.6    with sd =  0.05 and cv of  0.09
The degrees of freedom are 12  and the fit is  0.01
The number of observations was  213  with Chi Square =  2.82  with prob <  1
The root mean square of the residuals is  0
The df corrected root mean square of the residuals is  0.01
RMSEA index =  0  and the 90 % confidence intervals are  0 0.023
BIC =  -61.51
Compare this with the adequacy of just a general factor and no group factors
The degrees of freedom for just the general factor are 27  and the fit is  1.48
The number of observations was  213  with Chi Square =  307.1  with prob <  2.8e-49
The root mean square of the residuals is  0.1
The df corrected root mean square of the residuals is  0.16
RMSEA index =  0.224  and the 90 % confidence intervals are  0.223 0.226
BIC =  162.35
Measures of factor score adequacy
                                                 g  F1*  F2*  F3*
Correlation of scores with factors            0.86 0.73 0.72 0.75
Multiple R square of scores with factors      0.74 0.54 0.52 0.56
Minimum correlation of factor score estimates 0.49 0.08 0.03 0.11
```

A correlation plot to show a general factor



Two ways of showing a hierarchical structure





But remember that the SL solution has proportionality constraints.
G is mediated by lower level factors.

Exploratory versus confirmatory bifactor model

- `omega` will do exploratory bifactoring
- `omegaSem` will do exploratory and then confirmatory based upon that solution.
    - This is not true "confirmatory" in that the solution was decided on in an exploratory fashion.
    - True confirmatory tests a prior hypothesis
- `omegaSem` reports both exploratory and confirmatory results.
- The sem model is hidden as part of the structure but may be found from the `str` command or just ask for the names of the output,
- `omegaFromSem` will take the *lavaan* output and do the omega analysis
- `omegaDirect` (adapted from Niels Waller) does a direct solution

omegaSem

From omega (SL)

```
Alpha:                 0.89
G.6:                   0.91
Omega Hierarchical:    0.74
Omega H asymptotic:    0.79
Omega Total            0.93
```

Schmid Leiman Factor loadings greater than 0.2

	g	F1*	F2*	F3*	h2	u2	p2
Sentences	0.71	0.57			0.82	0.18	0.61
Vocabulary	0.73	0.55			0.84	0.16	0.63
Sent.Completion	0.68	0.52			0.73	0.27	0.63
First.Letters	0.65		0.56		0.73	0.27	0.57
4.Letter.Words	0.62		0.49		0.63	0.37	0.61
Suffixes	0.56		0.41		0.50	0.50	0.63
Letter.Series	0.59			0.61	0.72	0.28	0.48
Pedigrees	0.58	0.23		0.34	0.50	0.50	0.66
Letter.Group	0.54			0.46	0.53	0.47	0.56

With eigenvalues of:

g	F1*	F2*	F3*
3.58	0.96	0.74	0.71

From omegaSem

```
Omega Hierarchical from a confirmatory mode
     using sem =  0.79
Omega Total  from a confirmatory model
     using sem =  0.93
```

With loadings of

	g	F1*	F2*	F3*	h2
Sentences	0.77	0.49			0.83
Vocabulary	0.79	0.45			0.83
Sent.Completion	0.75	0.40			0.73
First.Letters	0.61		0.61		0.75
4.Letter.Words	0.60		0.51		0.61
Suffixes	0.57		0.39		0.48
Letter.Series	0.57			0.73	0.85
Pedigrees	0.66			0.25	0.50
Letter.Group	0.53			0.41	0.45

With eigenvalues of:

g	F1*	F2*	F3*
3.88	0.61	0.79	0.76

omegaDirect on Thurstone data set

R code

```
omegaDirect(Thurstone,3)
```

```
Omega from direct Schmid Leiman =  0.7

Call: omegaDirect(m = Thurstone, nfactors = 3)
Standardized loadings (pattern matrix) based upon correlation matrix
                     g   F1*   F2*   F3*   h2   u2
Sentences         0.68  0.59  0.01  0.08 0.82 0.18
Vocabulary        0.70  0.58  0.08  0.03 0.84 0.16
Sent.Completion   0.66  0.55  0.05  0.05 0.74 0.26
First.Letters     0.62  0.02  0.58  0.03 0.73 0.27
Four.Letter.Words 0.60  0.00  0.51  0.10 0.63 0.37
Suffixes          0.54  0.13  0.43 -0.03 0.50 0.50
Letter.Series     0.59 -0.01 -0.02  0.62 0.73 0.27
Pedigrees         0.57  0.23 -0.02  0.36 0.51 0.49
Letter.Group      0.54 -0.06  0.14  0.46 0.52 0.48

                         g  F1*  F2*  F3*
SS loadings           3.39 1.06 0.81 0.74
Proportion Var        0.38 0.12 0.09 0.08
Cumulative Var        0.38 0.49 0.58 0.67
Proportion Explained  0.56 0.18 0.14 0.12
Cumulative Proportion 0.56 0.74 0.88 1.00

With eigenvalues of:
   g  F1*  F2*  F3*
3.39 1.06 0.81 0.74
The degrees of freedom for the model is 12  and the fit was  0.01
```

OmegaDirect (continued)

```
                         g  F1*  F2*  F3*
SS loadings           3.39 1.06 0.81 0.74
Proportion Var        0.38 0.12 0.09 0.08
Cumulative Var        0.38 0.49 0.58 0.67
Proportion Explained  0.56 0.18 0.14 0.12
Cumulative Proportion 0.56 0.74 0.88 1.00

With eigenvalues of:
   g  F1*  F2*  F3*
3.39 1.06 0.81 0.74
The degrees of freedom for the model is 12  and the fit was  0.01

The root mean square of the residuals is  0.01
The df corrected root mean square of the residuals is  0.01

 Total, General and Subset omega for each subset
                                                 g  F1*  F2*  F3*
Omega total for total scores and subscales    0.93 0.92 0.82 0.79
Omega general for total scores and subscales  0.70 0.53 0.47 0.46
Omega group for total scores and subscales    0.17 0.38 0.35 0.33
```

The sem model is an object in omegaSem output

```
omts <- omegaSem(Thurstone,n.obs=213)
omts$omegaSem$model
```

```
     Path                                  Parameter                Initial Value
 [1,] "g->Sentences"                        "Sentences"               NA
 [2,] "g->Vocabulary"                       "Vocabulary"              NA
 [...
 [9,] "g->Letter.Group"                     "Letter.Group"            NA
[10,] "F1*->Sentences"                      "F1*Sentences"            NA
[11,] "F1*->Vocabulary"                     "F1*Vocabulary"           NA
[12,] "F1*->Sent.Completion"                "F1*Sent.Completion"      NA
[13,] "F2*->First.Letters"                  "F2*First.Letters"        NA
[14,] "F2*->4.Letter.Words"                 "F2*4.Letter.Words"       NA
[15,] "F2*->Suffixes"                       "F2*Suffixes"             NA
[16,] "F3*->Letter.Series"                  "F3*Letter.Series"        NA
[17,] "F3*->Pedigrees"                      "F3*Pedigrees"            NA
[18,] "F3*->Letter.Group"                   "F3*Letter.Group"         NA
[19,] "Sentences<->Sentences"               "e1"                      NA
[20,] "Vocabulary<->Vocabulary"             "e2"                      NA
...
[27,] "Letter.Group<->Letter.Group"         "e9"                      NA
[28,] "F1*<->F1*"                          NA                         "1"
[29,] "F2*<->F2*"                          NA                         "1"
[30,] "F3*<->F3*"                          NA                         "1"
[31,] "g <->g"                             NA                         "1"
```

Using the sem model from omegaSem

R code

```
> sem.t.om <- sem(omts$omegaSem$model,Thurstone,N=213)
> summary(sem.t.om)
```

```
 Model Chisquare =  24.216   Df =  18 Pr(>Chisq) = 0.14807
 Chisquare (null model) =  1101.9   Df =  36
 Goodness-of-fit index =  0.97578
 Adjusted goodness-of-fit index =  0.93944
 RMSEA index =  0.040361   90% CI: (NA, 0.077994)
 Bentler-Bonnett NFI =  0.97802
 Tucker-Lewis NNFI =  0.98834
 Bentler CFI =  0.99417
 SRMR =  0.034895
 BIC =  -72.287

 Normalized Residuals
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.8210 -0.3340  0.0000  0.0282  0.1560  1.8000
```

sem parameter estimates

```
Parameter Estimates
                    Estimate Std Error z value  Pr(>|z|)
Sentences           0.76787  0.072626  10.57291 0.0000e+00 Sentences <--- g
Vocabulary          0.79092  0.072418  10.92170 0.0000e+00 Vocabulary <--- g
Sent.Completion     0.75362  0.073402  10.26709 0.0000e+00 Sent.Completion <--- g
First.Letters       0.60838  0.072201   8.42617 0.0000e+00 First.Letters <--- g
4.Letter.Words      0.59733  0.073851   8.08843 6.6613e-16 4.Letter.Words <--- g
Suffixes            0.57179  0.071492   7.99792 1.3323e-15 Suffixes <--- g
Letter.Series       0.56689  0.074271   7.63282 2.2871e-14 Letter.Series <--- g
Pedigrees           0.66233  0.069321   9.55455 0.0000e+00 Pedigrees <--- g
Letter.Group        0.52995  0.078985   6.70955 1.9522e-11 Letter.Group <--- g
F1*Sentences        0.48787  0.085457   5.70898 1.1366e-08 Sentences <--- F1*
F1*Vocabulary       0.45232  0.090422   5.00233 5.6640e-07 Vocabulary <--- F1*
F1*Sent.Completion  0.40445  0.093402   4.33024 1.4895e-05 Sent.Completion <--- F1*
F2*First.Letters    0.61405  0.085794   7.15733 8.2268e-13 First.Letters <--- F2*
F2*4.Letter.Words   0.50581  0.084848   5.96130 2.5024e-09 4.Letter.Words <--- F2*
F2*Suffixes         0.39432  0.078289   5.03671 4.7359e-07 Suffixes <--- F2*
F3*Letter.Series    0.72730  0.159499   4.55988 5.1184e-06 Letter.Series <--- F3*
F3*Pedigrees        0.24684  0.089011   2.77317 5.5513e-03 Pedigrees <--- F3*
F3*Letter.Group     0.40915  0.122180   3.34875 8.1177e-04 Letter.Group <--- F3*
e1                  0.17236  0.034113   5.05265 4.3571e-07 Sentences <--> Sentences
e2                  0.16984  0.030037   5.65438 1.5641e-08 Vocabulary <--> Vocabulary
e3                  0.26847  0.033188   8.08958 6.6613e-16 Sent.Completion <--> Sent.Compl
e4                  0.25281  0.079472   3.18115 1.4669e-03 First.Letters <--> First.Letter
e5                  0.38735  0.063194   6.12960 8.8103e-10 4.Letter.Words <--> 4.Letter.Wo
e6                  0.51757  0.059639   8.67838 0.0000e+00 Suffixes <--> Suffixes
e7                  0.14967  0.223242   0.67044 5.0257e-01 Letter.Series <--> Letter.Serie
e8                  0.50039  0.059655   8.38800 0.0000e+00 Pedigrees <--> Pedigrees
e9                  0.55175  0.084725   6.51223 7.4044e-11 Letter.Group <--> Letter.Group
```

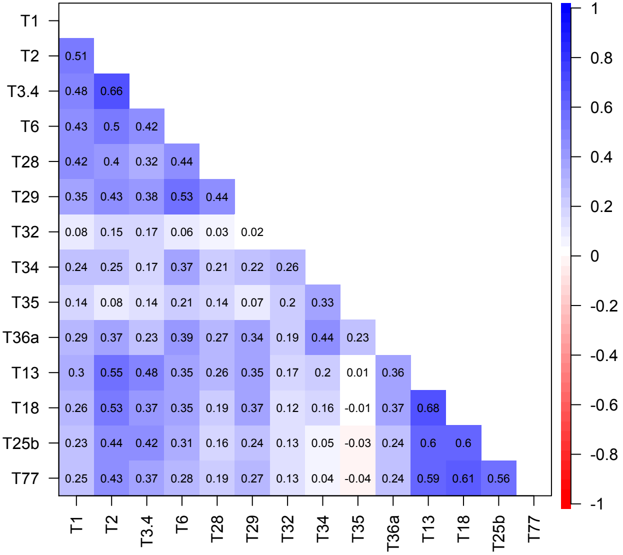
14 cognitive variables from Holzinger

R code

```
lowerMat(Holzinger)
 #we already have the correlations, just show them
```

```
      T1    T2    T3.4  T6    T28   T29   T32   T34   T35   T36a  T13   T18   T25b  T77
T1    1.00
T2    0.51  1.00
T3.4  0.48  0.66  1.00
T6    0.43  0.50  0.42  1.00
T28   0.42  0.40  0.32  0.44  1.00
T29   0.35  0.43  0.38  0.53  0.44  1.00
T32   0.08  0.15  0.17  0.06  0.03  0.02  1.00
T34   0.24  0.25  0.17  0.37  0.21  0.22  0.26  1.00
T35   0.14  0.08  0.14  0.21  0.14  0.07  0.20  0.33  1.00
T36a  0.29  0.37  0.23  0.39  0.27  0.34  0.19  0.44  0.23  1.00
T13   0.30  0.54  0.48  0.35  0.26  0.35  0.17  0.20  0.01  0.36  1.00
T18   0.26  0.53  0.37  0.35  0.19  0.37  0.12  0.16 -0.01  0.37  0.68  1.00
T25b  0.23  0.44  0.42  0.31  0.16  0.24  0.13  0.05 -0.03  0.23  0.60  0.60  1.00
T77   0.25  0.43  0.37  0.28  0.19  0.27  0.13  0.04 -0.04  0.24  0.59  0.61  0.56  1.00
>
```

Holzinger 14 cognitive variables



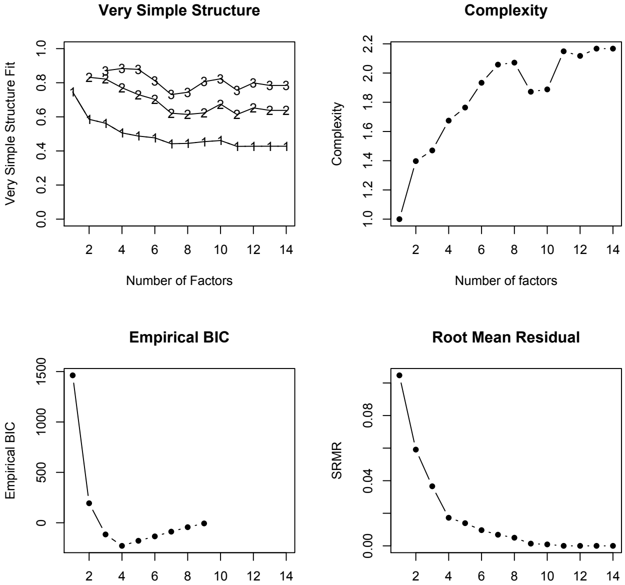
How many factors

R code

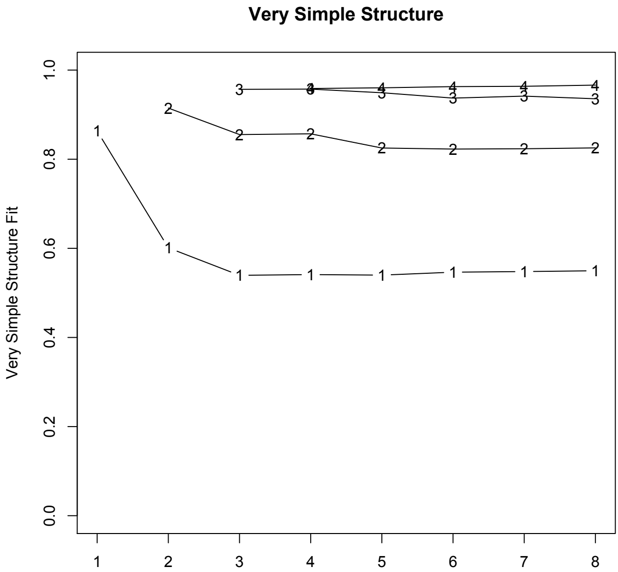
```
nfactors(Holzinger,n.obs=355)
```

```
Number of factors
Call: vss(x = x, n = n, rotate = rotate, diagonal = diagonal, fm = fm,
    n.obs = n.obs, plot = FALSE, title = title, use = use, cor = cor)
VSS complexity 1 achieves a maximimum of 0.75  with  1  factors
VSS complexity 2 achieves a maximimum of 0.83  with  2  factors
The Velicer MAP achieves a minimum of 0.03  with  2  factors
Empirical BIC achieves a minimum of  -221.54  with  4  factors
Sample Size adjusted BIC achieves a minimum of  -69.03  with  4  factors

Statistics by number of factors
   vss1 vss2   map dof   chisq    prob sqresid  fit RMSEA    BIC SABIC complex  eChisq
1  0.75 0.00 0.034  77 5.4e+02 2.6e-71     8.8 0.75 0.132   91.2 335.5     1.0 7.1e+02 1.
2  0.59 0.83 0.026  64 2.3e+02 7.3e-21     5.9 0.83 0.087 -144.0  59.1     1.4 2.3e+02 5.
3  0.56 0.82 0.029  52 1.2e+02 4.7e-07     4.5 0.87 0.061 -187.3 -22.3     1.5 8.6e+01 3.
4  0.51 0.77 0.036  41 4.2e+01 4.4e-01     3.6 0.90 0.011 -199.1 -69.0     1.7 1.9e+01 1.
5  0.49 0.73 0.061  31 2.7e+01 6.6e-01     3.5 0.90 0.000 -154.9 -56.5     1.8 1.3e+01 1.
6  0.48 0.70 0.071  22 1.4e+01 9.2e-01     3.0 0.91 0.000 -115.6 -45.8     1.9 6.0e+00 9.
7  0.44 0.62 0.102  14 5.8e+00 9.7e-01     2.9 0.92 0.000  -76.4 -32.0     2.1 3.0e+00 6.
8  0.44 0.61 0.141   7 2.7e+00 9.2e-01     2.8 0.92 0.000  -38.5 -16.2     2.1 1.6e+00 5.
9  0.45 0.62 0.183   1 1.9e-01 6.6e-01     2.4 0.93 0.000   -5.7  -2.5     1.9 1.2e-01 1.
10 0.46 0.67 0.235  -4 9.9e-02      NA     2.2 0.94    NA     NA    NA     1.9 5.1e-02 8.
11 0.43 0.61 0.369  -8 1.9e-07      NA     2.2 0.94    NA     NA    NA     2.1 1.0e-07 1.
12 0.43 0.65 0.529 -11 1.0e-08      NA     2.3 0.93    NA     NA    NA     2.1 5.1e-09 2.
13 0.43 0.64 1.000 -13 6.0e-13      NA     2.3 0.93    NA     NA    NA     2.2 3.5e-16 7.
14 0.43 0.64    NA -14 0.0e+00      NA     2.3 0.93    NA     NA    NA     2.2 2.3e-26 5.
```

Holzinger 14 variable problem: nfactors

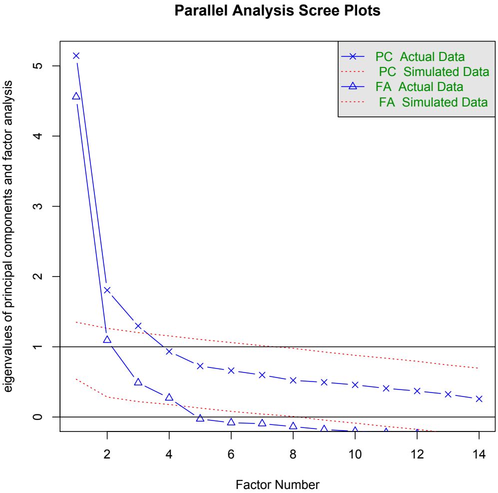


Holzinger 14 variable problem

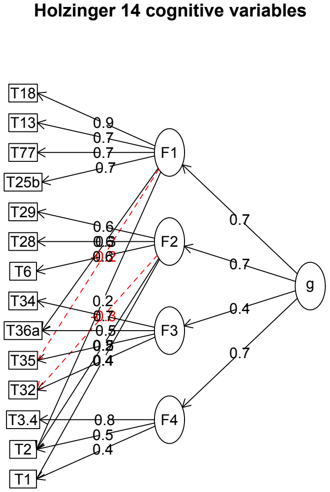


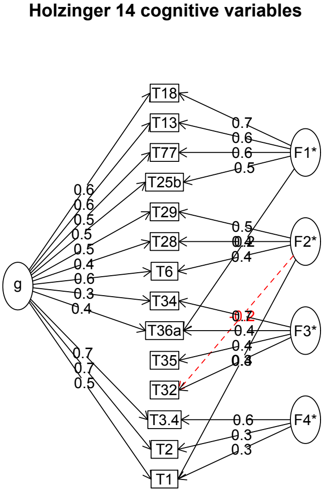
Parallel analysis of Holzinger 14 variables

> fa.parallel(Holzinger,n.obs=355) Parallel analysis suggests that the number of factors = 4 and the number of components = 3



Exploratory Omega solution for Holzinger 14 variable problem





Omega for Holzinger – exploratory and confirmatory

```
Call: omegaSem(m = Holzinger, nfactors = 4, n.obs = 355)
Omega

Alpha:                  0.85
G.6:                    0.88
Omega Hierarchical:     0.64
Omega H asymptotic:     0.71
Omega Total             0.9

Schmid Leiman Factor loadings greater than  0.2
         g   F1*   F2*   F3*   F4*   h2   u2   p2
T1     0.52        0.27        0.27 0.42 0.58 0.64
T2     0.69                    0.34 0.66 0.34 0.72
T3.4   0.65                    0.57 0.75 0.25 0.56
T6     0.57        0.42             0.54 0.46 0.59
T28    0.44        0.44             0.40 0.60 0.49
T29    0.51        0.47             0.50 0.50 0.54
T32               -0.25  0.38       0.26 0.74 0.13
T34    0.32              0.66       0.54 0.46 0.19
T35                      0.44       0.25 0.75 0.10
T36a   0.43  0.20        0.44       0.45 0.55 0.41
T13    0.59  0.55                   0.67 0.33 0.52
T18    0.55  0.65                   0.74 0.26 0.42
T25b   0.49  0.53                   0.53 0.47 0.43
T77    0.47  0.55                   0.53 0.47 0.41

With eigenvalues of:
   g  F1*  F2*  F3*  F4*
3.41 1.44 0.78 1.02 0.58
```

```
 Omega Hierarchical from a confirmatory mod
 Omega Total  from a confirmatory model usi
With loadings of
        g  F1*  F2*  F3*  F4*   h2   u2
T1   0.59                     0.37 0.63
T2   0.80                0.26 0.71 0.29
T3.4 0.64                0.57 0.74 0.26
T6   0.65      0.36           0.55 0.45
T28  0.50      0.34           0.36 0.64
T29  0.57      0.45           0.53 0.47
T32                 0.32      0.13 0.87
T34  0.36           0.65      0.55 0.45
T35                 0.44      0.22 0.78
T36a 0.51           0.38      0.41 0.59
T13  0.64 0.51                0.67 0.33
T18  0.60 0.57                0.68 0.32
T25b 0.49 0.55                0.55 0.45
T77  0.48 0.57                0.55 0.45

With eigenvalues of:
   g  F1*  F2*  F3*  F4*
4.06 1.21 0.45 0.87 0.43
```

lavaan: Latent Variable Analysis

- The *lavaan* package Rosseel (2012) is a very powerful sem/cfa modeling package.
- Mimics input of LISREL and MPlus
  - Easier to set up than sem Fox et al. (2013)
  - Clearly targets MPlus
  - Further help available at `http://lavaan.ugent.be/`
  - Example data sets from MPlus

An example from lavaan

1. Holzinger Swineford data set from two schools
   - First describe the data
   - Descriptive stats and graphics
2. Then consider the structure of the 9 variables
   - Using fa
   - Using sem/lavaan
3. Full HS data set is part of *psychTools*
   - As pointed out by Keith Widaman, the HS data in lavaan have been rescaled 0-10.
   - Note the Holzinger Swineford are *tests* not items. See (Widaman and Revelle, 2023a,b)

Holzinger descriptive statistics

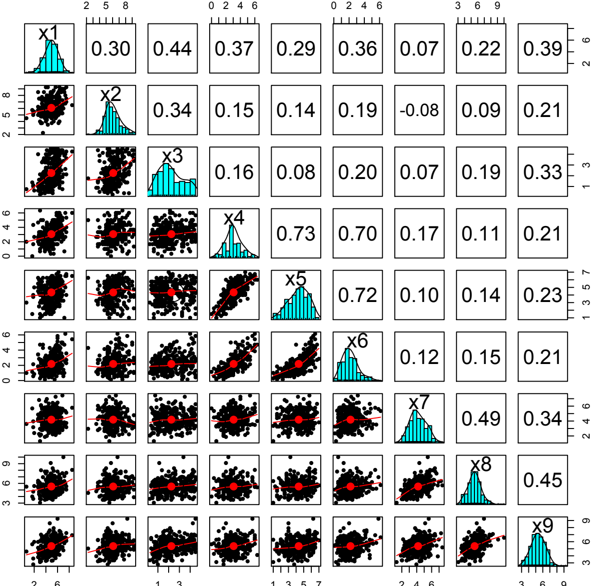
See also `holzinger.swineford` in *psychTools*

R code

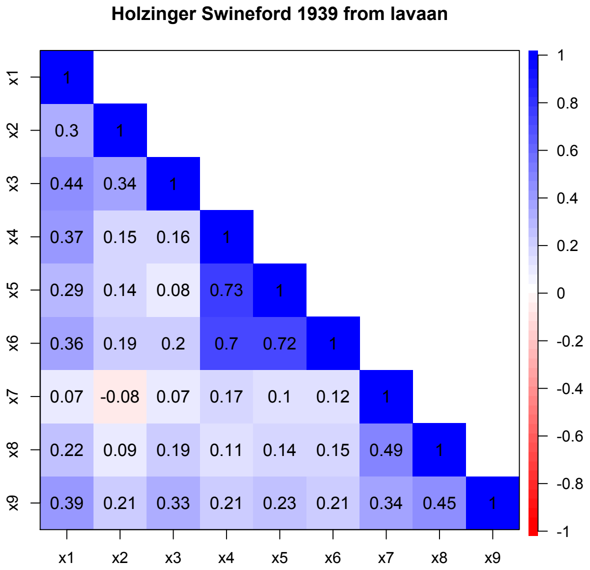
```
describe(HolzingerSwineford1939)
```

	var	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	
id	1	301	176.55	105.94	163.00	176.78	140.85	1.00	351.00	350.00	-0.01	-1.36	6.
sex	2	301	1.51	0.50	2.00	1.52	0.00	1.00	2.00	1.00	-0.06	-2.00	0.
ageyr	3	301	13.00	1.05	13.00	12.89	1.48	11.00	16.00	5.00	0.69	0.20	0.
agemo	4	301	5.38	3.45	5.00	5.32	4.45	0.00	11.00	11.00	0.09	-1.22	0.
school*	5	301	1.52	0.50	2.00	1.52	0.00	1.00	2.00	1.00	-0.07	-2.00	0.
grade	6	300	7.48	0.50	7.00	7.47	0.00	7.00	8.00	1.00	0.09	-2.00	0.
x1	7	301	4.94	1.17	5.00	4.96	1.24	0.67	8.50	7.83	-0.25	0.31	0.
x2	8	301	6.09	1.18	6.00	6.02	1.11	2.25	9.25	7.00	0.47	0.33	0.
x3	9	301	2.25	1.13	2.12	2.20	1.30	0.25	4.50	4.25	0.38	-0.91	0.
x4	10	301	3.06	1.16	3.00	3.02	0.99	0.00	6.33	6.33	0.27	0.08	0.
x5	11	301	4.34	1.29	4.50	4.40	1.48	1.00	7.00	6.00	-0.35	-0.55	0.
x6	12	301	2.19	1.10	2.00	2.09	1.06	0.14	6.14	6.00	0.86	0.82	0.
x7	13	301	4.19	1.09	4.09	4.16	1.10	1.30	7.43	6.13	0.25	-0.31	0.
x8	14	301	5.53	1.01	5.50	5.49	0.96	3.05	10.00	6.95	0.53	1.17	0.
x9	15	301	5.37	1.01	5.42	5.37	0.99	2.78	9.25	6.47	0.20	0.29	0.

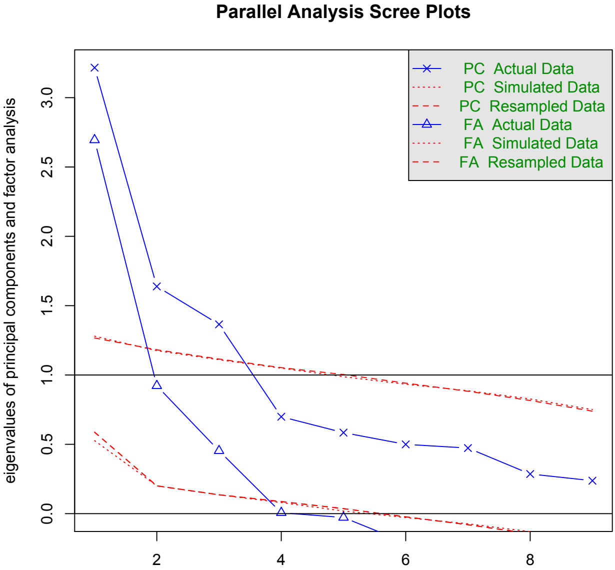
Pairs panels of HolzingerSwineford1939



Weaker correlations than the Thurstone data set



Parallel Analysis of the Holzlinger Swineford data set



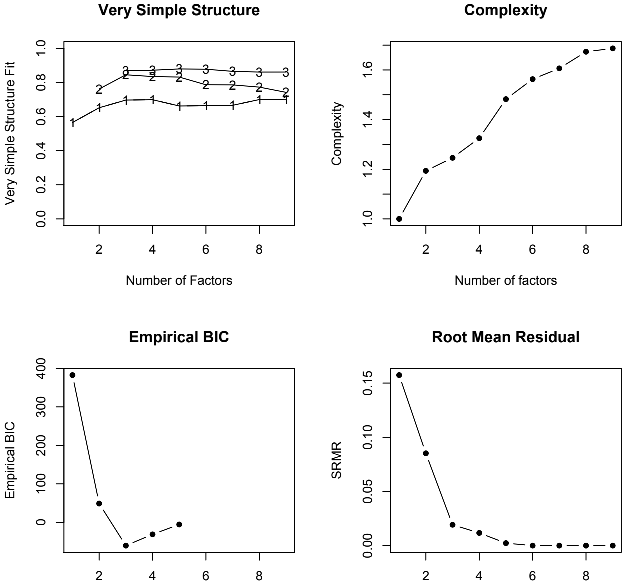
Holzinger Swineford 1939 Number of factors?

R code

```
nfactors(HolzingerSwineford1939[7:15])
```

```
Number of factors
Call: vss(x = x, n = n, rotate = rotate, diagonal = diagonal, fm = fm,
    n.obs = n.obs, plot = FALSE, title = title, use = use, cor = cor)
VSS complexity 1 achieves a maximimum of 0.7  with  8  factors
VSS complexity 2 achieves a maximimum of 0.85  with  3  factors
The Velicer MAP achieves a minimum of 0.06  with  2  factors
Empirical BIC achieves a minimum of  -60.46  with  3  factors
Sample Size adjusted BIC achieves a minimum of  -9.55  with  4  factors

Statistics by number of factors
  vss1 vss2   map dof   chisq     prob sqresid  fit RMSEA   BIC SABIC complex  eChisq    S
1 0.57 0.00 0.080  27 3.1e+02 2.9e-49     7.1 0.57 0.187 152.9 238.6     1.0 5.4e+02 1.6e
2 0.65 0.76 0.063  19 1.3e+02 4.1e-18     3.9 0.76 0.139  19.2  79.5     1.2 1.6e+02 8.5e
3 0.70 0.85 0.068  12 2.2e+01 3.4e-02     2.1 0.87 0.055 -46.1  -8.1     1.2 8.0e+00 1.9e
4 0.70 0.83 0.120   6 5.7e+00 4.6e-01     2.0 0.88 0.000 -28.6  -9.6     1.3 3.0e+00 1.2e
5 0.66 0.83 0.207   1 2.5e-01 6.2e-01     1.7 0.90 0.000  -5.5  -2.3     1.5 1.1e-01 2.3e
6 0.66 0.79 0.343  -3 4.0e-08       NA     1.6 0.90    NA    NA    NA     1.6 3.0e-08 1.2e
7 0.67 0.79 0.443  -6 2.8e-11       NA     1.5 0.91    NA    NA    NA     1.6 1.8e-11 2.9e
8 0.70 0.77 1.000  -8 0.0e+00       NA     1.4 0.91    NA    NA    NA     1.7 2.9e-14 1.2e
9 0.70 0.74    NA  -9 0.0e+00       NA     1.4 0.91    NA    NA    NA     1.7 1.1e-26 7.1e
```

Parallel Analysis of the Holzlinger Swineford data set



Extract 3 factors from Holzinger Swineford data set

R code

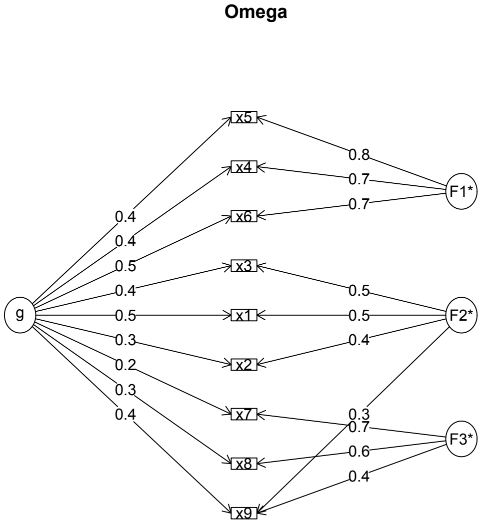
```
hs.fa <- fa(HolzingerSwineford1939[,7:15],3)
 hs.fa
```

```
Factor Analysis using method =  minres
Call: fa(r = HolzingerSwineford1939[, 7:15], nfactors = 3)
Standardized loadings (pattern matrix) based upon correlation matrix
     MR1   MR3   MR2   h2   u2
x1  0.19  0.60  0.03 0.49 0.51
x2  0.04  0.51 -0.12 0.25 0.75
x3 -0.07  0.69  0.02 0.46 0.54
x4  0.84  0.02  0.01 0.72 0.28
x5  0.89 -0.07  0.01 0.76 0.24
x6  0.81  0.08 -0.01 0.69 0.31
x7  0.04 -0.15  0.72 0.50 0.50
x8 -0.03  0.10  0.70 0.53 0.47
x9  0.03  0.37  0.46 0.46 0.54

                       MR1  MR3  MR2
SS loadings           2.24 1.34 1.28
Proportion Var        0.25 0.15 0.14
Cumulative Var        0.25 0.40 0.54
Proportion Explained  0.46 0.28 0.26
Cumulative Proportion 0.46 0.74 1.00

 With factor correlations of
     MR1  MR3  MR2
MR1 1.00 0.33 0.22
MR3 0.33 1.00 0.27
MR2 0.22 0.27 1.00
```

With goodness of fit stats

```
Test of the hypothesis that 3 factors are sufficient.

The degrees of freedom for the null model are  36  and the objective function was  3.05 w
The degrees of freedom for the model are 12  and the objective function was  0.08

The root mean square of the residuals (RMSR) is  0.01
The df corrected root mean square of the residuals is  0.03
The number of observations was  301  with Chi Square =  22.38  with prob <  0.034

Tucker Lewis Index of factoring reliability =  0.964
RMSEA index =  0.055  and the 90 % confidence intervals are  0.015 0.088
BIC =  -46.11
Fit based upon off diagonal values = 1
Measures of factor score adequacy
                                                 MR1  MR3  MR2
Correlation of scores with factors              0.94 0.84 0.85
Multiple R square of scores with factors        0.89 0.71 0.72
Minimum correlation of possible factor scores   0.78 0.42 0.45
```

Weak evidence for hierarchical structure



Omega analysis of Holzinger Swineford data

```
> om<- omega(HolzingerSwineford1939[,7:15])
> om
Omega
Call: omega(m = HolzingerSwineford1939[, 7:15])
Alpha:                  0.76
G.6:                    0.81
Omega Hierarchical:     0.45
Omega H asymptotic:     0.53
Omega Total             0.85

Schmid Leiman Factor loadings greater than  0.2
      g   F1*   F2*   F3*   h2   u2   p2
x1 0.49        0.46        0.49 0.51 0.50
x2 0.30        0.39        0.25 0.75 0.35
x3 0.41        0.53        0.46 0.54 0.38
x4 0.45  0.72              0.72 0.28 0.28
x5 0.41  0.76              0.76 0.24 0.23
x6 0.46  0.69              0.69 0.31 0.30
x7 0.23              0.66 0.50 0.50 0.11
x8 0.35              0.64 0.53 0.47 0.23
x9 0.45        0.28  0.42 0.46 0.54 0.44

With eigenvalues of:
   g  F1*  F2*  F3*
1.46 1.62 0.75 1.02
```

Omega fit statistics for Holzinger Swineford

```
general/max  0.9   max/min =   2.15
mean percent general =  0.31    with sd =  0.12 and cv of  0.38

The degrees of freedom are 12  and the fit is  0.08
The number of observations was  301  with Chi Square =  22.38  with prob <  0.034
The root mean square of the residuals is  0.01
The df corrected root mean square of the residuals is  0.03
RMSEA index =  0.055  and the 90 % confidence intervals are  0.015 0.088
BIC =  -46.11

Compare this with the adequacy of just a general factor and no group factors
The degrees of freedom for just the general factor are 27  and the fit is  1.75
The number of observations was  301  with Chi Square =  517.18  with prob <  4.4e-92
The root mean square of the residuals is  0.14
The df corrected root mean square of the residuals is  0.23

RMSEA index =  0.248  and the 90 % confidence intervals are  0.227 0.264
BIC =  363.09

Measures of factor score adequacy
                                                 g  F1*   F2*  F3*
Correlation of scores with factors            0.68 0.84  0.67 0.78
Multiple R square of scores with factors      0.46 0.71  0.44 0.61
Minimum correlation of factor score estimates -0.07 0.43 -0.11 0.22
```

The lavaan commands

R code

```
# The Holzinger and Swineford (1939) example
HS.model <- ' visual  =~ x1 + x2 + x3
              textual =~ x4 + x5 + x6
              speed   =~ x7 + x8 + x9 '

fit <- lavaan(HS.model, data=HolzingerSwineford1939,
              auto.var=TRUE, auto.fix.first=TRUE,
              auto.cov.lv.x=TRUE)
summary(fit, fit.measures=TRUE)
```

lavaan output

```
lavaan (0.4-14) converged normally after 41 iterations

  Number of observations                           301

  Estimator                                         ML
  Minimum Function Chi-square                   85.306
  Degrees of freedom                                24
  P-value                                        0.000
Chi-square test baseline model:
  Minimum Function Chi-square                  918.852
  Degrees of freedom                                36
  P-value                                        0.000
Full model versus baseline model:
  Comparative Fit Index (CFI)                    0.931
  Tucker-Lewis Index (TLI)                       0.896
Loglikelihood and Information Criteria:
  Loglikelihood user model (H0)              -3737.745
  Loglikelihood unrestricted model (H1)      -3695.092
  Number of free parameters                         21
  Akaike (AIC)                                7517.490
  Bayesian (BIC)                              7595.339
  Sample-size adjusted Bayesian (BIC)         7528.739
Root Mean Square Error of Approximation:
  RMSEA                                          0.092
```

and the parameter estimates – default defines one variable/factor to variance = 1

```
Parameter estimates:
  Information                                        Expected
  Standard Errors                                    Standard
                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  visual =~
    x1                1.000
    x2                0.553    0.100    5.554    0.000
    x3                0.729    0.109    6.685    0.000
  textual =~
    x4                1.000
    x5                1.113    0.065   17.014    0.000
    x6                0.926    0.055   16.703    0.000
  speed =~
    x7                1.000
    x8                1.180    0.165    7.152    0.000
    x9                1.082    0.151    7.155    0.000
Covariances:
  visual ~~
    textual           0.408    0.074    5.552    0.000
    speed             0.262    0.056    4.660    0.000
  textual ~~
    speed             0.173    0.049    3.518    0.000
Variances:
    x1                0.549    0.114
    x2                1.134    0.102
    x3                0.844    0.091
    x4                0.371    0.048
    x5                0.446    0.058
    x6                0.356    0.043
    x7                0.799    0.081
```

Do this again, but fix the latent variable variance to 1

```
                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  visual =~
    x1                0.900    0.081   11.127    0.000
    x2                0.498    0.077    6.429    0.000
    x3                0.656    0.074    8.817    0.000
  textual =~
    x4                0.990    0.057   17.474    0.000
    x5                1.102    0.063   17.576    0.000
    x6                0.917    0.054   17.082    0.000
  speed =~
    x7                0.619    0.070    8.903    0.000
    x8                0.731    0.066   11.090    0.000
    x9                0.670    0.065   10.305    0.000
Covariances:
  visual ~~
    textual           0.459    0.064    7.189    0.000
    speed             0.471    0.073    6.461    0.000
  textual ~~
    speed             0.283    0.069    4.117    0.000
Variances:
    x1                0.549    0.114
    x2                1.134    0.102
    x3                0.844    0.091
    x4                0.371    0.048
    x5                0.446    0.058
    x6                0.356    0.043
    x7                0.799    0.081
    x8                0.488    0.074
    x9                0.566    0.071
    visual            1.000
    textual           1.000
```

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