Chapter 4: sem in R and in LISREL

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There are many programs that allow one to analyze latent variable models. Almost all statistical packages will include the ability to do exploratory factor analysis and many allow for confirmatory analysis. Commerically available sem programs include AMOS, EQS, LISREL, MPlus, and SAS. Open source programs include R and Mx. The Loehlin text gives sample code for many problems in LISREL and EQS syntax, Raykov and Marcoulides (2006) give examples in EQS, LISREL and Mplus. In this chapter we compare the set up and output of the sem package in R with the unix version of LISREL for several problems.

4.1 Example data set 1: 9 cognitive variables (from Rakov and Marcoulides)

Tenko Raykov and George Marcoulides, in their textbook on SEM (Rakov and Marcoulides, 2006), present a data set based upon 220 high school students on 9 cognitive measures. They report three measures of Induction taken in the junior year, three measures of Figural Relations in the junior year, and three measures of figural relations in the senior year.

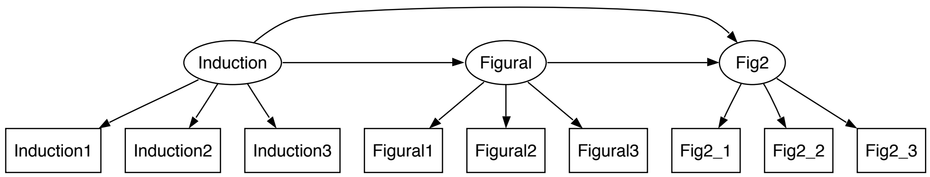

Figure 4.1: 9 cognitive variables (adapted from Raykov and Marcoulides, 2006)

They present the data set as a lower triangular covariance matrix which we can read this into R using the **scan** function embedded in a function to convert the data to a rectangular matrix:

```
> lower.triangle <- function(nrow = 2, data = NULL) {
+     if (is.null(data))
+         data <- scan(pipe("pbpaste"))
+     mat <- diag(0, nrow)
+     k <- 1
+     for (i in 1:nrow) {
+         for (j in 1:i) {
+             mat[i, j] <- data[k]
+             k <- k + 1
+         }
+     }
+     mat <- mat + t(mat)
+     diag(mat) <- diag(mat)/2
+     return(mat)
+ }
```

```
p5.in <- scan("")
56.21
31.55 75.55
23.27 28.30 44.45
24.48 32.24 22.56 84.64
22.51 29.54 20.61 57.61 78.93
22.65 27.56 15.33 53.57 49.27 73.76
33.24 46.49 31.44 67.81 54.76 54.58 141.77
32.56 40.37 25.58 55.82 52.33 47.74 98.62 117.33
30.32 40.44 27.69 54.78 53.44 59.52 96.95 84.87 106.35

prob5 <- lower.triangle(9,p5.in)
```

```
> colnames(prob5) <- rownames(prob5) <- c("Induct1", "Induct2", "Induct3", "Figural1", "Figur
+     "Figural3", "Fig2.1", "Fig2.2", "Fig2.3")
> prob5
```

```
        Induct1 Induct2 Induct3 Figural1 Figural2 Figural3 Fig2.1 Fig2.2 Fig2.3
Induct1   56.21   31.55   23.27    24.48    22.51    22.65  33.24  32.56  30.32
Induct2   31.55   75.55   28.30    32.24    29.54    27.56  46.49  40.37  40.44
```

Induct3	23.27	28.30	44.45	22.56	20.61	15.33	31.44	25.58	27.69
Figural1	24.48	32.24	22.56	84.64	57.61	53.57	67.81	55.82	54.78
Figural2	22.51	29.54	20.61	57.61	78.93	49.27	54.76	52.33	53.44
Figural3	22.65	27.56	15.33	53.57	49.27	73.76	54.58	47.74	59.52
Fig2.1	33.24	46.49	31.44	67.81	54.76	54.58	141.77	98.62	96.95
Fig2.2	32.56	40.37	25.58	55.82	52.33	47.74	98.62	117.33	84.87
Fig2.3	30.32	40.44	27.69	54.78	53.44	59.52	96.95	84.87	106.35

4.2 Using R to analyze the data set

The model proposed for this is that Induction in year1 predicts Figural Ability in Year 1 and Year 2 and that Figural Ability in Year 1 predicts Figural Ability in Year 2.

4.2.1 An initial formulation is empirically underidentified

The R code for doing the basic analysis is straightforward:

```
      path                                label initial estimate
 [1,] "Induction -> Induct1"              NA    "1"
 [2,] "Induction -> Induct2"              "2"   NA
 [3,] "Induction -> Induct3"              "3"   NA
 [4,] "Figural -> Figural1"               NA    "1"
 [5,] "Figural -> Figural2"               "5"   NA
 [6,] "Figural -> Figural3"               "6"   NA
 [7,] "Figural.time2 -> Fig2.1"           NA    "1"
 [8,] "Figural.time2 -> Fig2.2"           "8"   NA
 [9,] "Figural.time2 -> Fig2.3"           "9"   NA
[10,] "Induction -> Figural"              "i"   NA
[11,] "Induction -> Figural.time2"        "j"   NA
[12,] "Figural -> Figural.time2"          "k"   NA
[13,] "Induct1 <-> Induct1"               "u"   NA
[14,] "Induct2 <-> Induct2"               "v"   NA
[15,] "Induct3 <-> Induct3"               "w"   NA
[16,] "Figural1 <-> Figural1"             "x"   NA
[17,] "Figural2 <-> Figural2"             "y"   NA
[18,] "Figural3 <-> Figural3"             "z"   NA
[19,] "Fig2.1 <-> Fig2.1"                 "q"   NA
[20,] "Fig2.2 <-> Fig2.2"                 "r"   NA
[21,] "Fig2.3 <-> Fig2.3"                 "s"   NA
[22,] "Induction <-> Induction"           "A"   "1"
[23,] "Figural <-> Figural"               "B"   "1"
[24,] "Figural.time2 <-> Figural.time2"   "C"   "1"

 Model Chisquare =  124   Df =  24 Pr(>Chisq) = 2.1e-15
 Chisquare (null model) =  1177   Df =  36
 Goodness-of-fit index =  0.88
```

```
 Adjusted goodness-of-fit index =  0.78
 RMSEA index =  0.14   90% CI: (0.11, 0.16)
 Bentler-Bonnett NFI =  0.9
 Tucker-Lewis NNFI =  0.87
 Bentler CFI =  0.91
 BIC =  -5.7

 Normalized Residuals
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-1.55000 -0.47200  0.00098  0.14300  0.55700  3.20000

 Parameter Estimates
  Estimate Std Error z value Pr(>|z|)
2  1.3e+00 0.118     10.6    0.0e+00  Induct2 <--- Induction
3  8.5e-01 0.100      8.5    0.0e+00  Induct3 <--- Induction
5  9.3e-01 0.026     35.3    0.0e+00  Figural2 <--- Figural
6  8.8e-01 0.021     42.0    0.0e+00  Figural3 <--- Figural
8  8.8e-01 0.039     22.4    0.0e+00  Fig2.2 <--- Figural.time2
9  8.8e-01 0.028     31.8    0.0e+00  Fig2.3 <--- Figural.time2
i  2.0e+00   NaN      NaN        NaN  Figural <--- Induction
j -2.0e+03   NaN      NaN        NaN  Figural.time2 <--- Induction
k  1.0e+03   NaN      NaN        NaN  Figural.time2 <--- Figural
u  4.2e+01 4.210     10.0    0.0e+00  Induct1 <--> Induct1
v  5.3e+01 5.350      9.9    0.0e+00  Induct2 <--> Induct2
w  3.4e+01 3.391     10.0    0.0e+00  Induct3 <--> Induct3
x  2.6e+01 3.040      8.5    0.0e+00  Figural1 <--> Figural1
y  2.9e+01 3.535      8.2    2.2e-16  Figural2 <--> Figural2
z  2.8e+01 3.382      8.3    0.0e+00  Figural3 <--> Figural3
q  3.2e+01 4.232      7.5    8.5e-14  Fig2.1 <--> Fig2.1
r  3.2e+01 4.058      8.0    1.8e-15  Fig2.2 <--> Fig2.2
s  2.0e+01 3.050      6.6    3.6e-11  Fig2.3 <--> Fig2.3
A  1.4e+01   NaN      NaN        NaN  Induction <--> Induction
B -7.0e-04   NaN      NaN        NaN  Figural <--> Figural
C  7.4e+02   NaN      NaN        NaN  Figural.time2 <--> Figural.time2

 Iterations =  500

 Aliased parameters: i j k A B C
```

4.2.2 Adjusting to model to converge

Unfortunately, the estimation in 4.2.1 does not converge and failed after 500 iterations. This is not an unusual problem in estimation. By specifying start values for the Induction -> Figural.time2 path, we can get a satsifactory solution:

```
     path                                 label initial estimate
 [1,] "Induction -> Induct1"            NA    "1"
 [2,] "Induction -> Induct2"            "2"   NA
 [3,] "Induction -> Induct3"            "3"   NA
 [4,] "Figural -> Figural1"             NA    "1"
 [5,] "Figural -> Figural2"             "5"   NA
 [6,] "Figural -> Figural3"             "6"   NA
 [7,] "Figural.time2 -> Fig2.1"         NA    "1"
 [8,] "Figural.time2 -> Fig2.2"         "8"   NA
 [9,] "Figural.time2 -> Fig2.3"         "9"   NA
[10,] "Induction -> Figural"            "i"   NA
[11,] "Induction -> Figural.time2"      "j"   NA
[12,] "Figural -> Figural.time2"        "k"   "0.75"
[13,] "Induct1 <-> Induct1"             "u"   NA
[14,] "Induct2 <-> Induct2"             "v"   NA
[15,] "Induct3 <-> Induct3"             "w"   NA
[16,] "Figural1 <-> Figural1"           "x"   NA
[17,] "Figural2 <-> Figural2"           "y"   NA
[18,] "Figural3 <-> Figural3"           "z"   NA
[19,] "Fig2.1 <-> Fig2.1"               "q"   NA
[20,] "Fig2.2 <-> Fig2.2"               "r"   NA
[21,] "Fig2.3 <-> Fig2.3"               "s"   NA
[22,] "Induction <-> Induction"         "A"   "1"
[23,] "Figural <-> Figural"             "B"   "1"
[24,] "Figural.time2 <-> Figural.time2" "C"   "1"

 Model Chisquare =  52   Df =  24 Pr(>Chisq) = 0.00076
 Chisquare (null model) =  1177   Df =  36
 Goodness-of-fit index =  0.95
 Adjusted goodness-of-fit index =  0.91
 RMSEA index =  0.073   90% CI: (0.046, 0.1)
 Bentler-Bonnett NFI =  0.96
 Tucker-Lewis NNFI =  0.96
 Bentler CFI =  0.98
 BIC =  -77

 Normalized Residuals
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-9.5e-01 -8.9e-02 -7.3e-05 -1.2e-02  1.4e-01  1.3e+00

 Parameter Estimates
  Estimate Std Error z value Pr(>|z|)
2  1.27    0.159      8.0    1.3e-15  Induct2 <--- Induction
3  0.89    0.114      7.8    7.1e-15  Induct3 <--- Induction
5  0.92    0.066     13.9    0.0e+00  Figural2 <--- Figural
6  0.88    0.066     13.4    0.0e+00  Figural3 <--- Figural
8  0.88    0.052     16.9    0.0e+00  Fig2.2 <--- Figural.time2
```

```
9  0.88    0.048     18.2    0.0e+00  Fig2.3 <--- Figural.time2
i  0.98    0.147      6.6    3.6e-11  Figural <--- Induction
j  0.60    0.178      3.4    7.2e-04  Figural.time2 <--- Induction
k  0.81    0.110      7.4    1.5e-13  Figural.time2 <--- Figural
u 30.90    3.891      7.9    2.0e-15  Induct1 <--> Induct1
v 34.83    5.067      6.9    6.2e-12  Induct2 <--> Induct2
w 24.49    3.075      8.0    1.8e-15  Induct3 <--> Induct3
x 22.83    3.450      6.6    3.7e-11  Figural1 <--> Figural1
y 26.87    3.459      7.8    8.0e-15  Figural2 <--> Figural2
z 26.33    3.353      7.9    4.0e-15  Figural3 <--> Figural3
q 31.31    4.451      7.0    2.0e-12  Fig2.1 <--> Fig2.1
r 32.17    4.043      8.0    1.8e-15  Fig2.2 <--> Fig2.2
s 20.44    3.213      6.4    2.0e-10  Fig2.3 <--> Fig2.3
A 25.31    5.156      4.9    9.1e-07  Induction <--> Induction
B 37.70    6.085      6.2    5.8e-10  Figural <--> Figural
C 36.00    6.017      6.0    2.2e-09  Figural.time2 <--> Figural.time2

 Iterations =  168

     Std. Estimate
1    0.67105       Induct1 <--- Induction
2  2 0.73412       Induct2 <--- Induction
3  3 0.67018       Induct3 <--- Induction
4    0.85457       Figural1 <--- Figural
5  5 0.81215       Figural2 <--- Figural
6  6 0.80191       Figural3 <--- Figural
7    0.88269       Fig2.1 <--- Figural.time2
8  8 0.85197       Fig2.2 <--- Figural.time2
9  9 0.89877       Fig2.3 <--- Figural.time2
10 i 0.62464       Figural <--- Induction
11 j 0.28886       Figural.time2 <--- Induction
12 k 0.60902       Figural.time2 <--- Figural

         Induct1 Induct2 Induct3 Figural1 Figural2 Figural3 Fig2.1 Fig2.2 Fig2.3
Induct1     0.00   -0.55    0.79    -0.23    -0.17     1.01  -2.15   1.49  -0.89
Induct2    -0.55    0.00   -0.21     0.90     0.78     0.11   1.61   0.96   0.86
Induct3     0.79   -0.21    0.00     0.62     0.47    -3.89   0.01  -2.02  -0.03
Figural1   -0.23    0.90    0.62     0.00     0.88    -0.58   2.58  -1.46  -2.75
Figural2   -0.17    0.78    0.47     0.88     0.00    -0.42  -5.11  -0.24   0.64
Figural3    1.01    0.11   -3.89    -0.58    -0.42     0.00  -2.56  -2.43   9.13
Fig2.1     -2.15    1.61    0.01     2.58    -5.11    -2.56   0.00   1.63  -0.46
Fig2.2      1.49    0.96   -2.02    -1.46    -0.24    -2.43   1.63   0.00  -0.67
Fig2.3     -0.89    0.86   -0.03    -2.75     0.64     9.13  -0.46  -0.67   0.00
```

4.2.3 Modifying the model to improve the fit

We see from the residuals (and Rakov and Marcoulides) that the fit is not very good and that we should allow for correlated errors for Figural3 in the junior year with Fig2.3 in the senior year. We adjust the model (and thus are no longer strictly doing a confirmatory analysis) to allow for these correlated errors.

```
      path                                label initial estimate
 [1,] "Induction -> Induct1"              NA    "1"
 [2,] "Induction -> Induct2"              "2"   NA
 [3,] "Induction -> Induct3"              "3"   NA
 [4,] "Figural -> Figural1"               NA    "1"
 [5,] "Figural -> Figural2"               "5"   NA
 [6,] "Figural -> Figural3"               "6"   NA
 [7,] "Figural.time2 -> Fig2.1"           NA    "1"
 [8,] "Figural.time2 -> Fig2.2"           "8"   NA
 [9,] "Figural.time2 -> Fig2.3"           "9"   NA
[10,] "Induction -> Figural"              "i"   NA
[11,] "Induction -> Figural.time2"        "j"   NA
[12,] "Figural -> Figural.time2"          "k"   NA
[13,] "Figural3 <-> Fig2.3"               "10"  NA
[14,] "Induct1 <-> Induct1"               "u"   NA
[15,] "Induct2 <-> Induct2"               "v"   NA
[16,] "Induct3 <-> Induct3"               "w"   NA
[17,] "Figural1 <-> Figural1"             "x"   NA
[18,] "Figural2 <-> Figural2"             "y"   NA
[19,] "Figural3 <-> Figural3"             "z"   NA
[20,] "Fig2.1 <-> Fig2.1"                 "q"   NA
[21,] "Fig2.2 <-> Fig2.2"                 "r"   NA
[22,] "Fig2.3 <-> Fig2.3"                 "s"   NA
[23,] "Induction <-> Induction"           "A"   "1"
[24,] "Figural <-> Figural"               "B"   "1"
[25,] "Figural.time2 <-> Figural.time2" "C"   "1"

 Model Chisquare =  21   Df =  23 Pr(>Chisq) = 0.61
 Chisquare (null model) =  1177   Df =  36
 Goodness-of-fit index =  0.98
 Adjusted goodness-of-fit index =  0.96
 RMSEA index =  0   90% CI: (NA, 0.049)
 Bentler-Bonnett NFI =  0.98
 Tucker-Lewis NNFI =  1
 Bentler CFI =  1
 BIC =  -104

 Normalized Residuals
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-8.3e-01 -8.4e-02  1.6e-04 -9.5e-05  1.5e-01  4.5e-01
```

```
 Parameter Estimates
   Estimate Std Error z value Pr(>|z|)
2   1.27    0.159      8.0    1.1e-15  Induct2 <--- Induction
3   0.89    0.115      7.8    6.9e-15  Induct3 <--- Induction
5   0.89    0.064     13.8    0.0e+00  Figural2 <--- Figural
6   0.83    0.062     13.4    0.0e+00  Figural3 <--- Figural
8   0.87    0.051     17.2    0.0e+00  Fig2.2 <--- Figural.time2
9   0.86    0.047     18.3    0.0e+00  Fig2.3 <--- Figural.time2
i   1.00    0.150      6.7    2.6e-11  Figural <--- Induction
j   0.67    0.181      3.7    2.1e-04  Figural.time2 <--- Induction
k   0.75    0.106      7.1    1.5e-12  Figural.time2 <--- Figural
10 12.27    2.488      4.9    8.2e-07  Fig2.3 <--> Figural3
u  31.04    3.891      8.0    1.6e-15  Induct1 <--> Induct1
v  34.91    5.060      6.9    5.2e-12  Induct2 <--> Induct2
w  24.32    3.068      7.9    2.2e-15  Induct3 <--> Induct3
x  19.67    3.398      5.8    7.1e-09  Figural1 <--> Figural1
y  27.71    3.554      7.8    6.4e-15  Figural2 <--> Figural2
z  28.54    3.484      8.2    2.2e-16  Figural3 <--> Figural3
q  29.40    4.300      6.8    8.1e-12  Fig2.1 <--> Fig2.1
r  31.34    3.954      7.9    2.2e-15  Fig2.2 <--> Fig2.2
s  22.50    3.296      6.8    8.6e-12  Fig2.3 <--> Fig2.3
A  25.17    5.140      4.9    9.8e-07  Induction <--> Induction
B  39.88    6.286      6.3    2.2e-10  Figural <--> Figural
C  39.37    6.072      6.5    9.0e-11  Figural.time2 <--> Figural.time2

 Iterations =  154

     Std. Estimate
1    0.66912       Induct1 <--- Induction
2  2 0.73342       Induct2 <--- Induction
3  3 0.67292       Induct3 <--- Induction
4    0.87615       Figural1 <--- Figural
5  5 0.80558       Figural2 <--- Figural
6  6 0.78263       Figural3 <--- Figural
7    0.89029       Fig2.1 <--- Figural.time2
8  8 0.85607       Fig2.2 <--- Figural.time2
9  9 0.88792       Fig2.3 <--- Figural.time2
10 i 0.62144       Figural <--- Induction
11 j 0.31746       Figural.time2 <--- Induction
12 k 0.56941       Figural.time2 <--- Figural

         Induct1 Induct2 Induct3 Figural1 Figural2 Figural3 Fig2.1 Fig2.2 Fig2.3
Induct1     0.00   -0.43    0.76    -0.65     0.20     1.71  -2.46   1.33  -0.52
Induct2    -0.43    0.00   -0.30     0.31     1.19     0.95   1.13   0.69   1.26
Induct3     0.76   -0.30    0.00     0.09     0.66    -3.40  -0.49  -2.35   0.11
Figural1   -0.65    0.31    0.09     0.00    -0.08    -0.57   2.30  -1.49  -1.81
Figural2    0.20    1.19    0.66    -0.08     0.00     1.20  -3.41   1.45   3.20
```

```
Figural3    1.71    0.95   -3.40    -0.57     1.20     0.10  -0.01  -0.01   0.10
Fig2.1     -2.46    1.13   -0.49     2.30    -3.41    -0.01   0.00   0.32  -0.11
Fig2.2      1.33    0.69   -2.35    -1.49     1.45    -0.01   0.32   0.00  -0.04
Fig2.3     -0.52    1.26    0.11    -1.81     3.20     0.10  -0.11  -0.04   0.01
```

4.2.4 Changing from a regression model to a correlation model

For theoretical reasons, the meaning of a regression model (X predicts Y or in the case of latent variables, latent X predicts latent Y) is very different than a simple correlation model. Both models fit the data equally well, but the path coefficients are very different. Compared the results from 4.2.3 with the results from a model that assumes just correlated latent variables:

```
      path                                  label initial estimate
 [1,] "Induction -> Induct1"                NA    "1"
 [2,] "Induction -> Induct2"                "2"   NA
 [3,] "Induction -> Induct3"                "3"   NA
 [4,] "Figural -> Figural1"                 NA    "1"
 [5,] "Figural -> Figural2"                 "5"   NA
 [6,] "Figural -> Figural3"                 "6"   NA
 [7,] "Figural.time2 -> Fig2.1"             NA    "1"
 [8,] "Figural.time2 -> Fig2.2"             "8"   NA
 [9,] "Figural.time2 -> Fig2.3"             "9"   NA
[10,] "Induction <-> Figural"               "i"   NA
[11,] "Induction <-> Figural.time2"         "j"   NA
[12,] "Figural <-> Figural.time2"           "k"   NA
[13,] "Figural3 <-> Fig2.3"                 "10"  NA
[14,] "Induct1 <-> Induct1"                 "u"   NA
[15,] "Induct2 <-> Induct2"                 "v"   NA
[16,] "Induct3 <-> Induct3"                 "w"   NA
[17,] "Figural1 <-> Figural1"               "x"   NA
[18,] "Figural2 <-> Figural2"               "y"   NA
[19,] "Figural3 <-> Figural3"               "z"   NA
[20,] "Fig2.1 <-> Fig2.1"                   "q"   NA
[21,] "Fig2.2 <-> Fig2.2"                   "r"   NA
[22,] "Fig2.3 <-> Fig2.3"                   "s"   NA
[23,] "Induction <-> Induction"             "A"   "1"
[24,] "Figural <-> Figural"                 "B"   "1"
[25,] "Figural.time2 <-> Figural.time2"     "C"   "1"

 Model Chisquare =  21   Df =  23 Pr(>Chisq) = 0.61
 Chisquare (null model) =  1177   Df =  36
 Goodness-of-fit index =  0.98
 Adjusted goodness-of-fit index =  0.96
 RMSEA index =  0   90% CI: (NA, 0.049)
 Bentler-Bonnett NFI =  0.98
 Tucker-Lewis NNFI =  1
```

```
 Bentler CFI =  1
 BIC =  -104

 Normalized Residuals
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-8.3e-01 -8.4e-02  2.3e-04 -3.9e-05  1.6e-01  4.5e-01

 Parameter Estimates
   Estimate Std Error z value Pr(>|z|)
2    1.27    0.159     8.0    1.3e-15  Induct2 <--- Induction
3    0.89    0.115     7.8    6.9e-15  Induct3 <--- Induction
5    0.89    0.064    13.8    0.0e+00  Figural2 <--- Figural
6    0.83    0.062    13.4    0.0e+00  Figural3 <--- Figural
8    0.87    0.051    17.2    0.0e+00  Fig2.2 <--- Figural.time2
9    0.86    0.047    18.3    0.0e+00  Fig2.3 <--- Figural.time2
i   25.13    4.361     5.8    8.3e-09  Figural <--> Induction
j   35.70    5.801     6.2    7.6e-10  Figural.time2 <--> Induction
k   65.51    8.340     7.9    4.0e-15  Figural.time2 <--> Figural
10  12.26    2.488     4.9    8.2e-07  Fig2.3 <--> Figural3
u   31.04    3.891     8.0    1.6e-15  Induct1 <--> Induct1
v   34.91    5.060     6.9    5.2e-12  Induct2 <--> Induct2
w   24.32    3.068     7.9    2.2e-15  Induct3 <--> Induct3
x   19.67    3.398     5.8    7.1e-09  Figural1 <--> Figural1
y   27.71    3.555     7.8    6.4e-15  Figural2 <--> Figural2
z   28.54    3.483     8.2    2.2e-16  Figural3 <--> Figural3
q   29.40    4.300     6.8    8.1e-12  Fig2.1 <--> Fig2.1
r   31.34    3.954     7.9    2.2e-15  Fig2.2 <--> Fig2.2
s   22.50    3.295     6.8    8.6e-12  Fig2.3 <--> Fig2.3
A   25.16    5.143     4.9    9.9e-07  Induction <--> Induction
B   64.97    8.369     7.8    8.2e-15  Figural <--> Figural
C  112.37   13.662     8.2    2.2e-16  Figural.time2 <--> Figural.time2

 Iterations =  215

    Std. Estimate
1   0.66911       Induct1 <--- Induction
2 2 0.73340       Induct2 <--- Induction
3 3 0.67292       Induct3 <--- Induction
4   0.87615       Figural1 <--- Figural
5 5 0.80555       Figural2 <--- Figural
6 6 0.78266       Figural3 <--- Figural
7   0.89029       Fig2.1 <--- Figural.time2
8 8 0.85608       Fig2.2 <--- Figural.time2
9 9 0.88793       Fig2.3 <--- Figural.time2

         Induct1 Induct2 Induct3 Figural1 Figural2 Figural3 Fig2.1 Fig2.2 Fig2.3
Induct1     0.00   -0.43    0.76    -0.65     0.20     1.71  -2.46   1.33  -0.52
Induct2    -0.43    0.00   -0.30     0.31     1.19     0.95   1.13   0.69   1.26
```

```
Induct3     0.76   -0.30    0.00     0.09     0.66    -3.40  -0.49  -2.35   0.11
Figural1   -0.65    0.31    0.09     0.00    -0.08    -0.57   2.30  -1.49  -1.81
Figural2    0.20    1.19    0.66    -0.08     0.00     1.20  -3.40   1.45   3.20
Figural3    1.71    0.95   -3.40    -0.57     1.20     0.10  -0.01  -0.02   0.10
Fig2.1     -2.46    1.13   -0.49     2.30    -3.40    -0.01   0.00   0.32  -0.11
Fig2.2      1.33    0.69   -2.35    -1.49     1.45    -0.02   0.32   0.00  -0.04
Fig2.3     -0.52    1.26    0.11    -1.81     3.20     0.10  -0.11  -0.04   0.01
```

Note that the coefficients i,j, and k are now covariances rather than beta weights.

4.3 Using LISREL to analyze the data set

The commerical computer package LISREL, developed by Karl Joreskog, was the first commerical program to do Linear Structural RELations. Although seemingly complicated than other packages, LISREL uses a matrix formulation that clearly shows the difference between observed and latent variables, the errors associated with each, and distinguishes between the predictor set of variables and the criterion set of variables.

The matrices are:

1. X variables (the observed variables)
2. Lambda X (LX:the factor loadings for the X variables on the eta factors)
3. Beta (BE:the beta weights linking the eta to the psi latent variables
4. Lambda Y (LY: the factor loadings for the Y variables on the psi factors)
5. Psi (PS: the dependent latent factor variances and covariances)
6. Theta and Epsilon (TE: the error variances and covariances for the X and Y variables).

LISRELis available for PCs as an add on to SPSS, but is also available as a stand alone package at the Northwestern Social Science Computing Cluster. To use LISREL at the SSCC it is necessary to have an account and then to log in as a remote user.

4.3.1 Instructions for using the SSCC

1. Log on to the system using SSH (see the "how to" for doing this)
2. upload the appropriate batch command file using a sftp connection.

   The file we will submit is taken from Raykov and Marcoulides (2006):

   ```
   STRUCTURAL REGRESSION MODEL
   DA NI=9 NO=220
   CM
   56.21
   31.55 75.55
   23.27 28.30 44.45
   24.48 32.24 22.56 84.64
   ```

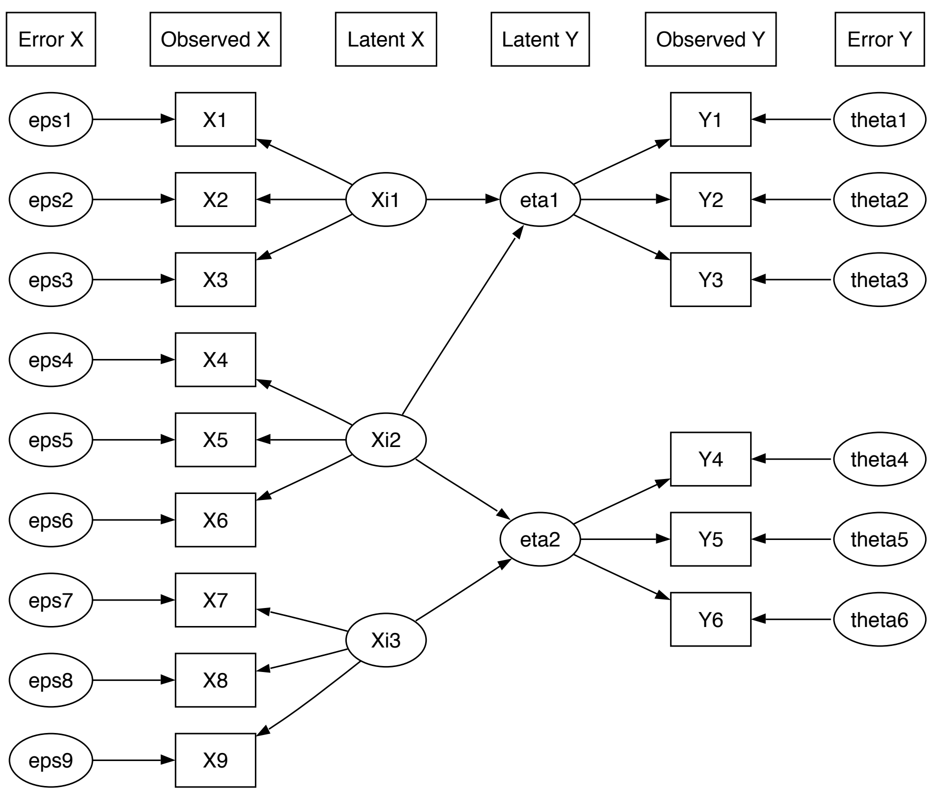

Figure 4.2: The Linear Structural Relations (LISREL) model integrates two measurement models with one regression model. How well are the X's represented by the latent variables (factors) Xi, and how well are the Y variables represented by the factors etas.

```
22.51 29.54 20.61 57.61 78.93
22.65 27.56 15.33 53.57 49.27 73.76
33.24 46.49 31.44 67.81 54.76 54.58 141.77
32.56 40.37 25.58 55.82 52.33 47.74 98.62 117.33
30.32 40.44 27.69 54.78 53.44 59.52 96.95 84.87 106.35
LA
IND1 IND2 IND3 FR11 FR12 FR13 FR21 FR22 FR23
MO NY=9 NE=3 PS=SY,FI TE=DI,FR LY=FU,FI BE=FU,FI
LE
INDUCTN FIGREL1 FIGREL2
FR LY(2, 1) LY(3, 1)
FR LY(5, 2) LY(6, 2)
FR LY(8, 3) LY(9, 3)
VA 1 LY(1, 1) LY(4, 2) LY(7, 3)
FR BE(2, 1) BE(3, 1) BE(3, 2)
FR PS(1, 1) PS(2, 2) PS(3, 3)
OU
```

This file is created (or in this case copied) and saved on the Mac/PC with a meaningful name, rm5.txt, and then uploaded to the SSCC using a sftp operation. (From my Mac I use Interarchy as my sftp client.)

3. submit the lisrel job by invoking lisrel8:

```
[revelle@hardin ~]$ lisrel8 rm5.txt rm5.out

          +----------------------------------+
          |                                  |
          |          L I S R E L  8.72       |
          |                                  |
          |                  by              |
          |                                  |
          | Karl G. Joreskog and Dag Sorbom  |
          |Available Workspace 16941056 bytes|
          +----------------------------------+
         This program is published exclusively by
         Scientific Software International, Inc.
         7383 N.Lincoln Avenue - Suite 100
         Lincolnwood, IL 60712-1704, U.S.A.
         Phone: (800)247-6113, (847)675-0720, Fax: (847)675-2140
         Copyright by Scientific Software International, Inc., 1981-2005
         Use of this program is subject to the terms specified in the
         Universal Copyright Convention.
         Website: www.ssicentral.com
         Revision LISREL872_03/28/2005
         Input file [INPUT]  :
         Input file [INPUT]  :
         rm5.txt
```

```
Output file [OUTPUT]:
Output file [OUTPUT]:
rm5.out
Reading input from file rm5.txt

STRUCTURAL REGRESSION MODEL
Computing Initial Estimates
Computing Information Matrix
Inverting Information Matrix
Iteration   1 for LISREL Estimates
Iteration   2 for LISREL Estimates
Iteration   3 for LISREL Estimates
Iteration   4 for LISREL Estimates
Iteration   5 for LISREL Estimates
Iteration   6 for LISREL Estimates
Computing Information Matrix
Inverting Information Matrix
Computing Goodness of Fit Statistics
```

```
[revelle@hardin ~]$
```

4. Transfer the output file (in this case “rm5.out”) back to your host machine (using sftp).
5. Examine the output

```
                              DATE:  2/12/2007
                                    TIME: 11:14


                                L I S R E L  8.72

                                       BY

                         Karl G. JÃ■reskog & Dag SÃ■rbom



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                          Website: www.ssicentral.com

The following lines were read from file rm5.txt:
```

```
STRUCTURAL REGRESSION MODEL
DA NI=9 NO=220
CM
56.21
31.55 75.55
23.27 28.30 44.45
24.48 32.24 22.56 84.64
22.51 29.54 20.61 57.61 78.93
22.65 27.56 15.33 53.57 49.27 73.76
33.24 46.49 31.44 67.81 54.76 54.58 141.77
32.56 40.37 25.58 55.82 52.33 47.74 98.62 117.33
30.32 40.44 27.69 54.78 53.44 59.52 96.95 84.87 106.35
LA
IND1 IND2 IND3 FR11 FR12 FR13 FR21 FR22 FR23
MO NY=9 NE=3 PS=SY,FI TE=DI,FR LY=FU,FI BE=FU,FI
LE
INDUCTN FIGREL1 FIGREL2
FR LY(2, 1) LY(3, 1)
FR LY(5, 2) LY(6, 2)
FR LY(8, 3) LY(9, 3)
VA 1 LY(1, 1) LY(4, 2) LY(7, 3)
FR BE(2, 1) BE(3, 1) BE(3, 2)
FR PS(1, 1) PS(2, 2) PS(3, 3)
OU
```

```
STRUCTURAL REGRESSION MODEL

                          Number of Input Variables  9
                          Number of Y - Variables    9
                          Number of X - Variables    0
                          Number of ETA - Variables  3
                          Number of KSI - Variables  0
                          Number of Observations   220

STRUCTURAL REGRESSION MODEL

        Covariance Matrix

               IND1       IND2       IND3       FR11       FR12       FR13
           --------   --------   --------   --------   --------   --------
    IND1      56.21
    IND2      31.55      75.55
    IND3      23.27      28.30      44.45
    FR11      24.48      32.24      22.56      84.64
    FR12      22.51      29.54      20.61      57.61      78.93
    FR13      22.65      27.56      15.33      53.57      49.27      73.76
```

FR21	33.24	46.49	31.44	67.81	54.76	54.58
FR22	32.56	40.37	25.58	55.82	52.33	47.74
FR23	30.32	40.44	27.69	54.78	53.44	59.52

Covariance Matrix

	FR21	FR22	FR23
FR21	141.77		
FR22	98.62	117.33	
FR23	96.95	84.87	106.35

STRUCTURAL REGRESSION MODEL

Parameter Specifications

LAMBDA-Y

	INDUCTN	FIGREL1	FIGREL2
IND1	0	0	0
IND2	1	0	0
IND3	2	0	0
FR11	0	0	0
FR12	0	3	0
FR13	0	4	0
FR21	0	0	0
FR22	0	0	5
FR23	0	0	6

BETA

	INDUCTN	FIGREL1	FIGREL2
INDUCTN	0	0	0
FIGREL1	7	0	0
FIGREL2	8	9	0

PSI

INDUCTN	FIGREL1	FIGREL2
10	11	12

THETA-EPS

```
          IND1       IND2       IND3       FR11       FR12       FR13
        --------   --------   --------   --------   --------   --------
            13         14         15         16         17         18
```

THETA-EPS

```
          FR21       FR22       FR23
        --------   --------   --------
            19         20         21
```

STRUCTURAL REGRESSION MODEL

Number of Iterations = 5

LISREL Estimates (Maximum Likelihood)

LAMBDA-Y

```
           INDUCTN    FIGREL1    FIGREL2
          --------   --------   --------
    IND1      1.00       - -        - -

    IND2      1.27       - -        - -
            (0.16)
              8.08

    IND3      0.89       - -        - -
            (0.12)
              7.70

    FR11       - -      1.00        - -

    FR12       - -      0.92        - -
                      (0.07)
                       13.76

    FR13       - -      0.88        - -
                      (0.06)
                       13.54

    FR21       - -       - -       1.00

    FR22       - -       - -       0.88
                                 (0.05)
                                  16.79
```

FR23	- -	- -	0.88
			(0.05)
			18.39

BETA

	INDUCTN	FIGREL1	FIGREL2
INDUCTN	- -	- -	- -
FIGREL1	0.98	- -	- -
	(0.15)		
	6.64		
FIGREL2	0.60	0.81	- -
	(0.18)	(0.11)	
	3.41	7.40	

Covariance Matrix of ETA

	INDUCTN	FIGREL1	FIGREL2
INDUCTN	25.31		
FIGREL1	24.71	61.81	
FIGREL2	35.39	65.23	110.46

PSI
Note: This matrix is diagonal.

INDUCTN	FIGREL1	FIGREL2
25.31	37.69	36.00
(5.14)	(6.10)	(5.92)
4.92	6.18	6.08

Squared Multiple Correlations for Structural Equations

INDUCTN	FIGREL1	FIGREL2
- -	0.39	0.67

THETA-EPS

IND1	IND2	IND3	FR11	FR12	FR13
30.90	34.84	24.49	22.83	26.87	26.33
(3.88)	(5.06)	(3.07)	(3.42)	(3.47)	(3.31)
7.97	6.89	7.98	6.67	7.75	7.95

THETA-EPS

FR21	FR22	FR23
31.31	32.17	20.44
(4.40)	(4.02)	(3.15)
7.12	7.99	6.50

Squared Multiple Correlations for Y - Variables

IND1	IND2	IND3	FR11	FR12	FR13
0.45	0.54	0.45	0.73	0.66	0.64

Squared Multiple Correlations for Y - Variables

FR21	FR22	FR23
0.78	0.73	0.81

Goodness of Fit Statistics

Degrees of Freedom = 24
Minimum Fit Function Chi-Square = 52.10 (P = 0.00076)
Normal Theory Weighted Least Squares Chi-Square = 48.28 (P = 0.0023)
Estimated Non-centrality Parameter (NCP) = 24.28
90 Percent Confidence Interval for NCP = (8.23 ; 48.09)

Minimum Fit Function Value = 0.24
Population Discrepancy Function Value (F0) = 0.11
90 Percent Confidence Interval for F0 = (0.038 ; 0.22)
Root Mean Square Error of Approximation (RMSEA) = 0.068
90 Percent Confidence Interval for RMSEA = (0.040 ; 0.096)
P-Value for Test of Close Fit (RMSEA < 0.05) = 0.14

Expected Cross-Validation Index (ECVI) = 0.41
90 Percent Confidence Interval for ECVI = (0.34 ; 0.52)
ECVI for Saturated Model = 0.41

```
                    ECVI for Independence Model = 9.49

Chi-Square for Independence Model with 36 Degrees of Freedom = 2060.02
                        Independence AIC = 2078.02
                            Model AIC = 90.28
                          Saturated AIC = 90.00
                       Independence CAIC = 2117.56
                           Model CAIC = 182.54
                         Saturated CAIC = 287.71

                      Normed Fit Index (NFI) = 0.97
                    Non-Normed Fit Index (NNFI) = 0.98
                Parsimony Normed Fit Index (PNFI) = 0.65
                    Comparative Fit Index (CFI) = 0.99
                    Incremental Fit Index (IFI) = 0.99
                     Relative Fit Index (RFI) = 0.96

                         Critical N (CN) = 181.68


               Root Mean Square Residual (RMR) = 1.99
                      Standardized RMR = 0.023
                  Goodness of Fit Index (GFI) = 0.95
           Adjusted Goodness of Fit Index (AGFI) = 0.91
          Parsimony Goodness of Fit Index (PGFI) = 0.51
```

4.3.2 Modify the model to allow for correlated errors

Just as we did for the sem using R, an examination of the residuals suggests that we need to modify the model to allow for correlated errors for the Figural3 at time 1 and time 2. This leads to the following LISREL commands:

```
STRUCTURAL REGRESSION MODEL
DA NI=9 NO=220
CM
56.21
31.55 75.55
23.27 28.30 44.45
24.48 32.24 22.56 84.64
22.51 29.54 20.61 57.61 78.93
22.65 27.56 15.33 53.57 49.27 73.76
33.24 46.49 31.44 67.81 54.76 54.58 141.77
32.56 40.37 25.58 55.82 52.33 47.74 98.62 117.33
30.32 40.44 27.69 54.78 53.44 59.52 96.95 84.87 106.35
LA
```

```
IND1 IND2 IND3 FR11 FR12 FR13 FR21 FR22 FR23
MO NY=9 NE=3 PS=SY,FI TE=SY,FI LY=FU,FI BE=FU,FI
LE
INDUCTN FIGREL1 FIGREL2
FR LY(2, 1) LY(3, 1)
FR LY(5, 2) LY(6, 2)
FR LY(8, 3) LY(9, 3)
VA 1 LY(1, 1) LY(4, 2) LY(7, 3)
FR BE(2, 1) BE(3, 1) BE(3, 2)
FR PS(1, 1) PS(2, 2) PS(3, 3)
FR TE(1,1) TE (2,2) TE(3,3) TE(4,4) TE(5,5) TE(6,6) TE(7,7) TE(8,8) TE(9,9) TE(9,6)
OU
```

Compare this set of commands to the previous set. What we have done is added a line to specify the errors in the "theta" matrix and specified that the 6th error correlates with the 9th error.

Uploading this revised command file to the SSCC and running it leads to the following output:

```
                              DATE:  2/12/2007
                                TIME: 11:37


                              L I S R E L  8.72

                                     BY

                        Karl G. JÃ■reskog & Dag SÃ■rbom



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                         Website: www.ssicentral.com

 The following lines were read from file rm5a.txt:

 STRUCTURAL REGRESSION MODEL
 DA NI=9 NO=220
 CM
```

```
56.21
31.55 75.55
23.27 28.30 44.45
24.48 32.24 22.56 84.64
22.51 29.54 20.61 57.61 78.93
22.65 27.56 15.33 53.57 49.27 73.76
33.24 46.49 31.44 67.81 54.76 54.58 141.77
32.56 40.37 25.58 55.82 52.33 47.74 98.62 117.33
30.32 40.44 27.69 54.78 53.44 59.52 96.95 84.87 106.35
LA
IND1 IND2 IND3 FR11 FR12 FR13 FR21 FR22 FR23
MO NY=9 NE=3 PS=SY,FI TE=SY,FI LY=FU,FI BE=FU,FI
LE
INDUCTN FIGREL1 FIGREL2
FR LY(2, 1) LY(3, 1)
FR LY(5, 2) LY(6, 2)
FR LY(8, 3) LY(9, 3)
VA 1 LY(1, 1) LY(4, 2) LY(7, 3)
FR BE(2, 1) BE(3, 1) BE(3, 2)
FR PS(1, 1) PS(2, 2) PS(3, 3)
FR TE(1,1) TE (2,2) TE(3,3) TE(4,4) TE(5,5) TE(6,6) TE(7,7) TE(8,8) TE(9,9) TE(9,6)
OU
```

```
STRUCTURAL REGRESSION MODEL

                          Number of Input Variables  9
                          Number of Y - Variables    9
                          Number of X - Variables    0
                          Number of ETA - Variables  3
                          Number of KSI - Variables  0
                          Number of Observations   220

STRUCTURAL REGRESSION MODEL

        Covariance Matrix
```

	IND1	IND2	IND3	FR11	FR12	FR13
	--------	--------	--------	--------	--------	--------
IND1	56.21					
IND2	31.55	75.55				
IND3	23.27	28.30	44.45			
FR11	24.48	32.24	22.56	84.64		
FR12	22.51	29.54	20.61	57.61	78.93	
FR13	22.65	27.56	15.33	53.57	49.27	73.76
FR21	33.24	46.49	31.44	67.81	54.76	54.58
FR22	32.56	40.37	25.58	55.82	52.33	47.74
FR23	30.32	40.44	27.69	54.78	53.44	59.52

Covariance Matrix

	FR21	FR22	FR23
FR21	141.77		
FR22	98.62	117.33	
FR23	96.95	84.87	106.35

STRUCTURAL REGRESSION MODEL

Parameter Specifications

LAMBDA-Y

	INDUCTN	FIGREL1	FIGREL2
IND1	0	0	0
IND2	1	0	0
IND3	2	0	0
FR11	0	0	0
FR12	0	3	0
FR13	0	4	0
FR21	0	0	0
FR22	0	0	5
FR23	0	0	6

BETA

	INDUCTN	FIGREL1	FIGREL2
INDUCTN	0	0	0
FIGREL1	7	0	0
FIGREL2	8	9	0

PSI

INDUCTN	FIGREL1	FIGREL2
10	11	12

THETA-EPS

	IND1	IND2	IND3	FR11	FR12	FR13
IND1	13					

```
   IND2          0         14
   IND3          0          0         15
   FR11          0          0          0         16
   FR12          0          0          0          0         17
   FR13          0          0          0          0          0         18
   FR21          0          0          0          0          0          0
   FR22          0          0          0          0          0          0
   FR23          0          0          0          0          0         21

       THETA-EPS

                FR21       FR22       FR23
            --------   --------   --------
   FR21         19
   FR22          0         20
   FR23          0          0         22


STRUCTURAL REGRESSION MODEL

Number of Iterations =  5

LISREL Estimates (Maximum Likelihood)

        LAMBDA-Y

             INDUCTN    FIGREL1    FIGREL2
            --------   --------   --------
   IND1         1.00        - -        - -

   IND2         1.27        - -        - -
              (0.16)
                8.07

   IND3         0.89        - -        - -
              (0.12)
                7.71

   FR11          - -       1.00        - -

   FR12          - -       0.89        - -
                         (0.06)
                          13.89

   FR13          - -       0.83        - -
                         (0.06)
```

```
                     13.46

   FR21         - -          - -        1.00

   FR22         - -          - -        0.87
                                      (0.05)
                                       17.20

   FR23         - -          - -        0.86
                                      (0.05)
                                       18.39


       BETA

            INDUCTN    FIGREL1    FIGREL2
           --------   --------   --------
INDUCTN         - -        - -        - -

FIGREL1        1.00        - -        - -
             (0.15)
               6.68

FIGREL2        0.67       0.75        - -
             (0.18)     (0.11)
               3.74       7.10


       Covariance Matrix of ETA

            INDUCTN    FIGREL1    FIGREL2
           --------   --------   --------
INDUCTN       25.17
FIGREL1       25.13      64.97
FIGREL2       35.70      65.51     112.37

       PSI
       Note: This matrix is diagonal.

            INDUCTN    FIGREL1    FIGREL2
           --------   --------   --------
              25.17      39.88      39.37
             (5.13)     (6.26)     (6.05)
               4.91       6.37       6.51


       Squared Multiple Correlations for Structural Equations
```

INDUCTN	FIGREL1	FIGREL2
- -	0.39	0.65

THETA-EPS

	IND1	IND2	IND3	FR11	FR12	FR13
IND1	31.04 (3.88) 8.00					
IND2	- -	34.91 (5.05) 6.91				
IND3	- -	- -	24.32 (3.06) 7.95			
FR11	- -	- -	- -	19.67 (3.35) 5.87		
FR12	- -	- -	- -	- -	27.71 (3.53) 7.84	
FR13	- -	- -	- -	- -	- -	28.54 (3.46) 8.24
FR21	- -	- -	- -	- -	- -	- -
FR22	- -	- -	- -	- -	- -	- -
FR23	- -	- -	- -	- -	- -	12.26 (2.46) 4.99

THETA-EPS

	FR21	FR22	FR23
FR21	29.40		

```
         (4.28)
           6.87

FR22        - -      31.34
                    (3.95)
                      7.93

FR23        - -        - -      22.50
                               (3.28)
                                 6.86
```

Squared Multiple Correlations for Y - Variables

IND1	IND2	IND3	FR11	FR12	FR13
0.45	0.54	0.45	0.77	0.65	0.61

Squared Multiple Correlations for Y - Variables

FR21	FR22	FR23
0.79	0.73	0.79

Goodness of Fit Statistics

Degrees of Freedom = 23
Minimum Fit Function Chi-Square = 20.55 (P = 0.61)
Normal Theory Weighted Least Squares Chi-Square = 20.01 (P = 0.64)
Estimated Non-centrality Parameter (NCP) = 0.0
90 Percent Confidence Interval for NCP = (0.0 ; 11.11)

Minimum Fit Function Value = 0.094
Population Discrepancy Function Value (F0) = 0.0
90 Percent Confidence Interval for F0 = (0.0 ; 0.051)
Root Mean Square Error of Approximation (RMSEA) = 0.0
90 Percent Confidence Interval for RMSEA = (0.0 ; 0.047)
P-Value for Test of Close Fit (RMSEA < 0.05) = 0.96

Expected Cross-Validation Index (ECVI) = 0.31
90 Percent Confidence Interval for ECVI = (0.31 ; 0.36)
ECVI for Saturated Model = 0.41
ECVI for Independence Model = 9.49

Chi-Square for Independence Model with 36 Degrees of Freedom = 2060.02
Independence AIC = 2078.02

```
                Model AIC = 64.01
             Saturated AIC = 90.00
        Independence CAIC = 2117.56
               Model CAIC = 160.67
           Saturated CAIC = 287.71

     Normed Fit Index (NFI) = 0.99
  Non-Normed Fit Index (NNFI) = 1.00
Parsimony Normed Fit Index (PNFI) = 0.63
  Comparative Fit Index (CFI) = 1.00
  Incremental Fit Index (IFI) = 1.00
    Relative Fit Index (RFI) = 0.98

           Critical N (CN) = 444.69


  Root Mean Square Residual (RMR) = 1.27
             Standardized RMR = 0.016
     Goodness of Fit Index (GFI) = 0.98
 Adjusted Goodness of Fit Index (AGFI) = 0.96
Parsimony Goodness of Fit Index (PGFI) = 0.50
```

4.4 Comparing the R and LISREL output

Each sem author has his or her own preferences about how to organize the output. Compare the LISREL output 4.3.2 with the R output for the prediction model 4.2.3 and the correlation model 4.2.4.

As one would hope, the chi square values and df are equal between the two programs. LISREL gives far more goodness of fit statistics and also has a more detailed output than sem.

4.5 Testing for factorial invariance

The models tested above measured Figural Relations in the Junior and Senior year. Were these tests measuring the same concept? If they were, then we would expect the factor loadings to be the same in both years. We can test this by constraining the equivalent loadings to be identical and comparing the differences in χ^2 for the two models. (The first model is discussed in section4.2.3

```
     path                                label initial estimate
[1,] "Induction -> Induct1"              NA    "1"
[2,] "Induction -> Induct2"              "2"   NA
[3,] "Induction -> Induct3"              "3"   NA
[4,] "Figural -> Figural1"               NA    "1"
```

```
 [5,] "Figural -> Figural2"              "5"   NA
 [6,] "Figural -> Figural3"              "6"   NA
 [7,] "Figural.time2 -> Fig2.1"          NA    "1"
 [8,] "Figural.time2 -> Fig2.2"          "5"   NA
 [9,] "Figural.time2 -> Fig2.3"          "6"   NA
[10,] "Induction -> Figural"             "i"   NA
[11,] "Induction -> Figural.time2"       "j"   NA
[12,] "Figural -> Figural.time2"         "k"   NA
[13,] "Figural3 <-> Fig2.3"              "10"  NA
[14,] "Induct1 <-> Induct1"              "u"   NA
[15,] "Induct2 <-> Induct2"              "v"   NA
[16,] "Induct3 <-> Induct3"              "w"   NA
[17,] "Figural1 <-> Figural1"            "x"   NA
[18,] "Figural2 <-> Figural2"            "y"   NA
[19,] "Figural3 <-> Figural3"            "z"   NA
[20,] "Fig2.1 <-> Fig2.1"                "q"   NA
[21,] "Fig2.2 <-> Fig2.2"                "r"   NA
[22,] "Fig2.3 <-> Fig2.3"                "s"   NA
[23,] "Induction <-> Induction"          "A"   "1"
[24,] "Figural <-> Figural"              "B"   "1"
[25,] "Figural.time2 <-> Figural.time2" "C"   "1"

 Model Chisquare =  21   Df =  25 Pr(>Chisq) = 0.7
 Chisquare (null model) =  1177   Df =  36
 Goodness-of-fit index =  0.98
 Adjusted goodness-of-fit index =  0.96
 RMSEA index =  0   90% CI: (NA, 0.043)
 Bentler-Bonnett NFI =  0.98
 Tucker-Lewis NNFI =  1
 Bentler CFI =  1
 BIC =  -114

 Normalized Residuals
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-9.1e-01 -1.1e-01  4.9e-05 -1.1e-02  1.7e-01  6.0e-01

 Parameter Estimates
   Estimate Std Error z value Pr(>|z|)
2   1.27    0.159      8.0    1.1e-15  Induct2 <--- Induction
3   0.89    0.115      7.8    6.9e-15  Induct3 <--- Induction
5   0.88    0.040     21.8    0.0e+00  Figural2 <--- Figural
6   0.86    0.042     20.5    0.0e+00  Figural3 <--- Figural
i   0.99    0.147      6.8    1.3e-11  Figural <--- Induction
j   0.67    0.181      3.7    2.0e-04  Figural.time2 <--- Induction
k   0.76    0.100      7.6    2.7e-14  Figural.time2 <--- Figural
10 12.23    2.481      4.9    8.3e-07  Fig2.3 <--> Figural3
u  31.03    3.891      8.0    1.6e-15  Induct1 <--> Induct1
```

```
v  34.90    5.060      6.9    5.3e-12  Induct2 <--> Induct2
w  24.34    3.069      7.9    2.2e-15  Induct3 <--> Induct3
x  19.85    3.234      6.1    8.3e-10  Figural1 <--> Figural1
y  28.00    3.461      8.1    6.7e-16  Figural2 <--> Figural2
z  28.19    3.417      8.2    2.2e-16  Figural3 <--> Figural3
q  29.21    4.243      6.9    5.9e-12  Fig2.1 <--> Fig2.1
r  31.14    3.917      8.0    1.8e-15  Fig2.2 <--> Fig2.2
s  22.74    3.275      6.9    3.8e-12  Fig2.3 <--> Fig2.3
A  25.18    5.141      4.9    9.7e-07  Induction <--> Induction
B  39.43    5.833      6.8    1.4e-11  Figural <--> Figural
C  39.62    5.962      6.6    3.0e-11  Figural.time2 <--> Figural.time2

 Iterations =  162

     Std. Estimate
1    0.66928       Induct1 <--- Induction
2  2 0.73353       Induct2 <--- Induction
3  3 0.67263       Induct3 <--- Induction
4    0.87392       Figural1 <--- Figural
5  5 0.79947       Figural2 <--- Figural
6  6 0.79051       Figural3 <--- Figural
7    0.89178       Fig2.1 <--- Figural.time2
8  5 0.85900       Fig2.2 <--- Figural.time2
9  6 0.88599       Fig2.3 <--- Figural.time2
10 i 0.62103       Figural <--- Induction
11 j 0.31730       Figural.time2 <--- Induction
12 k 0.57035       Figural.time2 <--- Figural
         Induct1 Induct2 Induct3 Figural1 Figural2 Figural3 Fig2.1 Fig2.2 Fig2.3
Induct1     0.00   -0.44    0.77    -0.48     0.56     1.29  -2.65   1.01  -0.38
Induct2    -0.44    0.00   -0.29     0.52     1.66     0.42   0.89   0.28   1.43
Induct3     0.77   -0.29    0.00     0.25     1.00    -3.76  -0.64  -2.62   0.25
Figural1   -0.48    0.52    0.25     0.61     1.19    -1.33   2.33  -1.74  -1.24
Figural2    0.56    1.66    1.00     1.19     1.33     1.01  -2.80   1.73   4.20
Figural3    1.29    0.42   -3.76    -1.33     1.01    -1.40  -1.44  -1.50  -0.62
Fig2.1     -2.65    0.89   -0.64     2.33    -2.80    -1.44  -0.89  -1.11  -0.10
Fig2.2      1.01    0.28   -2.62    -1.74     1.73    -1.50  -1.11  -1.49  -0.45
Fig2.3     -0.38    1.43    0.25    -1.24     4.20    -0.62  -0.10  -0.45   0.58
```

The difference in χ^2 is trivial and we have gained two degrees of freedom. This suggests that the two measures are factorially equivalent.

4.5.1 Testing for factorial equivalence in multiple groups

Not shown in this chapter is how to test for equivalence of measurement across different groups. This involves best fitting the model for multiple groups simultaneously and will be discussed in the next section (as yet unwritten).

4.6 References

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