

Psychology 405: Psychometric Theory

Reliability Theory: some problems

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Outline

Problems

parallel forms

Unequal forms

Orthogonal tests

Answers

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parallel forms

Unequal forms

Orthogonal tests



Consider two 5 item tests, A and B

1. average r within test A = .3
2. average r within test B = .3
3. average r between the tests = .3
4. What is the variance of test A
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6. What is the covariance of A and B
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8. What is the correlation of an item on Test A with Test A
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12. What is the α of this combined test?



Consider two 10 item tests, A and B

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Consider two 20 item tests, A and B

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Consider two 5 item tests, A and B

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3. average r between the tests = .1
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Consider two 5 item tests, A and B

1. average r within test A = .3
2. average r within test B = .3
3. average r between the tests = .3
4. What is the variance of test A $5 + 5*4 * .3 = 11$
5. What is the variance of test B $5 + 5*4 * .3 = 11$
6. What is the covariance of A and B $5*5 * .3 = 7.5$
7. What is the correlation of A and B? $7.5/\sqrt{11 * 11} = .68$
8. What is the correlation of an item on Test A with Test A
 $(1 + 4 * .3)/\sqrt{1 * 11} = .66$
9. What is the correlation of an item on Test A with Test B
 $5 * .3/\sqrt{11} = .45$
10. What is the α reliability of test A $\frac{5*.3}{1+4*.3} = \frac{4 \frac{11-5}{11}}{5} = .68$
11. What is the variance of test A and B? $11 + 7.5 + 7.5 + 11 = 37$
12. What is the α of this combined test? $\frac{10*.3}{1+9*.3} = \frac{10}{9} \frac{37-10}{37} = .81$



Consider two 10 item tests, A and B

1. average r within test A = .3
2. average r within test B = .3
3. average r between the tests = .3
4. What is the variance of test A $10 + 10*9* .3 = 37$
5. What is the variance of test B $10 + 10*9* .3 = 37$
6. What is the covariance of A and B $10 * 10 * .3 = 30$
7. What is the correlation of A and B? $30/\sqrt{37 * 37} = .81$
8. What is the correlation of an item on Test A with Test A
 $(1 + 9 * .3)/\sqrt{1 * 37} = .61$
9. What is the correlation of an item on Test A with Test B
 $10 * .3/\sqrt{37} = .49$
10. What is the α reliability of test A $\frac{10*.3}{1+8*.3} = \frac{3}{10} = .3$
11. What is the variance of test A and B? $37 + 30 + 30 + 37 = 134$
12. What is the α of this combined test?
 $\frac{20*.3}{1+19*.3} = \frac{20}{19} \frac{134-20}{134} = .896$



Consider two 20 item tests, A and B

- average r within test A = .3
- average r within test B = .3
- average r between the tests = .3
- What is the variance of test A = $20 * 19 * .3 + 20 = 134$
- What is the variance of test B
- What is the covariance of A and B $20 * 20 * .3 = 120$
- What is the correlation of A and B?

$$r = \frac{C_{xy}}{\sqrt{V_x * V_y}} = 120 / 134 = .8955$$
- What is the correlation of an item on Test A with Test A

$$\frac{19 * .3 + 1}{\sqrt{134 * 1}} = .579$$
- What is the correlation of an item on Test A with Test B

$$\frac{20 * .3}{\sqrt{134 * 1}} = .518$$
- What is the α reliability of test A $\frac{n\bar{r}}{1+(n-1)\bar{r}} = \frac{n}{n-1} \frac{V_x - \sum v_{x_i}}{V_x} =$

$$\frac{20 * .3}{1 + 19 * .3} = \frac{20}{19} * \frac{134 - 20}{134} = .8955$$
- What is the variance of test A and B?
- What is the α of this combined test?



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