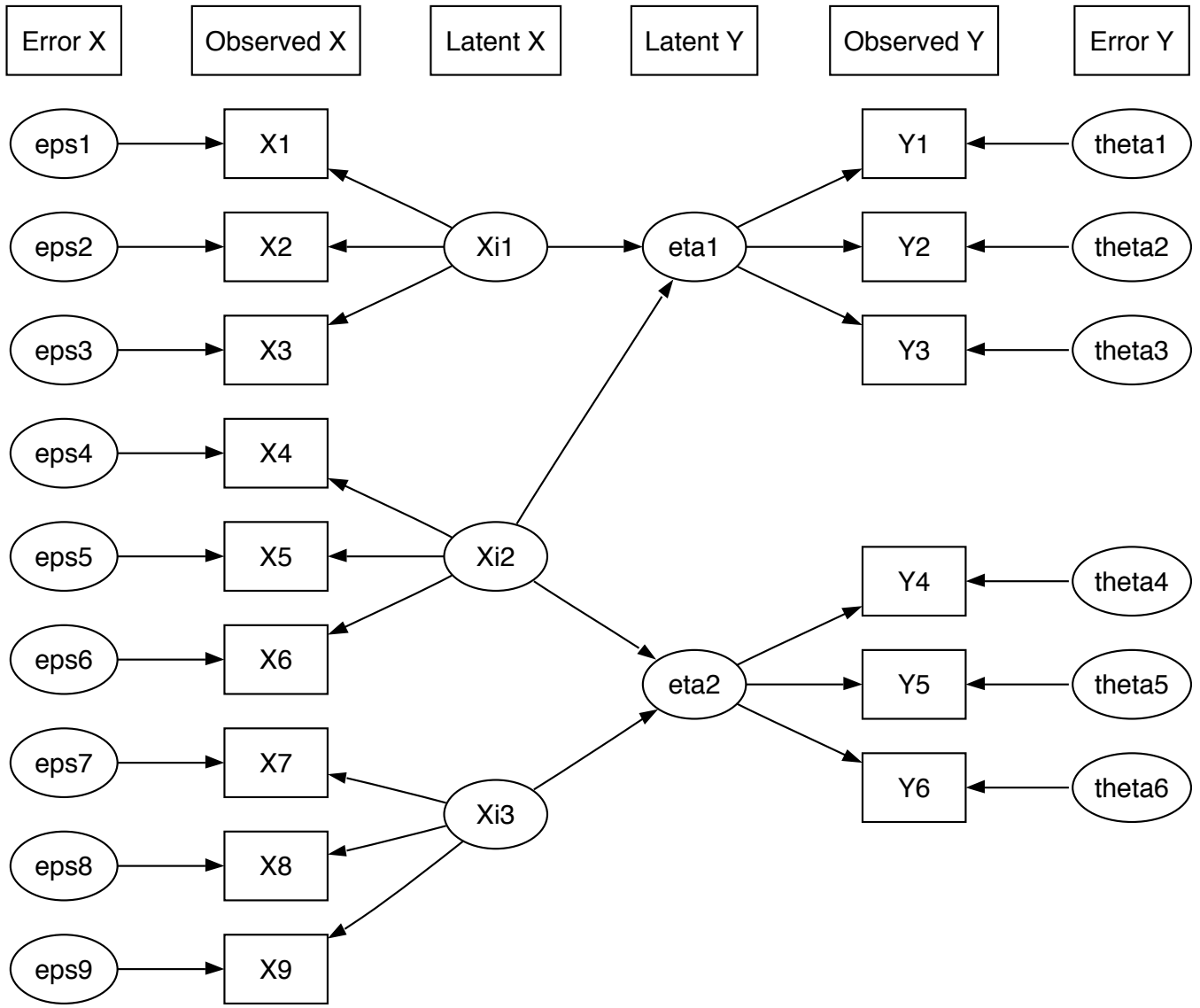
# Linear Structural Relations (LISREL and R)

Comparing results for various models

# Basic Lisrel model



# Multiple programs: multiple syntaxes

I. Commerical packages

   A. LISREL (Karl Jöreskog & Dag Sörbom)

   B. EQS (Peter Bentler)

   C. MPlus (Muthén & Muthén)

II. Open Source/Free

   A. sem (John Fox)

   B. Mx (Michael Neale)

# sem, R, LISREL

I. R is available for download at CRAN

   A. sem package needs to be installed

II. LISREL is available at Social Science Compute Cluster

   A. Can be run remotely

# Using the SSCC

```
vpn166129:~ bill$ ssh revelle@hardin.it.northwestern.edu
revelle@hardin.it.northwestern.edu's password:
Last login: Mon Feb 12 10:57:28 2007 from vpn166129.vpn.northwestern.edu

    Northwestern University Social Sciences Computing Cluster

        Academic Technologies / Research Computing


Please report system anomalies to action@hardin.it.northwestern.edu

--- Date --  -------------------- Description ------------------------
Feb 06 2007  AMD Core Math Library acml 3.6.0 installed
Connection to hardin.it.northwestern.edu closed.
vpn166129:~ bill$ ssh revelle@hardin.it.northwestern.edu
revelle@hardin.it.northwestern.edu's password:
Last login: Tue Feb 13 14:27:21 2007 from vpn166212.vpn.northwestern.edu

    Northwestern University Social Sciences Computing Cluster

        Academic Technologies / Research Computing


Please report system anomalies to action@hardin.it.northwestern.edu

--- Date --  -------------------- Description ------------------------
Feb 06 2007  AMD Core Math Library acml 3.6.0 installed
Feb 02 2007  matlab6 (32-bit, version 6.5, R13sp1) returned to service
Feb 01 2007  Stat/Transfer Upgraded to Version 8.2.7.0120
Jan 25 2007  X-Win32 8.1 Available http://www.at.northwestern.edu/x-win32/
Nov 16 2006  ox release 4.04 (x64) installed
Nov 02 2006  gauss release 8.0.0 (x64) installed
Oct 09 2006  SSCC hardware upgraded

See the complete list of Social Sciences Computing Cluster bulletins at
http://sscc.northwestern.edu/bull/index.cfm
```

# list the directory

```
[revelle@hardin ~]$ ls -l
total 156
-rw-------  1 revelle users   836 Dec 25 19:31 GUTHRIE.DSF
-rw-------  1 revelle users  2100 Dec 25 19:31 guthrie.FIT
-rw-------  1 revelle users  6736 Dec 25 19:31 guthrie.MSF
-rw-r--r--  1 revelle users   284 Dec 25 19:30 guthrie.txt
-rw-------  1 revelle users  8172 Dec 25 19:31 gut.out
-rw-------  1 revelle users   202 Dec 22 11:34 loehlin2.5.txt
-rw-------  1 revelle users   935 Dec 25 18:45 lout
-rw-------  1 revelle users  1656 Dec 25 18:32 lsq.out
-rw-------  1 revelle users   980 Dec 25 19:18 LSQUARE.DSF
-rw-------  1 revelle users  1299 Dec 25 19:26 lsquare.out
-rw-r--r--  1 revelle users   606 Dec 25 19:28 lsquare.txt
-rw-------  1 revelle users  4660 Dec 23 10:02 myfirstlisrel
-rw-r--r--  1 revelle users   434 Dec 25 08:00 prob2.5
-rw-------  1 revelle users  5037 Dec 25 07:59 prob2.5.out
-rw-------  1 revelle users   708 Dec 25 07:59 PROB2.DSF
-rw-------  1 revelle users  5604 Dec 25 07:59 prob2.MSF
-rw-------  1 revelle users  1316 Feb 12 11:37 RM5A.DSF
-rw-------  1 revelle users 15188 Feb 12 11:37 rm5a.MSF
-rw-------  1 revelle users 11561 Feb 12 11:37 rm5a.out
-rw-r--r--  1 revelle users   685 Feb 12 11:36 rm5a.txt
-rw-------  1 revelle users  1316 Feb 14 10:57 RM5.DSF
-rw-------  1 revelle users 15188 Feb 14 10:57 rm5.MSF
```

# submit a job

I. Upload the control statements using sftp

II. submit the job

   A. [revelle@hardin ~]$ lisrel8 rm5.txt rm5.out

III. move the output back to your desktop using sftp

IV. Repeat until satisfied

V. logout

# A specific model

I. Junior Year

   A. Induction

   B. Figural Ability

II. Senior Year

   A. Figural Ability

III. Prediction model

   A. Induction -> Figural Ability in both years

   B. Figural ability (jr) -> Figural ability (sr)

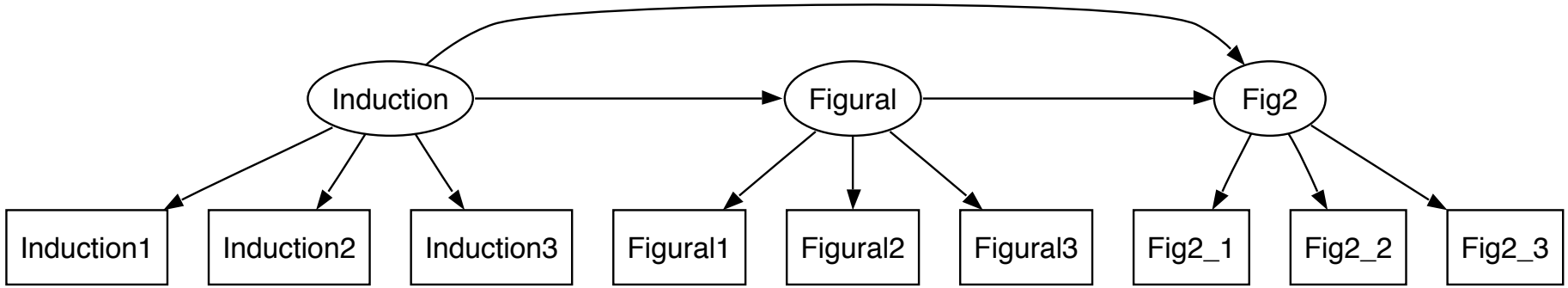
IV. Taken from Rakov and Marcoulides (2006)

# Measures are imperfect

I. Multiple measures of each construct

II. Construct is what ever is common to the multiple measures

# A specific model



# Covariance/ Correlations

56.21
31.55 75.55
23.27 28.30 44.45
24.48 32.24 22.56 84.64
22.51 29.54 20.61 57.61 78.93
22.65 27.56 15.33 53.57 49.27 73.76
33.24 46.49 31.44 67.81 54.76 54.58 141.77
32.56 40.37 25.58 55.82 52.33 47.74 98.62 117.33
30.32 40.44 27.69 54.78 53.44 59.52 96.95 84.87 106.35

# Rectangular or Triangular

prob5

| | Induct1 | Induct2 | Induct3 | Figural1 | Figural2 | Figural3 | Fig2.1 | Fig2.2 | Fig2.3 |
|---|---|---|---|---|---|---|---|---|---|
| Induct1 | 56.21 | 31.55 | 23.27 | 24.48 | 22.51 | 22.65 | 33.24 | 32.56 | 30.32 |
| Induct2 | 31.55 | 75.55 | 28.30 | 32.24 | 29.54 | 27.56 | 46.49 | 40.37 | 40.44 |
| Induct3 | 23.27 | 28.30 | 44.45 | 22.56 | 20.61 | 15.33 | 31.44 | 25.58 | 27.69 |
| Figural1 | 24.48 | 32.24 | 22.56 | 84.64 | 57.61 | 53.57 | 67.81 | 55.82 | 54.78 |
| Figural2 | 22.51 | 29.54 | 20.61 | 57.61 | 78.93 | 49.27 | 54.76 | 52.33 | 53.44 |
| Figural3 | 22.65 | 27.56 | 15.33 | 53.57 | 49.27 | 73.76 | 54.58 | 47.74 | 59.52 |
| Fig2.1 | 33.24 | 46.49 | 31.44 | 67.81 | 54.76 | 54.58 | 141.77 | 98.62 | 96.95 |
| Fig2.2 | 32.56 | 40.37 | 25.58 | 55.82 | 52.33 | 47.74 | 98.62 | 117.33 | 84.87 |
| Fig2.3 | 30.32 | 40.44 | 27.69 | 54.78 | 53.44 | 59.52 | 96.95 | 84.87 | 106.35 |

# sem model in R

```
   path                              label initial estimate
 [1,] "Induction -> Induct1"          NA    "1"
 [2,] "Induction -> Induct2"          "2"   NA
 [3,] "Induction -> Induct3"          "3"   NA
 [4,] "Figural -> Figural1"           NA    "1"
 [5,] "Figural -> Figural2"           "5"   NA
 [6,] "Figural -> Figural3"           "6"   NA
 [7,] "Figural.time2 -> Fig2.1"       NA    "1"
 [8,] "Figural.time2 -> Fig2.2"       "8"   NA
 [9,] "Figural.time2 -> Fig2.3"       "9"   NA
[10,] "Induction -> Figural"          "i"   NA
[11,] "Induction -> Figural.time2"    "j"   NA
[12,] "Figural -> Figural.time2"      "k"   NA
[13,] "Induct1 <-> Induct1"           "u"   NA
[14,] "Induct2 <-> Induct2"           "v"   NA
[15,] "Induct3 <-> Induct3"           "w"   NA
[16,] "Figural1 <-> Figural1"         "x"   NA
[17,] "Figural2 <-> Figural2"         "y"   NA
[18,] "Figural3 <-> Figural3"         "z"   NA
[19,] "Fig2.1 <-> Fig2.1"             "q"   NA
[20,] "Fig2.2 <-> Fig2.2"             "r"   NA
[21,] "Fig2.3 <-> Fig2.3"             "s"   NA
[22,] "Induction <-> Induction"       "A"   "1"
[23,] "Figural <-> Figural"           "B"   "1"
[24,] "Figural.time2 <-> Figural.time2" "C"   "1"
```

Fix some paths to 1

Estimate Regression model

Estimate errors

```
sem.prob5 <- sem(model.prob5,prob5,220)
```

# model in LISREL

```
STRUCTURAL REGRESSION MODEL
DA NI=9 NO=220
CM
56.21
31.55 75.55
23.27 28.30 44.45
24.48 32.24 22.56 84.64
22.51 29.54 20.61 57.61 78.93
22.65 27.56 15.33 53.57 49.27 73.76
33.24 46.49 31.44 67.81 54.76 54.58 141.77
32.56 40.37 25.58 55.82 52.33 47.74 98.62 117.33
30.32 40.44 27.69 54.78 53.44 59.52 96.95 84.87 106.35
LA
IND1 IND2 IND3 FR11 FR12 FR13 FR21 FR22 FR23
MO NY=9 NE=3 PS=SY,FI TE=DI,FR LY=FU,FI BE=FU,FI
LE
INDUCTN FIGREL1 FIGREL2
FR LY(2, 1) LY(3, 1)
FR LY(5, 2) LY(6, 2)
FR LY(8, 3) LY(9, 3)
VA 1 LY(1, 1) LY(4, 2) LY(7, 3)
FR BE(2, 1) BE(3, 1) BE(3, 2)
FR PS(1, 1) PS(2, 2) PS(3, 3)
OU
```

# oops, sem doesn't converge

```
Warning messages:
1: Optimization may not have converged; nlm return code = 4. Consult ?nlm.
 in: sem.default(ram = ram, S = S, N = N, param.names = pars, var.names = vars,
2: Negative parameter variances.
Model is probably underidentified.
 in: sem.default(ram = ram, S = S, N = N, param.names = pars, var.names = vars,
> summary(sem.prob5,digits=2)

 Model Chisquare =  124   Df =  24 Pr(>Chisq) = 2.1e-15
 Chisquare (null model) =  1177   Df =  36
 Goodness-of-fit index =  0.88
 Adjusted goodness-of-fit index =  0.78
 RMSEA index =  0.14   90% CI: (0.11, 0.16)
 Bentler-Bonnett NFI =  0.9
 Tucker-Lewis NNFI =  0.87
 Bentler CFI =  0.91
 BIC =  -5.7
```

# Parameters are weird

```
 Parameter Estimates
  Estimate Std Error z value Pr(>|z|)
2  1.3e+00 0.118     10.6    0.0e+00  Induct2 <--- Induction
3  8.5e-01 0.100      8.5    0.0e+00  Induct3 <--- Induction
5  9.3e-01 0.026     35.3    0.0e+00  Figural2 <--- Figural
6  8.8e-01 0.021     42.0    0.0e+00  Figural3 <--- Figural
8  8.8e-01 0.039     22.4    0.0e+00  Fig2.2 <--- Figural.time2
9  8.8e-01 0.028     31.8    0.0e+00  Fig2.3 <--- Figural.time2
i  2.0e+00   NaN      NaN       NaN  Figural <--- Induction
j -2.0e+03   NaN      NaN       NaN  Figural.time2 <--- Induction
k  1.0e+03   NaN      NaN       NaN  Figural.time2 <--- Figural
u  4.2e+01 4.210     10.0    0.0e+00  Induct1 <--> Induct1
v  5.3e+01 5.350      9.9    0.0e+00  Induct2 <--> Induct2
w  3.4e+01 3.391     10.0    0.0e+00  Induct3 <--> Induct3
x  2.6e+01 3.040      8.5    0.0e+00  Figural1 <--> Figural1
y  2.9e+01 3.535      8.2    2.2e-16  Figural2 <--> Figural2
z  2.8e+01 3.382      8.3    0.0e+00  Figural3 <--> Figural3
q  3.2e+01 4.232      7.5    8.5e-14  Fig2.1 <--> Fig2.1
r  3.2e+01 4.058      8.0    1.8e-15  Fig2.2 <--> Fig2.2
s  2.0e+01 3.050      6.6    3.6e-11  Fig2.3 <--> Fig2.3
A  1.4e+01   NaN      NaN       NaN  Induction <--> Induction
B -7.0e-04   NaN      NaN       NaN  Figural <--> Figural
C  7.4e+02   NaN      NaN       NaN  Figural.time2 <--> Figural.time2

 Iterations =  500

 Aliased parameters: i j k A B C
Warning message:
NaNs produced in: sqrt(diag(object$cov))
```

```
    path                                label initial estimate
 [1,] "Induction -> Induct1"              NA    "1"
 [2,] "Induction -> Induct2"              "2"   NA
 [3,] "Induction -> Induct3"              "3"   NA
 [4,] "Figural -> Figural1"               NA    "1"
 [5,] "Figural -> Figural2"               "5"   NA
 [6,] "Figural -> Figural3"               "6"   NA
 [7,] "Figural.time2 -> Fig2.1"           NA    "1"
 [8,] "Figural.time2 -> Fig2.2"           "8"   NA
 [9,] "Figural.time2 -> Fig2.3"           "9"   NA
[10,] "Induction -> Figural"              "i"   NA
[11,] "Induction -> Figural.time2"        "j"   NA
[12,] "Figural -> Figural.time2"          "k"   "0.75"
[13,] "Induct1 <-> Induct1"               "u"   NA
[14,] "Induct2 <-> Induct2"               "v"   NA
[15,] "Induct3 <-> Induct3"               "w"   NA
[16,] "Figural1 <-> Figural1"             "x"   NA
[17,] "Figural2 <-> Figural2"             "y"   NA
[18,] "Figural3 <-> Figural3"             "z"   NA
[19,] "Fig2.1 <-> Fig2.1"                 "q"   NA
[20,] "Fig2.2 <-> Fig2.2"                 "r"   NA
[21,] "Fig2.3 <-> Fig2.3"                 "s"   NA
[22,] "Induction <-> Induction"           "A"   "1"
[23,] "Figural <-> Figural"               "B"   "1"
[24,] "Figural.time2 <-> Figural.time2"   "C"   "1"
```

# Respecify start values

# sem fit statistics

```
summary(sem.prob5,digits=2)

 Model Chisquare =  52   Df =  24 Pr(>Chisq) = 0.00076
 Chisquare (null model) =  1177   Df =  36
 Goodness-of-fit index =  0.95
 Adjusted goodness-of-fit index =  0.91
 RMSEA index =  0.073   90% CI: (0.046, 0.1)
 Bentler-Bonnett NFI =  0.96
 Tucker-Lewis NNFI =  0.96
 Bentler CFI =  0.98
 BIC =  -77

 Normalized Residuals
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-9.5e-01 -8.9e-02 -7.3e-05 -1.2e-02  1.4e-01  1.3e+00
```

Goodness of Fit Statistics

Degrees of Freedom = 24
Minimum Fit Function Chi-Square = 52.10 (P = 0.00076)
Normal Theory Weighted Least Squares Chi-Square = 48.28 (P = 0.0023)
Estimated Non-centrality Parameter (NCP) = 24.28
90 Percent Confidence Interval for NCP = (8.23 ; 48.09)

Minimum Fit Function Value = 0.24
Population Discrepancy Function Value (F0) = 0.11
90 Percent Confidence Interval for F0 = (0.038 ; 0.22)
Root Mean Square Error of Approximation (RMSEA) = 0.068
90 Percent Confidence Interval for RMSEA = (0.040 ; 0.096)
P-Value for Test of Close Fit (RMSEA < 0.05) = 0.14

Expected Cross-Validation Index (ECVI) = 0.41
90 Percent Confidence Interval for ECVI = (0.34 ; 0.52)
ECVI for Saturated Model = 0.41
ECVI for Independence Model = 9.49

Chi-Square for Independence Model with 36 Degrees of Freedom = 2060.02
Independence AIC = 2078.02
Model AIC = 90.28
Saturated AIC = 90.00
Independence CAIC = 2117.56
Model CAIC = 182.54
Saturated CAIC = 287.71

Normed Fit Index (NFI) = 0.97
Non-Normed Fit Index (NNFI) = 0.98
Parsimony Normed Fit Index (PNFI) = 0.65
Comparative Fit Index (CFI) = 0.99
Incremental Fit Index (IFI) = 0.99
Relative Fit Index (RFI) = 0.96

Critical N (CN) = 181.68

Root Mean Square Residual (RMR) = 1.99
Standardized RMR = 0.023
Goodness of Fit Index (GFI) = 0.95
Adjusted Goodness of Fit Index (AGFI) = 0.91
Parsimony Goodness of Fit Index (PGFI) = 0.51

# LISREL fits

# sem-R parameters

| | Estimate | Std Error | z value | Pr(>\|z\|) | |
|---|---|---|---|---|---|
| 2 | 1.27 | 0.159 | 8.0 | 1.3e-15 | Induct2 <--- Induction |
| 3 | 0.89 | 0.114 | 7.8 | 7.1e-15 | Induct3 <--- Induction |
| 5 | 0.92 | 0.066 | 13.9 | 0.0e+00 | Figural2 <--- Figural |
| 6 | 0.88 | 0.066 | 13.4 | 0.0e+00 | Figural3 <--- Figural |
| 8 | 0.88 | 0.052 | 16.9 | 0.0e+00 | Fig2.2 <--- Figural.time2 |
| 9 | 0.88 | 0.048 | 18.2 | 0.0e+00 | Fig2.3 <--- Figural.time2 |
| i | 0.98 | 0.147 | 6.6 | 3.6e-11 | Figural <--- Induction |
| j | 0.60 | 0.178 | 3.4 | 7.2e-04 | Figural.time2 <--- Induction |
| k | 0.81 | 0.110 | 7.4 | 1.5e-13 | Figural.time2 <--- Figural |
| u | 30.90 | 3.891 | 7.9 | 2.0e-15 | Induct1 <--> Induct1 |
| v | 34.83 | 5.067 | 6.9 | 6.2e-12 | Induct2 <--> Induct2 |
| w | 24.49 | 3.075 | 8.0 | 1.8e-15 | Induct3 <--> Induct3 |
| x | 22.83 | 3.450 | 6.6 | 3.7e-11 | Figural1 <--> Figural1 |
| y | 26.87 | 3.459 | 7.8 | 8.0e-15 | Figural2 <--> Figural2 |
| z | 26.33 | 3.353 | 7.9 | 4.0e-15 | Figural3 <--> Figural3 |
| q | 31.31 | 4.451 | 7.0 | 2.0e-12 | Fig2.1 <--> Fig2.1 |
| r | 32.17 | 4.043 | 8.0 | 1.8e-15 | Fig2.2 <--> Fig2.2 |
| s | 20.44 | 3.213 | 6.4 | 2.0e-10 | Fig2.3 <--> Fig2.3 |
| A | 25.31 | 5.156 | 4.9 | 9.1e-07 | Induction <--> Induction |
| B | 37.70 | 6.085 | 6.2 | 5.8e-10 | Figural <--> Figural |
| C | 36.00 | 6.017 | 6.0 | 2.2e-09 | Figural.time2 <--> Figural.time2 |

# LISREL parameters

LAMBDA-Y

| | INDUCTN | FIGREL1 | FIGREL2 |
|---|---|---|---|
| IND1 | 1.00 | - - | - - |
| IND2 | 1.27 | - - | - - |
| | | (0.16) | |
| | | 8.08 | |
| IND3 | 0.89 | - - | - - |
| | | (0.12) | |
| | | 7.70 | |
| FR11 | - - | 1.00 | - - |
| FR12 | - - | 0.92 | - - |
| | | (0.07) | |
| | | 13.76 | |
| FR13 | - - | 0.88 | - - |
| | | (0.06) | |
| | | 13.54 | |
| FR21 | - - | - - | 1.00 |
| FR22 | - - | - - | 0.88 |
| | | | (0.05) |
| | | | 16.79 |
| FR23 | - - | - - | 0.88 |
| | | | (0.05) |
| | | | 18.39 |

BETA

| | INDUCTN | FIGREL1 | FIGREL2 |
|---|---|---|---|
| INDUCTN | - - | - - | - - |
| FIGREL1 | 0.98 | - - | - - |
| | | (0.15) | |
| | | 6.64 | |
| FIGREL2 | 0.60 | 0.81 | - - |
| | (0.18) | (0.11) | |
| | 3.41 | 7.40 | |

Covariance Matrix of ETA

| | INDUCTN | FIGREL1 | FIGREL2 |
|---|---|---|---|
| INDUCTN | 25.31 | | |
| FIGREL1 | 24.71 | 61.81 | |
| FIGREL2 | 35.39 | 65.23 | 110.46 |

PSI

Note: This matrix is diagonal.

| INDUCTN | FIGREL1 | FIGREL2 |
|---|---|---|
| 25.31 | 37.69 | 36.00 |
| (5.14) | (6.10) | (5.92) |
| 4.92 | 6.18 | 6.08 |

# sem-R standarized

| | | | |
|---|---|---|---|
| 1 | | 0.67105 | Induct1 <--- Induction |
| 2 | 2 | 0.73412 | Induct2 <--- Induction |
| 3 | 3 | 0.67018 | Induct3 <--- Induction |
| 4 | | 0.85457 | Figural1 <--- Figural |
| 5 | 5 | 0.81215 | Figural2 <--- Figural |
| 6 | 6 | 0.80191 | Figural3 <--- Figural |
| 7 | | 0.88269 | Fig2.1 <--- Figural.time2 |
| 8 | 8 | 0.85197 | Fig2.2 <--- Figural.time2 |
| 9 | 9 | 0.89877 | Fig2.3 <--- Figural.time2 |
| 10 | i | 0.62464 | Figural <--- Induction |
| 11 | j | 0.28886 | Figural.time2 <--- Induction |
| 12 | k | 0.60902 | Figural.time2 <--- Figural |

# Residuals

round(residuals(sem.prob5),2)

| | Induct1 | Induct2 | Induct3 | Figural1 | Figural2 | Figural3 | Fig2.1 | Fig2.2 | Fig2.3 |
|---|---|---|---|---|---|---|---|---|---|
| Induct1 | 0.00 | -0.55 | 0.79 | -0.23 | -0.17 | 1.01 | -2.15 | 1.49 | -0.89 |
| Induct2 | -0.55 | 0.00 | -0.21 | 0.90 | 0.78 | 0.11 | 1.61 | 0.96 | 0.86 |
| Induct3 | 0.79 | -0.21 | 0.00 | 0.62 | 0.47 | -3.89 | 0.01 | -2.02 | -0.03 |
| Figural1 | -0.23 | 0.90 | 0.62 | 0.00 | 0.88 | -0.58 | 2.58 | -1.46 | -2.75 |
| Figural2 | -0.17 | 0.78 | 0.47 | 0.88 | 0.00 | -0.42 | -5.11 | -0.24 | 0.64 |
| Figural3 | 1.01 | 0.11 | -3.89 | -0.58 | -0.42 | 0.00 | -2.56 | -2.43 | 9.13 |
| Fig2.1 | -2.15 | 1.61 | 0.01 | 2.58 | -5.11 | -2.56 | 0.00 | 1.63 | -0.46 |
| Fig2.2 | 1.49 | 0.96 | -2.02 | -1.46 | -0.24 | -2.43 | 1.63 | 0.00 | -0.67 |
| Fig2.3 | -0.89 | 0.86 | -0.03 | -2.75 | 0.64 | 9.13 | -0.46 | -0.67 | 0.00 |

Suggests correlated errors from Figural 3 across time

```
    path                                label initial estimate
 [1,] "Induction -> Induct1"               NA    "1"
 [2,] "Induction -> Induct2"               "2"   NA
 [3,] "Induction -> Induct3"               "3"   NA
 [4,] "Figural -> Figural1"               NA    "1"
 [5,] "Figural -> Figural2"               "5"   NA
 [6,] "Figural -> Figural3"               "6"   NA
 [7,] "Figural.time2 -> Fig2.1"            NA    "1"
 [8,] "Figural.time2 -> Fig2.2"            "8"   NA
 [9,] "Figural.time2 -> Fig2.3"            "9"   NA
[10,] "Induction -> Figural"               "i"   NA
[11,] "Induction -> Figural.time2"         "j"   NA
[12,] "Figural -> Figural.time2"           "k"   NA
[13,] "Figural3 <-> Fig2.3"                "10"  NA
[14,] "Induct1 <-> Induct1"                "u"   NA
[15,] "Induct2 <-> Induct2"                "v"   NA
[16,] "Induct3 <-> Induct3"                "w"   NA
[17,] "Figural1 <-> Figural1"              "x"   NA
[18,] "Figural2 <-> Figural2"              "y"   NA
[19,] "Figural3 <-> Figural3"              "z"   NA
[20,] "Fig2.1 <-> Fig2.1"                  "q"   NA
[21,] "Fig2.2 <-> Fig2.2"                  "r"   NA
[22,] "Fig2.3 <-> Fig2.3"                  "s"   NA
[23,] "Induction <-> Induction"            "A"   "1"
[24,] "Figural <-> Figural"                "B"   "1"
[25,] "Figural.time2 <-> Figural.time2"    "C"   "1"
```

# Redo the model

allow correlated error

# LISREL respecified

```
STRUCTURAL REGRESSION MODEL
DA NI=9 NO=220
CM
56.21
31.55   75.55
23.27   28.30   44.45
24.48   32.24   22.56   84.64
22.51   29.54   20.61   57.61   78.93
22.65   27.56   15.33   53.57   49.27   73.76
33.24   46.49   31.44   67.81   54.76   54.58   141.77
32.56   40.37   25.58   55.82   52.33   47.74   98.62   117.33
30.32   40.44   27.69   54.78   53.44   59.52   96.95   84.87   106.35
LA
IND1 IND2 IND3 FR11 FR12 FR13 FR21 FR22 FR23
MO NY=9 NE=3 PS=SY,FI TE=SY,FI LY=FU,FI BE=FU,FI
LE
INDUCTN FIGREL1 FIGREL2
FR LY(2, 1) LY(3, 1)
FR LY(5, 2) LY(6, 2)
FR LY(8, 3) LY(9, 3)
VA 1 LY(1, 1) LY(4, 2) LY(7, 3)
FR BE(2, 1) BE(3, 1) BE(3, 2)
FR PS(1, 1) PS(2, 2) PS(3, 3)
FR TE(1,1) TE (2,2) TE(3,3) TE(4,4) TE(5,5) TE(6,6) TE(7,7) TE(8,8) TE(9,9) TE(9,6)
OU
```

correlated error

# Much better fit

```
Model Chisquare =  21   Df =  23 Pr(>Chisq) = 0.61
Chisquare (null model) =  1177   Df =  36
Goodness-of-fit index =  0.98
Adjusted goodness-of-fit index =  0.96
RMSEA index =  0   90% CI: (NA, 0.049)
Bentler-Bonnett NFI =  0.98
Tucker-Lewis NNFI =  1
Bentler CFI =  1
BIC =  -104

Normalized Residuals
   Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-8.3e-01 -8.4e-02  1.6e-04 -9.5e-05  1.5e-01  4.5e-01
```

Chisquare from 52 to 21 with df from 24 to 23 but, is this still confirmatory?

# sem parameters

Parameter Estimates

| | Estimate | Std Error | z value | Pr(>\|z\|) | |
|---|---|---|---|---|---|
| 2 | 1.27 | 0.159 | 8.0 | 1.3e-15 | Induct2 <--- Induction |
| 3 | 0.89 | 0.114 | 7.8 | 7.1e-15 | Induct3 <--- Induction |
| 5 | 0.92 | 0.066 | 13.9 | 0.0e+00 | Figural2 <--- Figural |
| 6 | 0.88 | 0.066 | 13.4 | 0.0e+00 | Figural3 <--- Figural |
| 8 | 0.88 | 0.052 | 16.9 | 0.0e+00 | Fig2.2 <--- Figural.time2 |
| 9 | 0.88 | 0.048 | 18.2 | 0.0e+00 | Fig2.3 <--- Figural.time2 |
| i | 0.98 | 0.147 | 6.6 | 3.6e-11 | Figural <--- Induction |
| j | 0.60 | 0.178 | 3.4 | 7.2e-04 | Figural.time2 <--- Induction |
| k | 0.81 | 0.110 | 7.4 | 1.5e-13 | Figural.time2 <--- Figural |
| u | 30.90 | 3.891 | 7.9 | 2.0e-15 | Induct1 <--> Induct1 |
| v | 34.83 | 5.067 | 6.9 | 6.2e-12 | Induct2 <--> Induct2 |
| w | 24.49 | 3.075 | 8.0 | 1.8e-15 | Induct3 <--> Induct3 |
| x | 22.83 | 3.450 | 6.6 | 3.7e-11 | Figural1 <--> Figural1 |
| y | 26.87 | 3.459 | 7.8 | 8.0e-15 | Figural2 <--> Figural2 |
| z | 26.33 | 3.353 | 7.9 | 4.0e-15 | Figural3 <--> Figural3 |
| q | 31.31 | 4.451 | 7.0 | 2.0e-12 | Fig2.1 <--> Fig2.1 |
| r | 32.17 | 4.043 | 8.0 | 1.8e-15 | Fig2.2 <--> Fig2.2 |
| s | 20.44 | 3.213 | 6.4 | 2.0e-10 | Fig2.3 <--> Fig2.3 |
| A | 25.31 | 5.156 | 4.9 | 9.1e-07 | Induction <--> Induction |
| B | 37.70 | 6.085 | 6.2 | 5.8e-10 | Figural <--> Figural |
| C | 36.00 | 6.017 | 6.0 | 2.2e-09 | Figural.time2 <--> Figural.time2 |

# LISREL parameters to estimate

LAMBDA-Y

| | INDUCTN | FIGREL1 | FIGREL2 |
|---|---|---|---|
| IND1 | 0 | 0 | 0 |
| IND2 | 1 | 0 | 0 |
| IND3 | 2 | 0 | 0 |
| FR11 | 0 | 0 | 0 |
| FR12 | 0 | 3 | 0 |
| FR13 | 0 | 4 | 0 |
| FR21 | 0 | 0 | 0 |
| FR22 | 0 | 0 | 5 |
| FR23 | 0 | 0 | 6 |

BETA

| | INDUCTN | FIGREL1 | FIGREL2 |
|---|---|---|---|
| INDUCTN | 0 | 0 | 0 |
| FIGREL1 | 7 | 0 | 0 |
| FIGREL2 | 8 | 9 | 0 |

PSI

| INDUCTN | FIGREL1 | FIGREL2 |
|---|---|---|
| 10 | 11 | 12 |

THETA-EPS

| | IND1 | IND2 | IND3 | FR11 | FR12 | FR13 |
|---|---|---|---|---|---|---|
| IND1 | 13 | | | | | |
| IND2 | 0 | 14 | | | | |
| IND3 | 0 | 0 | 15 | | | |
| FR11 | 0 | 0 | 0 | 16 | | |
| FR12 | 0 | 0 | 0 | 0 | 17 | |
| FR13 | 0 | 0 | 0 | 0 | 0 | 18 |
| FR21 | 0 | 0 | 0 | 0 | 0 | 0 |
| FR22 | 0 | 0 | 0 | 0 | 0 | 0 |
| FR23 | 0 | 0 | 0 | 0 | 0 | 21 |

THETA-EPS

| | FR21 | FR22 | FR23 |
|---|---|---|---|
| FR21 | 19 | | |
| FR22 | 0 | 20 | |
| FR23 | 0 | 0 | 22 |

# LISREL parameters

LAMBDA-Y

| | INDUCTN | FIGREL1 | FIGREL2 |
|---|---|---|---|
| IND1 | 1.00 | - - | - - |
| IND2 | 1.27 | - - | - - |
| | (0.16) | | |
| | 8.07 | | |
| IND3 | 0.89 | - - | - - |
| | (0.12) | | |
| | 7.71 | | |
| FR11 | - - | 1.00 | - - |
| FR12 | - - | 0.89 | - - |
| | | (0.06) | |
| | | 13.89 | |
| FR13 | - - | 0.83 | - - |
| | | (0.06) | |
| | | 13.46 | |
| FR21 | - - | - - | 1.00 |
| FR22 | - - | - - | 0.87 |
| | | | (0.05) |
| | | | 17.20 |
| FR23 | - - | - - | 0.86 |
| | | | (0.05) |
| | | | 18.39 |

BETA

| | INDUCTN | FIGREL1 | FIGREL2 |
|---|---|---|---|
| INDUCTN | - - | - - | - - |
| FIGREL1 | 1.00 | - - | - - |
| | | (0.15) | |
| | | 6.68 | |
| FIGREL2 | 0.67 | 0.75 | - - |
| | (0.18) | (0.11) | |
| | 3.74 | 7.10 | |

Covariance Matrix of ETA

| | INDUCTN | FIGREL1 | FIGREL2 |
|---|---|---|---|
| INDUCTN | 25.17 | | |
| FIGREL1 | 25.13 | 64.97 | |
| FIGREL2 | 35.70 | 65.51 | 112.37 |

PSI

Note: This matrix is diagonal.

| INDUCTN | FIGREL1 | FIGREL2 |
|---|---|---|
| 25.17 | 39.88 | 39.37 |

```
    path                                label initial estimate
 [1,] "Induction -> Induct1"              NA    "1"
 [2,] "Induction -> Induct2"              "2"   NA
 [3,] "Induction -> Induct3"              "3"   NA
 [4,] "Figural -> Figural1"               NA    "1"
 [5,] "Figural -> Figural2"               "5"   NA
 [6,] "Figural -> Figural3"               "6"   NA
 [7,] "Figural.time2 -> Fig2.1"           NA    "1"
 [8,] "Figural.time2 -> Fig2.2"           "8"   NA
 [9,] "Figural.time2 -> Fig2.3"           "9"   NA
[10,] "Induction <-> Figural"             "i"   NA
[11,] "Induction <-> Figural.time2"       "j"   NA
[12,] "Figural <-> Figural.time2"         "k"   NA
[13,] "Figural3 <-> Fig2.3"               "10"  NA
[14,] "Induct1 <-> Induct1"               "u"   NA
[15,] "Induct2 <-> Induct2"               "v"   NA
[16,] "Induct3 <-> Induct3"               "w"   NA
[17,] "Figural1 <-> Figural1"             "x"   NA
[18,] "Figural2 <-> Figural2"             "y"   NA
[19,] "Figural3 <-> Figural3"             "z"   NA
[20,] "Fig2.1 <-> Fig2.1"                 "q"   NA
[21,] "Fig2.2 <-> Fig2.2"                 "r"   NA
[22,] "Fig2.3 <-> Fig2.3"                 "s"   NA
[23,] "Induction <-> Induction"           "A"   "1"
[24,] "Figural <-> Figural"               "B"   "1"
[25,] "Figural.time2 <-> Figural.time2"   "C"   "1"
```

# sem - correlation model

# fit is the same

Model Chisquare = 21 Df = 23 Pr(>Chisq) = 0.61
Chisquare (null model) = 1177 Df = 36
Goodness-of-fit index = 0.98
Adjusted goodness-of-fit index = 0.96
RMSEA index = 0 90% CI: (NA, 0.049)
Bentler-Bonnett NFI = 0.98
Tucker-Lewis NNFI = 1
Bentler CFI = 1
BIC = -104

# but some paths differ

| | Estimate | Std Error | z value | Pr(>\|z\|) | |
|---|---|---|---|---|---|
| 2 | 1.27 | 0.159 | 8.0 | 1.3e-15 | Induct2 <--- Induction |
| 3 | 0.89 | 0.115 | 7.8 | 6.9e-15 | Induct3 <--- Induction |
| 5 | 0.89 | 0.064 | 13.8 | 0.0e+00 | Figural2 <--- Figural |
| 6 | 0.83 | 0.062 | 13.4 | 0.0e+00 | Figural3 <--- Figural |
| 8 | 0.87 | 0.051 | 17.2 | 0.0e+00 | Fig2.2 <--- Figural.time2 |
| 9 | 0.86 | 0.047 | 18.3 | 0.0e+00 | Fig2.3 <--- Figural.time2 |
| i | 25.13 | 4.361 | 5.8 | 8.3e-09 | Figural <--> Induction |
| j | 35.70 | 5.801 | 6.2 | 7.6e-10 | Figural.time2 <--> Induction |
| k | 65.51 | 8.340 | 7.9 | 4.0e-15 | Figural.time2 <--> Figural |
| 10 | 12.26 | 2.488 | 4.9 | 8.2e-07 | Fig2.3 <--> Figural3 |
| u | 31.04 | 3.891 | 8.0 | 1.6e-15 | Induct1 <--> Induct1 |
| v | 34.91 | 5.060 | 6.9 | 5.2e-12 | Induct2 <--> Induct2 |
| w | 24.32 | 3.068 | 7.9 | 2.2e-15 | Induct3 <--> Induct3 |
| x | 19.67 | 3.398 | 5.8 | 7.1e-09 | Figural1 <--> Figural1 |
| y | 27.71 | 3.555 | 7.8 | 6.4e-15 | Figural2 <--> Figural2 |
| z | 28.54 | 3.483 | 8.2 | 2.2e-16 | Figural3 <--> Figural3 |
| q | 29.40 | 4.300 | 6.8 | 8.1e-12 | Fig2.1 <--> Fig2.1 |
| r | 31.34 | 3.954 | 7.9 | 2.2e-15 | Fig2.2 <--> Fig2.2 |
| s | 22.50 | 3.295 | 6.8 | 8.6e-12 | Fig2.3 <--> Fig2.3 |
| A | 25.16 | 5.143 | 4.9 | 9.9e-07 | Induction <--> Induction |
| B | 64.97 | 8.369 | 7.8 | 8.2e-15 | Figural <--> Figural |
| C | 112.37 | 13.662 | 8.2 | 2.2e-16 | Figural.time2 <--> Figural.time2 |

# Factorial Invariance

I. Are the constructs measured equally well for different groups or for different occasions?

II. Are the factor loadings equivalent across time?

A. test for equality (versus non equality) of factor loadings

```
    path                                label initial estimate
 [1,] "Induction -> Induct1"              NA    "1"
 [2,] "Induction -> Induct2"              "2"   NA
 [3,] "Induction -> Induct3"              "3"   NA
 [4,] "Figural -> Figural1"               NA    "1"
 [5,] "Figural -> Figural2"               "5"   NA
 [6,] "Figural -> Figural3"               "6"   NA
 [7,] "Figural.time2 -> Fig2.1"           NA    "1"
 [8,] "Figural.time2 -> Fig2.2"           "5"   NA
 [9,] "Figural.time2 -> Fig2.3"           "6"   NA
[10,] "Induction -> Figural"              "i"   NA
[11,] "Induction -> Figural.time2"        "j"   NA
[12,] "Figural -> Figural.time2"          "k"   NA
[13,] "Figural3 <-> Fig2.3"               "10"  NA
[14,] "Induct1 <-> Induct1"               "u"   NA
[15,] "Induct2 <-> Induct2"               "v"   NA
[16,] "Induct3 <-> Induct3"               "w"   NA
[17,] "Figural1 <-> Figural1"             "x"   NA
[18,] "Figural2 <-> Figural2"             "y"   NA
[19,] "Figural3 <-> Figural3"             "z"   NA
[20,] "Fig2.1 <-> Fig2.1"                 "q"   NA
[21,] "Fig2.2 <-> Fig2.2"                 "r"   NA
[22,] "Fig2.3 <-> Fig2.3"                 "s"   NA
[23,] "Induction <-> Induction"           "A"   "1"
[24,] "Figural <-> Figural"               "B"   "1"
[25,] "Figural.time2 <-> Figural.time2"   "C"   "1"
```

# specify equality constraints

# Test the model compare fits

Equality

```
sem.prob5e <- sem(model.prob5e,prob5,220)
> summary(sem.prob5e,digits=2)

Model Chisquare =  21   Df =  25 Pr(>Chisq) = 0.7
Chisquare (null model) =  1177   Df =  36
Goodness-of-fit index =  0.98
Adjusted goodness-of-fit index =  0.96
RMSEA index =  0   90% CI: (NA, 0.043)
Bentler-Bonnett NFI =  0.98
Tucker-Lewis NNFI =  1
Bentler CFI =  1
BIC =  -114
```

# Model: Factors are different

Model Chisquare = 21 Df = 23 Pr(>Chisq) = 0.61
Chisquare (null model) = 1177 Df = 36
Goodness-of-fit index = 0.98
Adjusted goodness-of-fit index = 0.96
RMSEA index = 0 90% CI: (NA, 0.049)
Bentler-Bonnett NFI = 0.98
Tucker-Lewis NNFI = 1
Bentler CFI = 1
BIC = -104

Chi square is not noticeably better but use up 2 df

# Sensitivity to size

Model Chisquare = 191.11 Df = 25 Pr(>Chisq) = 0
Chisquare (null model) = 10747 Df = 36
Goodness-of-fit index = 0.9798
Adjusted goodness-of-fit index = 0.96365
RMSEA index = 0.057652 90% CI: (0.050178, 0.065419)
Bentler-Bonnett NFI = 0.98222
Tucker-Lewis NNFI = 0.97767
Bentler CFI = 0.9845
BIC = 1.084

Pretend we have 2000 participants

# What do we mean when we fit a model?

I. Special issue of Personality and Individual Difference devoted to this topic

A. Barratt, P. (2007) **Structural equation modelling: Adjudging model fit. Personality and individual differences, 815-824.**

B. **http://www.sciencedirect.com/science/journal/01918869**

II. Paul Barratt: Target article

A. replies by Bentler, Steiger, Mulaik