## Package 'psych'

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Title Procedures for Psychological, Psychometric, and Personality Research
Author William Revelle [revelle@northwestern.edu](mailto:revelle@northwestern.edu)
Maintainer William Revelle [revelle@northwestern.edu](mailto:revelle@northwestern.edu)
Description A number of routines for personality, psychometrics and experimental psychology. Functions are primarily for scale construction using factor analysis, cluster analysis and reliability analysis, although others provide basic descriptive statistics. Functions for simulating particular item and test structures are included. Several functions serve as a useful front end for structural equation modeling. Graphical displays of path diagrams, factor analysis and structural equation models are created using basic graphics. Some of the functions are written to support a book on psychometrics as well as publications in personality research. For more information, see the personality-project.org/r webpage.

## License GPL (>= 2)

Suggests polycor, GPArotation, sem, MASS,graph,Rgraphviz,mvtnorm, Rcsdp
URL http://personality-project.org/r, http://personality-project.org/r/psych.manual.pdf

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00 .psychA package for personality, psychometric, and psychological research

## Description

Overview of the psych package.
The psych package has been developed at Northwestern University to include functions most useful for personality and psychological research. Some of the functions (e.g., read.clipboard, describe, pairs.panels, error.bars) are useful for basic data entry and descriptive analyses. Use help(package="psych") for a list of all functions.

Psychometric applications include routines (fa for principal axes (factor.pa), minimum residual (minres: factor.minres), and weighted least squares (link \{factor.wls \} factor analysis as well as functions to do Schmid Leiman transformations (schmid) to transform a hierarchical factor structure into a bifactor solution. Factor or components transformations to a target matrix include the standard Promax transformation (Promax), a transformation to a cluster target, or to any simple target matrix (target.rot) as well as the ability to call many of the GPArotation functions. Functions for determining the number of factors in a data matrix include Very Simple Structure (VSS) and Minimum Average Partial correlation (MAP). An alternative approach to factor analysis is Item Cluster Analysis (ICLUST). Reliability coefficients alpha (score.items, score.multiple.choice), beta (ICLUST) and McDonald's omega (omega and omega.graph) as well as Guttman's six estimates of internal consistency reliability (guttman) and the six measures of Intraclass correlation coefficients (ICC) discussed by Shrout and Fleiss are also available.
The score.items, and score.multiple.choice functions may be used to form single or multiple scales from sets of dichotomous, multilevel, or multiple choice items by specifying scoring keys.
Additional functions make for more convenient descriptions of item characteristics. Functions under development include 1 and 2 parameter Item Response measures. The tetrachoric, polychoric and irt.fa functions are used to find 2 parameter descriptions of item functioning.
A number of procedures have been developed as part of the Synthetic Aperture Personality Assessment (SAPA) project. These routines facilitate forming and analyzing composite scales equivalent to using the raw data but doing so by adding within and between cluster/scale item correlations. These functions include extracting clusters from factor loading matrices (factor2cluster), synthetically forming clusters from correlation matrices (cluster.cor), and finding multiple ((mat.regress) and partial ((partial.r) correlations from correlation matrices.
Functions to generate simulated data with particular structures include sim.circ (for circumplex structures), sim.item (for general structures) and sim. congeneric (for a specific demonstration of congeneric measurement). The functions sim.congeneric and sim.hierarchical can be used to create data sets with particular structural properties. A more general form for all of these is sim. structural for generating general structural models. These are discussed in more detail in the vignette (psych_for_sem).
Functions to apply various standard statistical tests include p. rep and its variants for testing the probability of replication, $r$. con for the confidence intervals of a correlation, and r.test to test single, paired, or sets of correlations.
In order to study diurnal or circadian variations in mood, it is helpful to use circular statistics. Functions to find the circular mean (circadian.mean), circular (phasic) correlations (circadian. cor) and the correlation between linear variables and circular variables (circadian.linear.cor) supplement a function to find the best fitting phase angle (cosinor) for measures taken with a fixed period (e.g., 24 hours).
The most recent development version of the package is always available for download as a source file from the repository at http://personality-project.org/r/src/contrib/.

## Details

Two vignettes (overview.pdf) and psych_for_sem.pdf) are useful introductions to the package. They may be found as vignettes in R or may be downloaded from http: / / personality-project.

```
org/r/book/overview.pdf and http://personality-project.org/r/book/psych_
for_sem.pdf.
```

The psych package was originally a combination of multiple source files maintained at the http: / /personality-project.org/r repository: "useful.r", VSS.r., ICLUST.r, omega.r, etc."useful.r" is a set of routines for easy data entry (read.clipboard), simple descriptive statistics (describe), and splom plots combined with correlations (pairs.panels, adapted from the help files of pairs). It is now a single package.
The VSS routines allow for testing the number of factors (VSS), showing plots (VSS.plot) of goodness of fit, and basic routines for estimating the number of factors/components to extract by using the MAP's procedure, the examining the scree plot (VSS. scree) or comparing with the scree of an equivalent matrix of random numbers (VSS. parallel).

In addition, there are routines for hierarchical factor analysis using Schmid Leiman tranformations (omega, omega.graph) as well as Item Cluster analysis (ICLUST, ICLUST.graph).
The more important functions in the package are for the analysis of multivariate data, with an emphasis upon those functions useful in scale construction of item composites.
When given a set of items from a personality inventory, one goal is to combine these into higher level item composites. This leads to several questions:

1) What are the basic properties of the data? describe reports basic summary statistics (mean, sd, median, mad, range, minimum, maximum, skew, kurtosis, standard error) for vectors, columns of matrices, or data.frames. describe.by provides descriptive statistics, organized by one or more grouping variables. pairs.panels shows scatter plot matrices (SPLOMs) as well as histograms and the Pearson correlation for scales or items. error.bars will plot variable means with associated confidence intervals. error.bars will plot confidence intervals for both the x and y coordinates. corr.test will find the significance values for a matrix of correlations.
2) What is the most appropriate number of item composites to form? After finding either standard Pearson correlations, or finding tetrachoric or polychoric correlations using a wrapper (poly .mat) for John Fox's hetcor function, the dimensionality of the correlation matrix may be examined. The number of factors/components problem is a standard question of factor analysis, cluster analysis, or principal components analysis. Unfortunately, there is no agreed upon answer. The Very Simple Structure (VSS) set of procedures has been proposed as on answer to the question of the optimal number of factors. Other procedures (VSS.scree, VSS.parallel, fa.parallel, and MAP) also address this question.
3) What are the best composites to form? Although this may be answered using principal components (principal), principal axis (factor.pa) or minimum residual (factor.minres) factor analysis (all part of the fa function) and to show the results graphically (fa.graph), it is sometimes more useful to address this question using cluster analytic techniques. (Some would argue that better yet is to use maximum likelihood factor analysis using factanal from the stats package.) Previous versions of ICLUST (e.g., Revelle, 1979) have been shown to be particularly successful at forming maximally consistent and independent item composites. Graphical output from ICLUST. graph uses the Graphviz dot language and allows one to write files suitable for Graphviz. If Rgraphviz is available, these graphs can be done in R.
Graphical organizations of cluster and factor analysis output can be done using cluster.plot which plots items by cluster/factor loadings and assigns items to that dimension with the highest loading.
4) How well does a particular item composite reflect a single construct? This is a question of reliability and general factor saturation. Multiple solutions for this problem result in (Cronbach's) alpha
(alpha, score.items), (Revelle's) Beta (ICLUST), and (McDonald's) omega (both omega hierarchical and omega total). Additional reliability estimates may be found in the guttman function.
This can also be examined for a single dimension by applying irt.fa Item Response Theory techniques using factor analysis of the tetrachoric or polychoric correlation matrices and converting the results into the standard two parameter parameterization of item difficulty and item discrimination. Information functions for the items suggest where they are most effective.
5) For some applications, data matrices are synthetically combined from sampling different items for different people. So called Synthetic Aperture Personality Assessement (SAPA) techniques allow the formation of large correlation or covariance matrices even though no one person has taken all of the items. To analyze such data sets, it is easy to form item composites based upon the covariance matrix of the items, rather than original data set. These matrices may then be analyzed using a number of functions (e.g., cluster.cor, factor.pa, ICLUST, principal, mat.regress, and factor2cluster.
6) More typically, one has a raw data set to analyze. alpha will report several reliablity estimates as well as item-whole correlations for items forming a single scale, score. items will score data sets on multiple scales, reporting the scale scores, item-scale and scale-scale correlations, as well as coefficient alpha, alpha-1 and G6+. Using a 'keys' matrix (created by make . keys or by hand), scales can have overlapping or independent items. score.multiple.choice scores multiple choice items or converts multiple choice items to dichtomous ( $0 / 1$ ) format for other functions.
An additional set of functions generate simulated data to meet certain structural properties. sim. anova produces data simulating a 3 way analysis of variance (ANOVA) or linear model with or with out repeated measures. sim. item creates simple structure data, sim.circ will produce circumplex structured data, sim. dichot produces circumplex or simple structured data for dichotomous items. These item structures are useful for understanding the effects of skew, differential item endorsement on factor and cluster analytic soutions. sim. structural will produce correlation matrices and data matrices to match general structural models. (See the vignette).

When examining personality items, some people like to discuss them as representing items in a two dimensional space with a circumplex structure. Tests of circumplex fit circ.tests have been developed. When representing items in a circumplex, it is convenient to view them in polar coordinates.
Additional functions for testing the difference between two independent or dependent correlation $r$. test, to find the phi or Yule coefficients from a two by table, or to find the confidence interval of a correlation coefficient.
Ten data sets are included: bfi represents 25 personality items thought to represent five factors of personality, iqitems has 14 multiple choice iq items. sat. act has data on self reported test scores by age and gender. galton Galton's data set of the heights of parents and their children. peas recreates the original Galton data set of the genetics of sweet peas. heights and cubits provide even more Galton data, vegetables provides the Guilford preference matrix of vegetables. cities provides airline miles between 11 US cities (demo data for multidimensional scaling).

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| :--- | :--- |
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## Index:

psych A package for personality, psychometric, and psychological research.

Useful data entry and descriptive statistics

| read.clipboard | shortcut for reading from the clipboard |
| :--- | :--- |
| read.clipboard.csv | shortcut for reading comma delimited files from clipboard <br> read.clipboard.lower <br> shortcut for reading lower triangular matrices from the clipboard <br> read.clipboard.upper <br> shortcut for reading upper triangular matrices from the clipboard |
| sescribe | Basic descriptive statistics useful for psychometrics |
| describe.by | Find summary statistics by groups |
| headtail | combines the head and tail functions for showing data sets |
| pairs.panels | SPLOM and correlations for a data matrix |
| corr.test | Correlations, sample sizes, and p values for a data matrix |
| cor.plot | graphically show the size of correlations in a correlation matrix |
| multi.hist | Histograms and densities of multiple variables arranged in matrix form |
| skew | Calculate skew for a vector, each column of a matrix, or data.frame |
| kurtosi | Calculate kurtosis for a vector, each column of a matrix or dataframe |
| geometric.mean | Find the geometric mean of a vector or columns of a data.frame |
| harmonic.mean | Find the harmonic mean of a vector or columns of a data.frame |
| error.bars | Plot means and error bars |
| error.bars.by | Plot means and error bars for separate groups |
| error.crosses | Two way error bars |
| interp.median | Find the interpolated median, quartiles, or general quantiles. |
| rescale | Rescale data to specified mean and standard deviation |
| table2df | Convert a two dimensional table of counts to a matrix or data frame |

Data reduction through cluster and factor analysis

| fa | Combined function for principal axis, minimum residual, weighted least squares, and maximum likeli |
| :--- | :--- |
| factor.pa | Do a principal Axis factor analysis (deprecated) |
| factor.minres | Do a minimum residual factor analysis (deprecated) |
| factor.wls | Do a weighted least squares factor analysis (deprecated) |
| fa.graph | Show the results of a factor analysis or principal components analysis graphically |
| fa.diagram | Show the results of a factor analysis without using Rgraphviz |
| fa.sort | Sort a factor or principal components output |
| principal | Do an eigen value decomposition to find the principal components of a matrix |
| fa.parallel | Scree test and Parallel analysis |
| factor.scores | Estimate factor scores given a data matrix and factor loadings |
| guttman | 8 different measures of reliability (6 from Guttman (1945) |
| irt.fa | Apply factor analysis to dichotomous items to get IRT parameters |
| iclust | Apply the ICLUST algorithm |
| ICLUST.graph | Graph the output from ICLUST using the dot language |


| ICLUST.rgraph | Graph the output from ICLUST using rgraphviz |
| :--- | :--- |
| polychoric | Find the polychoric correlations for items and find item thresholds (uses J. Fox's polycor) |
| poly.mat | Find the polychoric correlations for items (uses J. Fox's hetcor) |
| omega | Calculate the omega estimate of factor saturation (requires the GPArotation package) |
| omega.graph | Draw a hierarchical or Schmid Leiman orthogonalized solution (uses Rgraphviz) |
| schmid | Apply the Schmid Leiman transformation to a correlation matrix |
| score.items | Combine items into multiple scales and find alpha |
| score.multiple.choice | Combine items into multiple scales and find alpha and basic scale statistics |
| smc | Find the Squared Multiple Correlation (used for initial communality estimates) |
| tetrachoric | Find tetrachoric correlations and item thresholds |
| polyserial | Find polyserial and biserial correlations for item validity studies |
| VSS | Apply the Very Simple Structure criterion to determine the appropriate number of factors. |
| VSS.parallel | Do a parallel analysis to determine the number of factors for a random matrix |
| VSS.plot | Plot VSS output |
| VSS.scree | Show the scree plot of the factor/principal components |
| MAP | Apply the Velicer Minimum Absolute Partial criterion for number of factors |

Functions for reliability analysis (some are listed above as well).

| alpha | Find coefficient alpha and Guttman Lambda 6 for a scale (see also score.items) |
| :--- | :--- |
| guttman | 8 different measures of reliability (6 from Guttman (1945) |
| omega | Calculate the omega estimates of reliability (requires the GPArotation package) |
| ICC | Intraclass correlation coefficients |
| score.items | Combine items into multiple scales and find alpha |
| glb.algebraic | The greates lower bound found by an algebraic solution (requires Rcsdp). Written by Andreas Moeltner |

Procedures particularly useful for Synthetic Aperture Personality Assessment

| alpha | Find coefficient alpha and Guttman Lambda 6 for a scale (see also score.items) |
| :--- | :--- |
| make.keys | Create the keys file for score.items or cluster.cor |
| correct.cor | Correct a correlation matrix for unreliability |
| count.pairwise <br> cluster.cor | Count the number of complete cases when doing pair wise correlations |
| cluster.loadings | find correlations of composite variables from larger matrix |
| eigen.loadings | find correlations of items with composite variables from a larger matrix |
| fa | Do a minimal residual or principal axis factor analysis and estimate factor scores |
| factor.pa | Do a Principal Axis factor analysis and estimate factor scores |
| factor2cluster | extract cluster definitions from factor loadings |
| factor.congruence | Factor congruence coefficient |
| factor.fit | How well does a factor model fit a correlation matrix |
| factor.model | Reproduce a correlation matrix based upon the factor model |
| factor.residuals | Fit = data - model |
| factor.rotate | "hand rotate" factors |
| guttman | 8 different measures of reliability |
| mat.regress | standardized multiple regression from raw or correlation matrix input |

polyserial polyserial and biserial correlations with massive missing data

Functions for generating simulated data sets
sim The basic simulation functions
sim.anova Generate 3 independent variables and 1 or more dependent variables for demonstrating ANOVA and $\operatorname{lm}$ des
sim.circ Generate a two dimensional circumplex item structure
sim.item Generate a two dimensional simple structrue with particular item characteristics
sim.congeneric Generate a one factor congeneric reliability structure
sim.minor $\quad$ Simulate nfact major and nvar/2 minor factors
sim.structural Generate a multifactorial structural model
sim.irt Generate data for a 1, 2, 3 or 4 parameter logistic model
sim.VSS Generate simulated data for the factor model
phi.demo Create artificial data matrices for teaching purposes
sim.hierarchical Generate simulated correlation matrices with hierarchical or any structure

## Graphical functions (require Rgraphviz) - deprecated

structure.graph Draw a sem or regression graph
fa.graph Draw the factor structure from a factor or principal components analysis
omega.graph Draw the factor structure from an omega analysis (either with or without the Schmid Leiman transformation) ICLUST.graph Draw the tree diagram from ICLUST

Graphical functions that do not require Rgraphviz

| diagram | A general set of diagram functions. |
| :--- | :--- |
| structure.diagram | Draw a sem or regression graph |
| fa.diagram | Draw the factor structure from a factor or principal components analysis |
| omega.diagram | Draw the factor structure from an omega analysis (either with or without the Schmid Leiman transformati |
| ICLUST.diagram | Draw the tree diagram from ICLUST |
| plot.psych | A call to plot various types of output (e.g. from irt.fa, fa, omega, iclust |

Circular statistics (for circadian data analysis)

| circadian.cor | Find the correlation with e.g., mood and time of day |
| :--- | :--- |
| circadian.linear.cor | Correlate a circular value with a linear value |
| circadian.mean | Find the circular mean of each column of a a data set |
| cosinor | Find the best fitting phase angle for a circular data set |

Miscellaneous functions

| comorbidity | Convert base rate and comorbity to phi, Yule and tetrachoric |
| :--- | :--- |
| fisherz | Apply the Fisher r to z transform |
| fisherz2r | Apply the Fisher z to r transform |
| ICC | Intraclass correlation coefficients |
| cortest.mat | Test for equality of two matrices (see also cortest.normal, cortest.jennrich ) |
| cortest.bartlett | Test whether a matrix is an identity matrix |
| paired.r | Test for the difference of two paired or two independent correlations |
| r.con | Confidence intervals for correlation coefficients |
| r.test | Test of significance of r, differences between rs. |
| p.rep | The probability of replication given a p, r, t, or F |
| phi | Find the phi coefficient of correlation from a 2 x 2 table |
| phi.demo | Demonstrate the problem of phi coefficients with varying cut points |
| phi2poly | Given a phi coefficient, what is the polychoric correlation |
| phi2poly.matrix | Given a phi coefficient, what is the polychoric correlation (works on matrices) <br> polar |
| Convert 2 dimensional factor loadings to polar coordinates. |  |
| poly.mat | Use John Fox's hetcor to create a matrix of correlations from a data.frame or matrix of integer values |
| polychor.matrix | Use John Fox's polycor to create a matrix of polychoric correlations from a matrix of Yule correlations |
| scaling.fits | Compares alternative scaling solutions and gives goodness of fits |
| scrub | Basic data cleaning |
| tetrachor | Finds tetrachoric correlations |
| thurstone | Thurstone Case V scaling |
| tr | Find the trace of a square matrix |
| wkappa | weighted and unweighted versions of Cohen's kappa |
| Yule | Find the Yule Q coefficient of correlation |
| Yule.inv | What is the two by two table that produces a Yule Q with set marginals? |
| Yule2phi | What is the phi coefficient corresponding to a Yule Q with set marginals? |
| Yule2phi.matrix | Convert a matrix of Yule coefficients to a matrix of phi coefficients. |
| Yule2phi.matrix | Convert a matrix of Yule coefficients to a matrix of polychoric coefficients. |

Functions that are under development and not recommended for casual use
irt.item.diff.rasch IRT estimate of item difficulty with assumption that theta $=0$
irt.person.rasch Item Response Theory estimates of theta (ability) using a Rasch like model

Data sets included in the psych package
bfi represents 25 personality items thought to represent five factors of personality
bifactor 8 different data sets with a bifactor structure
cities The airline distances between 11 cities (used to demonstrate MDS)

| epi.bfi | 13 personality scales |
| :--- | :--- |
| iqitems | 14 multiple choice iq items |
| msq | 75 mood items |
| sat.act | Self reported ACT and SAT Verbal and Quantitative scores by age and gender |
| Tucker | Correlation matrix from Tucker |
| galton | Galton's data set of the heights of parents and their children |
| heights | Galton's data set of the relationship between height and forearm (cubit) length |
| cubits | Galton's data table of height and forearm length |
| peas | Galton's data set of the diameters of 700 parent and offspring sweet peas |
| vegetables | Guilford's preference matrix of vegetables (used for thurstone) |

A debugging function that may also be used as a demonstration of psych.
test.psych Run a test of the major functions on 5 different data sets. Primarily for development purposes. Although the outpu

## Note

Development versions (source code) of this package are maintained at the repository http:// personality-project.org/r along with further documentation. Specify that you are downloading a source package.
Some functions require other packages. Specifically, omega and schmid require the GPArotation package, and poly.mat, phi2poly and polychor.matrix requires John Fox's polychor package. ICLUST.rgraph and fa.graph require Rgraphviz but have alternatives using the diagram functions. i.e.:

| function | requires |
| :--- | :--- |
| omega | GPArotation |
| schmid | GPArotation |
| poly.mat | polychor |
| phi2poly | polychor |
| polychor.matrix | polychor |
| ICLUST.rgraph | Rgraphviz |
| fa.graph | Rgraphviz |
| structure.graph | Rgraphviz |
| glb.algebraic | Rcsdp |

## Author(s)

William Revelle
Department of Psychology
Northwestern University
Evanston, Illiniois
http://personality-project.org/revelle.html

Maintainer: William Revelle [revelle@northwestern.edu](mailto:revelle@northwestern.edu)

## References

A general guide to personality theory and research may be found at the personality-project http:
/ /personality-project.org. See also the short guide to R at http://personality-project.
org/r. In addition, see
Revelle, W. (in preparation) An Introduction to Psychometric Theory with applications in R. Springer. at http://personality-project.org/r/book/

## Examples

```
#See the separate man pages
test.psych()
```

$$
\begin{array}{ll}
\text { affect } & \text { Two data sets of affect and arousal scores as a function of personality } \\
\text { and movie conditions }
\end{array}
$$

## Description

A recurring question in the study of affect is the proper dimensionality and the relationship to various personality dimensions. Here are two data sets taken from studies of mood and arousal using movies to induce affective states.

## Usage

```
data(affect)
```


## Details

These are data from two studies conducted in the Personality, Motivation and Cognition Laboratory at Northwestern University. Both studies used a similar methodology:
Collection of pretest data using 5 scales from the Eysenck Personality Inventory and items taken from the Motivational State Questionnaire. In addition, state and trait anxiety measures were given. In the "maps" study, the Beck Depression Inventory was given also.
Then subjects were randomly assigned to one of four movie conditions: Frontline. A documentary about the liberation of the Bergen-Belsen concentration camp. Halloween. A horror film. 3: National Geographic, a nature film about the Serengeti plain. 4: Parenthood. A comedy. Each film clip was shown for 9 minutes. Following this the MSQ was given again.
Data from the MSQ was scored for Energetic and Tense Arousal (EA and TA) as well as Positive and Negative Affect (PA and NA).
Study flat had 170 participants, study maps had 160.
These studies are described in more detail in various publications from the PMC lab. In particular, Revelle and Anderson, 1997 and Rafaeli and Revelle (2006).

## Source

Data collecte at the Personality, Motivation, and Cognition Laboratory, Northwestern University.

## References

William Revelle and Kristen Joan Anderson (1997) Personality, motivation and cognitive performance: Final report to the Army Research Institute on contract MDA 903-93-K-0008
Rafaeli, Eshkol and Revelle, William (2006), A premature consensus: Are happiness and sadness truly opposite affects? Motivation and Emotion, 30, 1, 1-12.

## Examples

```
data(affect)
describe(flat)
pairs.panels(flat[15:18],bg=c("red","black","white","blue")[maps$Film],pch=21,main="Affect
describe(maps)
pairs.panels(maps[14:17],bg=c("red","black","white","blue")[maps$Film],pch=21,main="Affect
```

alpha Find two estimates of reliability: Cronbach's alpha and Guttman's Lambda 6.

## Description

Internal consistency measures of reliability range from $\omega_{h}$ to $\alpha$ to $\omega_{t}$. This function reports two estimates: Cronbach's coefficient $\alpha$ and Guttman's $\lambda_{6}$. Also reported are item - whole correlations, $\alpha$ if an item is omitted, and item means and standard deviations.

## Usage

alpha(x, keys=NULL, cumulative=FALSE, title=NULL, na.rm = TRUE)

## Arguments

$x \quad$ A data.frame or matrix of data, or a covariance or correlation matrix
keys If some items are to be reversed keyed, then the direction of all items must be specified in a keys vector
title Any text string to identify this run
cumulative should means reflect the sum of items or the mean of the items. The default value is means.
na.rm $\quad$ The default is to remove missing values and find pairwise correlations

## Details

Alpha is one of several estimates of the internal consistency reliability of a test.
Surprisingly, 105 years after Spearman (1904) introduced the concept of reliability to psychologists, there are still multiple approaches for measuring it. Although very popular, Cronbach's $\alpha$ (1951) underestimates the reliability of a test and over estimates the first factor saturation.
$\alpha$ (Cronbach, 1951) is the same as Guttman's $\lambda 3$ (Guttman, 1945) and may be found by

$$
\lambda_{3}=\frac{n}{n-1}\left(1-\frac{\operatorname{tr}(\vec{V})_{x}}{V_{x}}\right)=\frac{n}{n-1} \frac{V_{x}-\operatorname{tr}\left(\vec{V}_{x}\right)}{V_{x}}=\alpha
$$

Perhaps because it is so easy to calculate and is available in most commercial programs, alpha is without doubt the most frequently reported measure of internal consistency reliability. Alpha is the mean of all possible spit half reliabilities (corrected for test length). For a unifactorial test, it is a reasonable estimate of the first factor saturation, although if the test has any microstructure (i.e., if it is "lumpy") coefficients $\beta$ (Revelle, 1979; see ICLUST) and $\omega_{h}$ (see omega) are more appropriate estimates of the general factor saturation. $\omega_{t}$ (see omega) is a better estimate of the reliability of the total test.
Guttman's Lambda 6 (G6) considers the amount of variance in each item that can be accounted for the linear regression of all of the other items (the squared multiple correlation or smc), or more precisely, the variance of the errors, $e_{j}^{2}$, and is

$$
\lambda_{6}=1-\frac{\sum e_{j}^{2}}{V_{x}}=1-\frac{\sum\left(1-r_{s m c}^{2}\right)}{V_{x}}
$$

The squared multiple correlation is a lower bound for the item communality and as the number of items increases, becomes a better estimate.
G6 is also sensitive to lumpyness in the test and should not be taken as a measure of unifactorial structure. For lumpy tests, it will be greater than alpha. For tests with equal item loadings, alpha > G6, but if the loadings are unequal or if there is a general factor, G6 > alpha.
Alpha and G6 are both positive functions of the number of items in a test as well as the average intercorrelation of the items in the test. When calculated from the item variances and total test variance, as is done here, raw alpha is sensitive to differences in the item variances. Standardized alpha is based upon the correlations rather than the covariances.
More complete reliability analyses of a single scale can be done using the omega function which finds $\omega_{h}$ and $\omega_{t}$ based upon a hierarchical factor analysis.
Alternative functions score.items and cluster.cor will also score multiple scales and report more useful statistics. "Standardized" alpha is calculated from the inter-item correlations and will differ from raw alpha.
Three alternative item-whole correlations are reported, two are conventional, one unique. $r$ is the correlation of the item with the entire scale, not correcting for item overlap. r.drop is the correlation of the item with the scale composed of the remaining items. Although both of these are conventional statistics, they have the disadvantage that a) item overlap inflates the first and b) the scale is different for each item when an item is dropped. Thus, the third alternative, r.cor, corrects for the item overlap by subtracting the item variance but then replaces this with the best estimate of common variance, the smc.

## Value

| total | a list containing |
| :--- | :--- |
| raw_alpha | alpha based upon the covariances |
| std.alpha | The standarized alpha based upon the correlations |


| G6 (smc) | Guttman's Lambda 6 reliability |
| :--- | :--- |
| average_r | The average interitem correlation |
| mean | For data matrices, the mean of the scale formed by summing the items |
| sd | For data matrices, the standard deviation of the total score |
| alpha.drop | A data frame with all of the above for the case of each item being removed one |
| by one. |  |

## Author(s)

William Revelle

## References

Cronbach, L.J. (1951) Coefficient alpha and the internal strucuture of tests. Psychometrika, 16, 297-334.

Guttman, L. (1945). A basis for analyzing test-retest reliability. Psychometrika, 10 (4), 255-282.
Revelle, W. Hierarchical Cluster Analysis and the Internal Structure of Tests. Multivariate Behavioral Research, 1979, 14, 57-74.

Revelle, W. and Zinbarg, R. E. (2009) Coefficients alpha, beta, omega and the glb: comments on Sijtsma. Psychometrika, 2009.

## See Also

omega, ICLUST, guttman, score.items, cluster.cor

## Examples

```
r4 <- sim.congeneric()
alpha(r4)
r9 <- sim.hierarchical()
alpha(r9)
#an example of two independent factors that produce reasonable alphas
#this is a case where alpha is a poor indicator of unidimensionality
two.f <- sim.item(8)
alpha(two.f,keys=c(rep (1,4),rep (-1,4)))
#an example with discrete item responses -- show the frequencies
items <- sim.congeneric(N=500, short=FALSE,low=-2,high=2,categorical=TRUE) #500 responses to
a4 <- alpha(items$observed) #item response analysis of congeneric measures
```

a 4
\#summary just gives Alpha
summary (a4)

```
bfi 25 Personality items representing 5 factors
```


## Description

25 personality self report items taken from the International Personality Item Pool (ipip.ori.org) were included as part of the Synthetic Aperture Personality Assessment (SAPA) web based personality assessment project. The data from 2800 subjects are included here as a demonstration set for scale construction, factor analysis, and Item Response Theory analysis. Three additional demographic variables (sex, education, and age) are also included.

## Usage

data(bfi)

## Format

A data frame with 2800 observations on the following 28 variables. (The q numbers are the SAPA item numbers).

A1 Am indifferent to the feelings of others. (q_146)
A2 Inquire about others' well-being. (q_1162)
A3 Know how to comfort others. (q_1206)
A4 Love children. (q_1364)
A5 Make people feel at ease. (q_1419)
C1 Am exacting in my work. (q_124)
C2 Continue until everything is perfect. (q_530)
C3 Do things according to a plan. (q_619)
C4 Do things in a half-way manner. (q_626)
C5 Waste my time. (q_1949)
E1 Don't talk a lot. (q_712)
E2 Find it difficult to approach others. (q_901)
E3 Know how to captivate people. (q_1205)
E4 Make friends easily. (q_1410)
E5 Take charge. (q_1768)
N1 Get angry easily. (q_952)
N2 Get irritated easily. (q_974)
N3 Have frequent mood swings. (q_1099

N4 Often feel blue. (q_1479)
N5 Panic easily. (q_1505)
O1 Am full of ideas. (q_128)
02 Avoid difficult reading material.(q_316)
03 Carry the conversation to a higher level. (q_492)
04 Spend time reflecting on things. (q_1738)
O5 Will not probe deeply into a subject. (q_1964)
gender Males $=1$, Females $=2$
education $1=$ HS, $2=$ finished HS, $3=$ some college, $4=$ college graduate $5=$ graduate degree
age age in years

## Details

The first 25 items are organized by five putative factors: Agreeableness, Conscientiousness, Extraversion, Neuroticism, and Opennness. The scoring key is created using make . keys, the scores are found using score. items.

These five factors are a useful example of using irt. fa to do Item Response Theory based latent factor analysis of the polychoric correlation matrix. The endorsement plots for each item, as well as the item information functions reveal that the items differ in their quality.

## Source

The items are from the ipip (Goldberg, 1999). The data are from the SAPA project (Revelle, Wilt and Rosenthal, 2010), collected Spring, 2010.

## References

Goldberg, L.R. (1999) A broad-bandwidth, public domain, personality inventory measuring the lower-level facets of several five-factor models. In Mervielde, I. and Deary, I. and De Fruyt, F. and Ostendorf, F. (eds) Personality psychology in Europe. 7. Tilburg University Press. Tilburg, The Netherlands.

Revelle, W., Wilt, J., and Rosenthal, A. (2010) Personality and Cognition: The Personality-Cognition Link. In Gruszka, A. and Matthews, G. and Szymura, B. (Eds.) Handbook of Individual Differences in Cognition: Attention, Memory and Executive Control, Springer.

## Examples

```
data(bfi)
describe(bfi)
    keys.list <- list(Agree=c(-1,2:5), Conscientious=c (6:8,-9,-10), Extraversion=c(-11,-12,13:15)
    keys <- make.keys(28,keys.list, item.labels=colnames(bfi))
    score.items(keys,bfi)
    scores <- score.items(keys[1:27,],bfi[1:27]) #don't score age
    scores
```


## Description

Holzinger-Swineford (1937) introduced the bifactor model of a general factor and uncorrelated group factors. The Holzinger data sets are original $14 * 14$ matrix from their paper as well as a 9 *9 matrix used as an example by Joreskog. The Thurstone correlation matrix is a $9 * 9$ matrix of correlations of ability items. The Reise data set is 16 * 16 correlation matrix of mental health items. The Bechtholdt data sets are both $17 \times 17$ correlation matrices of ability tests.

## Usage

data(bifactor)

## Details

Holzinger and Swineford (1937) introduced the bifactor model (one general factor and several group factors) for mental abilities. This is a nice demonstration data set of a hierarchical factor structure that can be analyzed using the omega function or using sem. The bifactor model is typically used in measures of cognitive ability.
The 14 variables are ordered to reflect 3 spatial tests, 3 mental speed tests, 4 motor speed tests, and 4 verbal tests. The sample size is 355 .
Another data set from Holzinger (Holzinger.9) represents 9 cognitive abilities (Holzinger, 1939) and is used as an example by Karl Joreskog (2003) for factor analysis by the MINRES algorithm and also appears in the LISREL manual as example NPV.KM.
Another classic data set is the 9 variable Thurstone problem which is discussed in detail by R. P. McDonald $(1985,1999)$ and and is used as example in the sem package as well as in the PROC CALIS manual for SAS. These nine tests were grouped by Thurstone and Thurstone, 1941 (based on other data) into three factors: Verbal Comprehension, Word Fluency, and Reasoning. The original data came from Thurstone and Thurstone (1941) but were reanalyzed by Bechthold (1961) who broke the data set into two. McDonald, in turn, selected these nine variables from the larger set of 17 found in Bechtoldt.2. The sample size is 213.

Another set of 9 cognitive variables attributed to Thurstone (1933) is the data set of 4,175 students reported by Professor Brigham of Princeton to the College Entrance Examination Board. This set does not show a clear bifactor solution but is included as a demonstration of the differences between a maximimum likelihood factor analysis solution versus a principal axis factor solution.
More recent applications of the bifactor model are to the measurement of psychological status. The Reise data set is a correlation matrix based upon $>35,000$ observations to the Consumer Assessment of Health Care Provideers and Systems survey instrument. Reise, Morizot, and Hays (2007) describe a bifactor solution based upon 1,000 cases.

The five factors from Reise et al. reflect Getting care quickly (1-3), Doctor communicates well (47), Courteous and helpful staff $(8,9)$, Getting needed care (10-13), and Health plan customer service (14-16).

The two Bechtoldt data sets are two samples from Thurstone and Thurstone (1941). They include 17 variables, 9 of which were used by McDonald to form the Thurstone data set. The sample sizes are 212 and 213 respectively. The six proposed factors reflect memory, verbal, words, space, number and reasoning with three markers for all expect the rote memory factor. 9 variables from this set appear in the Thurstone data set.
Two more data sets with similar structures are found in the Harman data set.

- Bechtoldt.1: $17 \times 17$ correlation matrix of ability tests, $\mathrm{N}=212$.
- Bechtoldt.2: $17 \times 17$ correlation matrix of ability tests, $\mathrm{N}=213$.
- Holzinger: $14 \times 14$ correlation matrix of ability tests, $\mathrm{N}=355$
- Holzinger.9: $9 \times 9$ correlation matrix of ability tests, $\mathrm{N}=145$
- Reise: $16 \times 16$ correlation matrix of health satisfaction items. $\mathrm{N}=35,000$
- Thurstone: $9 \times 9$ correlation matrix of ability tests, $\mathrm{N}=213$
- Thurstone. 33: Another 9 x 9 correlation matrix of ability items, $\mathrm{N}=4175$


## Source

Holzinger: Holzinger and Swineford (1937)
Reise: Steve Reise (personal communication)
sem help page (for Thurstone)

## References

Bechtoldt, Harold, (1961). An empirical study of the factor analysis stability hypothesis. Psychometrika, 26, 405-432.
Holzinger, Karl and Swineford, Frances (1937) The Bi-factor method. Psychometrika, 2, 41-54
Holzinger, K., \& Swineford, F. (1939). A study in factor analysis: The stability of a bifactor solution. Supplementary Educational Monograph, no. 48. Chicago: University of Chicago Press.
McDonald, Roderick P. (1999) Test theory: A unified treatment. L. Erlbaum Associates. Mahwah, N.J.

Reise, Steven and Morizot, Julien and Hays, Ron (2007) The role of the bifactor model in resolving dimensionality issues in health outcomes measures. Quality of Life Research. 16, 19-31.
Thurstone, Louis Leon (1933) The theory of multiple factors. Edwards Brothers, Inc. Ann Arbor
Thurstone, Louis Leon and Thurstone, Thelma (Gwinn). (1941) Factorial studies of intelligence. The University of Chicago Press. Chicago, Il.

## Examples

```
data(bifactor)
if(!require(GPArotation)) {message("I am sorry, to run omega requires GPArotation") } else
holz <- omega(Holzinger,4, title = "14 ability tests from Holzinger-Swineford")
bf <- omega(Reise,5,title="16 health items from Reise")
omega(Reise,5,labels=colnames(Reise),title="16 health items from Reise")
thur.bf <- omega(Thurstone,title="9 variables from Thurstone")
}
```

block.random Create a block randomized structure for $n$ independent variables

## Description

Random assignment of n subjects with an equal number in all of N conditions may done by block randomization, where the block size is the number of experimental conditions. The number of Independent Variables and the number of levels in each IV are specified as input. The output is a the block randomized design.

## Usage

block.random(n, ncond $=$ NULL)

## Arguments

n
ncond The number of conditions for each IV. Defaults to 2 levels for one IV. If more than one IV, specify as a vector. If names are provided, they are used, otherwise the IVs are labeled as IV1 ... IVn

## Value

blocks A matrix of subject numbers, block number, and randomized levels for each IV

## Note

Prepared for a course on Research Methods in Psychology http://personality-project. org/revelle/syllabi/205/205.syllabus.html

## Author(s)

William Revelle

## Examples

```
br <- block.random(n=24,c(2,3))
pairs.panels(br)
br <- block.random(96,c(time=4,drug=3,sex=2))
pairs.panels(br)
```


## Description

35 items for 150 subjects from Bond's Logical Operations Test. A good example of Item Response Theory analysis using the Rasch model. One parameter (Rasch) analysis and two parameter IRT analyses produce somewhat different results.

## Usage

data(blot)

## Format

A data frame with 150 observations on 35 variables. The BLOT was developed as a paper and pencil test for children to measure Logical Thinking as discussed by Piaget and Inhelder.

## Details

Bond and Fox apply Rasch modeling to a variety of data sets. This one, Bond's Logical Operations Test, is used as an example of Rasch modeling for dichotomous items. In their text (p 56), Bond and Fox report the results using WINSTEPS. Those results are consistent (up to a scaling parameter) with those found by the rasch function in the ltm package. The WINSTEPS seem to produce difficulty estimates with a mean item difficulty of 0 , whereas rasch from ltm has a mean difficulty of -1.52 . In addition, rasch seems to reverse the signs of the difficulty estimates when reporting the coefficients and is effectively reporting "easiness".
However, when using a two parameter model, one of the items (V12) behaves very differently.
This data set is useful when comparing 1PL, 2PL and 2PN IRT models.

## Source

The data are taken (with kind permission from Trevor Bond) from the webpage http://homes . jcu.edu. au/~edtgb/book/data/Bond87.txt and read using read.fwf.

## References

T.G. Bond. BLOT:Bond's Logical Operations Test. Townsville, Australia: James Cook University. (Original work published 1976), 1995.
T. Bond and C. Fox. (2007) Applying the Rasch model: Fundamental measurement in the human sciences. Lawrence Erlbaum, Mahwah, NJ, US, 2 edition.

## See Also

See also the irt.fa and associated plot functions.

## Examples

```
data(blot)
#not run
#library(ltm)
#bblot.rasch <- rasch(blot, constraint = cbind(ncol(blot) + 1, 1)) #a 1PL model
#blot.2pl <- ltm(blot~z1) #a 2PL model
#do the same thing with functions in psych
#blot.fa <- irt.fa(blot) # a 2PN model
#plot(blot.fa)
```

bock Bock and Liberman (1970) data set of 1000 observations of the LSAT

## Description

An example data set used by McDonald (1999) as well as other discussions of Item Response Theory makes use of a data table on 10 items (two sets of 5) from the Law School Admissions Test (LSAT). Included in this data set is the original table as well as the reponses for 1000 subjects on the first set (Figure Classification) and second set (Debate).

## Usage

data(bock)

## Format

A data frame with 32 observations on the following 8 variables.
index 32 response patterns
Q1 Responses to item 1
Q2 Responses to item 2
Q3 Responses to item 3
Q4 Responses to item 4
Q5 Responses to item 5
Ob 6 count of observations for the section 6 test
Ob 7 count of observations for the section 7 test
Two other data sets are derived from the bock dataset. These are converted using the table 2 df function.

Isat6 reponses to 5 items for 1000 subjects on section 6
lsat 7 reponses to 5 items for 1000 subjects on section 7

## Details

The lsat6 data set is analyzed in the ltm package as well as by McDonald (1999). lsat7 is another 1000 subjects on part 7 of the LSAT. Both sets are described by Bock and Lieberman (1970). Both sets are useful examples of testing out IRT procedures and showing the use of tetrachoric correlations and item factor analysis using the irt. fa function.

## Source

R. Darrell Bock and M. Lieberman (1970). Fitting a response model for dichotomously scored items. Psychometrika, 35(2):179-197.

## References

R.P. McDonald. Test theory: A unified treatment. L. Erlbaum Associates, Mahwah, N.J., 1999.

## Examples

```
data(bock)
responses <- table2df(bock.table[,2:6],count=bock.table[,7],labs= paste("lsat6.",1:5,sep="")
describe(responses)
## maybe str(bock.table) ; plot(bock.table) ...
```

```
burt 11 emotional variables from Burt (1915)
```


## Description

Cyril Burt reported an early factor analysis with a circumplex structure of 11 emotional variables in 1915. 8 of these were subsequently used by Harman in his text on factor analysis. Unfortunately, it seems as if Burt made a mistake for the matrix is not positive definite. With one change from .87 to .81 the matrix is positive definite.

## Usage

data(burt)

## Format

A correlation matrix based upon 172 "normal school age children aged 9-12".
Sociality Sociality
Sorrow Sorrow
Tenderness Tenderness
Joy Joy
Wonder Wonder
Elation Elation

## Disgust Disgust

Anger Anger
Sex Sex
Fear Fear
Subjection Subjection

## Details

The Burt data set is interesting for several reasons. It seems to be an early example of the organizaton of emotions into an affective circumplex, a subset of it has been used for factor analysis examples (see Harman. Burt, and it is an example of how typos affect data. The original data matrix has one negative eigenvalue. With the replacement of the correlation between Sorrow and Tenderness from .87 to .81 , the matrix is positive definite.

## Source

(retrieved from the web at http://www.biodiversitylibrary.org/item/95822\#790) Following a suggestion by Jan DeLeeuw.

## References

Burt, C.General and Specific Factors underlying the Primary Emotions. Reports of the British Association for the Advancement of Science, 85th meeting, held in Manchester, September 7-11, 1915. London, John Murray, 1916, p. 694-696 (retrieved from the web at http://www.biodiversitylibrary.org/item/95822\#790)

## See Also

Harman. Burt in the Harman dataset.

## Examples

```
data(burt)
eigen(burt)$values #one is negative!
burt.new <- burt
burt.new[2,3] <- burt.new[3,2] <- . 80
eigen(burt.new)$values #all are positive
```


## Description

Rotations of factor analysis and principal components analysis solutions typically try to represent correlation matrices as simple structured. An alternative structure, appealing to some, is a circumplex structure where the variables are uniformly spaced on the perimeter of a circle in a two dimensional space. Generating these data is straightforward, and is useful for exploring alternative solutions to affect and personality structure.

## Usage

circ.tests(loads, loading = TRUE, sorting = TRUE)

## Arguments

loads A matrix of loadings loads here
loading Are these loadings or a correlation matrix loading
sorting Should the variables be sorted sorting

## Details

"A common model for representing psychological data is simple structure (Thurstone, 1947). According to one common interpretation, data are simple structured when items or scales have nonzero factor loadings on one and only one factor (Revelle \& Rocklin, 1979). Despite the commonplace application of simple structure, some psychological models are defined by a lack of simple structure. Circumplexes (Guttman, 1954) are one kind of model in which simple structure is lacking.
"A number of elementary requirements can be teased out of the idea of circumplex structure. First, circumplex structure implies minimally that variables are interrelated; random noise does not a circumplex make. Second, circumplex structure implies that the domain in question is optimally represented by two and only two dimensions. Third, circumplex structure implies that variables do not group or clump along the two axes, as in simple structure, but rather that there are always interstitial variables between any orthogonal pair of axes (Saucier, 1992). In the ideal case, this quality will be reflected in equal spacing of variables along the circumference of the circle (Gurtman, 1994; Wiggins, Steiger, \& Gaelick, 1981). Fourth, circumplex structure implies that variables have a constant radius from the center of the circle, which implies that all variables have equal communality on the two circumplex dimensions (Fisher, 1997; Gurtman, 1994). Fifth, circumplex structure implies that all rotations are equally good representations of the domain (Conte \& Plutchik, 1981; Larsen \& Diener, 1992). (Acton and Revelle, 2004)
Acton and Revelle reviewed the effectiveness of 10 tests of circumplex structure and found that four did a particularly good job of discriminating circumplex structure from simple structure, or circumplexes from ellipsoidal structures. Unfortunately, their work was done in Pascal and is not easily available. Here we release R code to do the four most useful tests:

1 The Gap test of equal spacing
2 Fisher's test of equality of axes
3 A test of indifference to Rotation
4 A test of equal Variance of squared factor loadings across arbitrary rotations.
To interpret the values of these various tests, it is useful to compare the particular solution to simulated solutions representing pure cases of circumplex and simple structure. See the example output from circ.simulation and compare these plots with the results of the circ.test.

## Value

A list of four items is returned. These are the gap, fisher, rotation and variance test results.
gaps gap.test
cities

| fisher | fisher.test |
| :--- | :--- |
| RT | rotation.test |
| VT | variance.test |

## Note

Of the 10 criterion discussed in Acton and Revelle (2004), these tests operationalize the four most useful.

## Author(s)

William Revelle

## References

Acton, G. S. and Revelle, W. (2004) Evaluation of Ten Psychometric Criteria for Circumplex Structure. Methods of Psychological Research Online, Vol. 9, No. 1 http: / /personality-project. org/revelle/publications/acton.revelle.mpr110_10.pdf

## See Also

```
circ.simulation,sim.circ
```


## Examples

```
circ.data <- circ.sim(24,500)
circ.fa <- factor.pa(circ.data,2)
#plot(circ.fa$loadings)
ct <- circ.tests(circ.fa)
#compare with non-circumplex data
simp.data <- item.sim(24,500)
simp.fa <- factor.pa(simp.data,2)
#plot(simp.fa$loadings)
st <- circ.tests(simp.fa)
print(rbind(ct,st), digits=2)
```

```
cities
Distances between 11 US cities
```


## Description

Airline distances between 11 US cities may be used as an example for multidimensional scaling or cluster analysis.

## Usage

data(cities)

## Format

A data frame with 11 observations on the following 11 variables.
ATL Atlana, Georgia
BOS Boston, Massachusetts
ORD Chicago, Illinois
DCA Washington, District of Columbia
DEN Denver, Colorado
LAX Los Angeles, California
MIA Miami, Florida
JFK New York, New York
SEA Seattle, Washington
SFO San Francisco, California
MSY New Orleans, Lousianna

## Details

An 11 x11 matrix of distances between major US airports. This is a useful demonstration of multiple dimensional scaling.
city.location is a dataframe of longitude and latitude for those cities.
Note that the 2 dimensional MDS solution does not perfectly capture the data from these city distances. Boston, New York and Washington, D.C. are located slightly too far west, and Seattle and LA are slightly too far south.

## Source

http://www.timeanddate.com/worldclock/distance.html

## Examples

```
data(cities)
city.location[,1] <- -city.location[,1]
if(require(maps)) {map("usa")
title("MultiDimensional Scaling of US cities")
points(city.location)} else {plot(city.location, xlab="Dimension 1", ylab="Dimension 2",main
city.loc <- cmdscale(cities, k=2) #ask for a 2 dimensional solution round(city.loc,0)
city.loc <- -city.loc
city.loc <- rescale(city.loc,mean(city.location),sd(city.location))
points(city.loc,type="n")
text(city.loc,labels=names(cities))
```


## Description

Given a $n \mathrm{x}$ c cluster definition matrix of $-1 \mathrm{~s}, 0 \mathrm{~s}$, and 1 s (the keys), and an $\mathrm{x} n$ correlation matrix, find the correlations of the composite clusters. The keys matrix can be entered by hand, copied from the clipboard (read.clipboard), or taken as output from the factor2cluster function. Similar functionality to score. items which also gives item by cluster correlations.

## Usage

cluster.cor(keys, r.mat, correct = TRUE, digits=2,SMC=TRUE)

## Arguments

keys A matrix of cluster keys
r.mat A correlation matrix
correct TRUE shows both raw and corrected for attenuation correlations
digits round off answer to digits
SMC Should squared multiple correlations be used as communality estimates for the correlation matrix?

## Details

This is one of the functions used in the SAPA procedures to form synthetic correlation matrices. Given any correlation matrix of items, it is easy to find the correlation matrix of scales made up of those items. This can also be done from the original data matrix or from the correlation matrix using score. items which is probably preferred.

A typical use in the SAPA project is to form item composites by clustering or factoring (see factor.pa, ICLUST, principal), extract the clusters from these results (factor2cluster), and then form the composite correlation matrix using cluster. cor. The variables in this reduced matrix may then be used in multiple correlatin procedures using mat.regress.
The original correlation is pre and post multiplied by the (transpose) of the keys matrix.
If some correlations are missing from the original matrix this will lead to missing values (NA) for scale intercorrelations based upon those lower level correlations.

Because the alpha estimate of reliability is based upon the correlations of the items rather than upon the covariances, this estimate of alpha is sometimes called "standardized alpha". If the raw items are available, it is useful to compare standardized alpha with the raw alpha found using score.items. They will differ substantially only if the items differ a great deal in their variances.

## Value

| cor | the (raw) correlation matrix of the clusters |
| :--- | :--- |
| sd | standard deviation of the cluster scores |
| corrected | raw correlations below the diagonal, alphas on diagonal, disattenuated above <br> diagonal |
| alpha | The (standardized) alpha reliability of each scale. <br> G6 |
| Guttman's Lambda 6 reliability estimate is based upon the smcs for each item <br> in a scale. G6 uses the smc based upon the entire item domain. |  |
| av.r | The average inter item correlation within a scale |
| size | How many items are in each cluster? |

## Note

See SAPA Revelle, W., Wilt, J., and Rosenthal, A. (2010) Personality and Cognition: The PersonalityCognition Link. In Gruszka, A. and Matthews, G. and Szymura, B. (Eds.) Handbook of Individual Differences in Cognition: Attention, Memory and Executive Control, Springer.

## Author(s)

Maintainer: William Revelle [revelle@northwestern.edu](mailto:revelle@northwestern.edu)

## See Also

```
factor2cluster,mat.regress,alpha.scale, score.items
```


## Examples

```
## Not run:
data(attitude)
keys <- matrix(c(1, 1, 1,0,0,0,0,
                    0,0,0,1,1,1,1),ncol=2)
colnames(keys) <- c("first","second")
r.mat <- cor(attitude)
cluster.cor(keys,r.mat)
## End(Not run)
#$cor
# first second
#first 1.0 0.6
#second 0.6 1.0
#
#$sd
# first second
# 2.57 3.01
#
#$corrected
# first second
#first 0.82 0.77
#second 0.60 0.74
```

```
#
#$size
# first second
# 4 4
```

cluster.fit cluster Fit: fit of the cluster model to a correlation matrix

## Description

How well does the cluster model found by ICLUST fit the original correlation matrix? A similar algorithm factor. fit is found in VSS. This function is internal to ICLUST but has more general use as well.
In general, the cluster model is a Very Simple Structure model of complexity one. That is, every item is assumed to represent only one factor/cluster. Cluster fit is an analysis of how well this model reproduces a correlation matrix. Two measures of fit are given: cluster fit and factor fit. Cluster fit assumes that variables that define different clusters are orthogonal. Factor fit takes the loadings generated by a cluster model, finds the cluster loadings on all clusters, and measures the degree of fit of this somewhat more complicated model. Because the cluster loadings are similar to, but not identical to factor loadings, the factor fits found here and by factor. fit will be similar.

## Usage

cluster.fit(original, load, clusters, diagonal = FALSE)

## Arguments

original The original correlation matrix being fit
load Cluster loadings - that is, the correlation of individual items with the clusters, corrected for item overlap
clusters The cluster structure
diagonal Should we fit the diagonal as well?

## Details

The cluster model is similar to the factor model: R is fitted by C ' C . Where $\mathrm{C}<-$ Cluster definition matrix x the loading matrix. How well does this model approximate the original correlation matrix and how does this compare to a factor model?
The fit statistic is a comparison of the original (squared) correlations to the residual correlations. Fit $=1-r^{*} 2 / r 2$ where $r^{*}$ is the residual correlation of data - model and model $=C^{\prime} \mathrm{C}$.

## Value

clusterfit The cluster model is a reduced form of the factor loading matrix. That is, it is the product of the elements of the cluster matrix * the loading matrix.
factorfit How well does the complete loading matrix reproduce the correlation matrix?

## Author(s)

Maintainer: William Revelle [revelle@northwestern.edu](mailto:revelle@northwestern.edu)

## References

```
http://personality-project.org/r/r.ICLUST.html
```


## See Also

```
VSS, ICLUST, factor2cluster, cluster.cor, factor.fit
```


## Examples

```
r.mat<- Harman74.cor$cov
iq.clus <- ICLUST(r.mat,nclusters =2)
fit <- cluster.fit(r.mat,iq.clus$loadings,iq.clus$clusters)
fit
```

    cluster.loadings Find item by cluster correlations, corrected for overlap and reliability
    
## Description

Given a $n \times n$ correlation matrix and a $n \times c$ matrix of $-1,0,1$ cluster weights for those $n$ items on c clusters, find the correlation of each item with each cluster. If the item is part of the cluster, correct for item overlap. Part of the ICLUST set of functions, but useful for many item analysis problems.

## Usage

cluster.loadings(keys, r.mat, correct = TRUE, SMC=TRUE)

## Arguments

keys $\quad$ Cluster keys: a matrix of $-1,0,1$ cluster weights
r.mat A correlation matrix
correct Correct for reliability
SMC Use the squared multiple correlation as a communality estimate, otherwise use the greatest correlation for each variable

## Details

Given a set of items to be scored as (perhaps overlapping) clusters and the intercorrelation matrix of the items, find the clusters and then the correlations of each item with each cluster. Correct for item overlap by replacing the item variance with its average within cluster inter-item correlation.
Although part of ICLUST, this may be used in any SAPA application where we are interested in item- whole correlations of items and composite scales.

These loadings are particularly interpretable when sorted by absolute magnitude for each cluster (see ICLUST. sort).

## Value

| loadings | A matrix of item-cluster correlations (loadings) |
| :--- | :--- |
| cor | Correlation matrix of the clusters |
| corrected | Correlation matrix of the clusters, raw correlations below the diagonal, alpha on <br> diagonal, corrected for reliability above the diagonal |
| sd | Cluster standard deviations |
| alpha | alpha reliabilities of the clusters |
| G6 | G6* Modified estimated of Guttman Lambda 6 |
| count | Number of items in the cluster |

## Note

Although part of ICLUST, this may be used in any SAPA application where we are interested in item- whole correlations of items and composite scales.

## Author(s)

Maintainer: William Revelle [revelle@northwestern.edu](mailto:revelle@northwestern.edu)

## References

ICLUST: http://personality-project.org/r/r.iclust.html

## See Also

ICLUST, factor2cluster, cluster.cor

## Examples

```
r.mat<- Harman74.cor$cov
clusters <- matrix(c(1, 1,1,rep (0, 24),1,1,1,1,rep (0,17)), ncol=2)
cluster.loadings(clusters,r.mat)
```

cluster.plot Plot factor/cluster loadings and assign items to clusters by their highest loading.

## Description

Cluster analysis and factor analysis are procedures for grouping items in terms of a smaller number of (latent) factors or (observed) clusters. Graphical presentations of clusters typically show tree structures, although they can be represented in terms of item by cluster correlations.

Cluster.plot plots items by their cluster loadings (taken, e.g., from ICLUST) or factor loadings (taken, eg., from factor.pa). Cluster membership may be assigned apriori or may be determined in terms of the highest (absolute) cluster loading for each item.
If the input is an object of class "kmeans", then the cluster centers are plotted.

## Usage

cluster.plot(ic.results, cluster $=$ NULL, cut $=0$, labels=NULL,title $=$ "Cluster plot
factor.plot(ic.results, cluster = NULL, cut = 0, labels=NULL,title,...)

## Arguments

ic.results A factor analysis or cluster analysis output including the loadings, or a matrix of item by cluster correlations. Or the output from a kmeans cluster analysis.
cluster A vector of cluster membership
cut $\quad$ Assign items to clusters if the absolute loadings are $>$ cut
labels If row.names exist they will be added to the plot, or, if they don't, labels can be specified. If labels $=$ NULL, and there are no row names, then variables are labeled by row number.)
title Any title
. . Further options to plot

## Details

Results of either a factor analysis or cluster analysis are plotted. Each item is assigned to its highest loading factor, and then identified by variable name as well as cluster (by color).
Both of these functions may be called directly or by calling the generic plot function. (see example).

## Value

Graphical output is presented.

## Author(s)

William Revelle

## See Also

ICLUST, ICLUST.graph, fa.graph, plot.psych

## Examples

```
circ.data <- circ.sim(24,500)
circ.fa <- fa(circ.data,2)
plot(circ.fa,cut=.5)
```

| cluster 2 keys | Convert a cluster vector (from e.g., kmeans) to a keys matrix suitable <br> for scoring item clusters. |
| :--- | :--- |

## Description

The output of the kmeans clustering function produces a vector of cluster membership. The score.items and cluster. cor functions require a matrix of keys. cluster2keys does this.
May also be used to take the output of an ICLUST analysis and find a keys matrix. (By doing a call to the factor 2 cluster function.

## Usage

cluster2keys(c)

## Arguments

C A vector of cluster assignments or an object of class "kmeans" that contains a vector of clusters.

## Details

Note that because kmeans will not reverse score items, the clusters defined by kmeans will not necessarily match those of ICLUST with the same number of clusters extracted.

## Value

keys A matrix of keys suitable for score.items or cluster.cor

## Author(s)

William Revelle

## See Also

cluster.cor,score.items, factor2cluster, make.keys

## Examples

```
test.data <- Harman74.cor$cov
kc <- kmeans(test.data,4)
keys <- cluster2keys(kc)
keys #these match those found by ICLUST
cluster.cor(keys,test.data)
```

cohen.kappa Find Cohen's kappa and weighted kappa coefficients for correlation
of two raters

## Description

Cohen's kappa (Cohen, 1960) and weighted kappa (Cohen, 1968) may be used to find the agreement of two raters when using nominal scores.
weighted.kappa is (probability of observed matches - probability of expected matches)/(1-probability of expected matches). Kappa just considers the matches on the main diagonal. Weighted kappa considers off diagonal elements as well.

## Usage

cohen.kappa(x, w=NULL, n.obs=NULL, alpha=.05)
wkappa $(x, w=N U L L) \quad$ \#deprectated

## Arguments

X
Either a two by n data with categorical values from 1 to p or a $\mathrm{p} x \mathrm{p}$ table. If a data array, a table will be found.

W
A pxp matrix of weights. If not specified, they are set to be 0 (on the diagonal) and (distance from diagonal) off the diagonal) $\wedge 2$.
n.obs Number of observations (if input is a square matrix.
alpha Probability level for confidence intervals

## Details

When cateogorical judgments are made with two cateories, a measure of relationship is the phi coefficient. However, some categorical judgments are made using more than two outcomes. For example, two diagnosticians might be asked to categorize patients three ways (e.g., Personality disorder, Neurosis, Psychosis) or to categorize the stages of a disease. Just as base rates affect observed cell frequencies in a two by two table, they need to be considered in the n-way table (Cohen, 1960).

Kappa considers the matches on the main diagonal. A penalty function (weight) may be applied to the off diagonal matches. If the weights increase by the square of the distance from the diagonal, weighted kappa is similar to an Intra Class Correlation (ICC).
Derivations of weighted kappa are sometimes expressed in terms of similarities, and sometimes in terms of dissimilarities. In the latter case, the weights on the diagonal are 1 and the weights off the diagonal are less than one. In this, if the weights are 1 -squared distance from the diagonal / k, then the result is similar to the ICC (for any positive k).
cohen.kappa may use either similarity weighting (diagonal $=0$ ) or dissimilarity weighting (diagonal $=1$ ) in order to match various published examples.
The input may be a two column data.frame or matrix with columns representing the two judges and rows the subjects being rated. Alternatively, the input may be a square n x n matrix of counts or proportion of matches. If proportions are used, it is necessary to specify the number of observations (n.obs) in order to correctly find the confidence intervals.

The confidence intervals are based upon the variance estimates discussed by Fleiss, Cohen, and Everitt who corrected the formulae of Cohen (1968) and Blashfield.

## Value

```
kappa Unweighted kappa
weighted.kappa
    The default weights are quadratric.
var.kappa Variance of kappa
var.weighted Variance of weighted kappa
n.obs number of observations
weight The weights used in the estimation of weighted kappa
confid The alpha/2 confidence intervals for unweighted and weighted kappa
plevel The alpha level used in determining the confidence limits
```


## Note

As is true of many R functions, there are alternatives in other packages. The Kappa function in the vcd package estimates unweighted and weighted kappa and reports the variance of the estimate. The input is a square matrix. The ckappa and wkappa functions in the psy package take raw data matrices.

To avoid confusion with Kappa (from vcd) or the kappa function from base, the function was originally named wkappa. With additional features modified from psy::ckappa to allow input with a different number of categories, the function has been renamed cohen.kappa.
Unfortunately, to make it more confusing, the weights described by Cohen are a function of the reciprocals of those discucssed by Fleiss and Cohen. The cohen.kappa function uses the appropriate formula for Cohen or Fleiss-Cohen weights.

## Author(s)

William Revelle

## References

Banerjee, M., Capozzoli, M., McSweeney, L and Sinha, D. (1999) Beyond Kappa: A review of interrater agreement measures The Canadian Journal of Statistics / La Revue Canadienne de Statistique, 27, 3-23
Cohen, J. (1960). A coefficient of agreement for nominal scales. Educational and Psychological Measurement, 20 37-46

Cohen, J. (1968). Weighted kappa: Nominal scale agreement provision for scaled disagreement or partial credit. Psychological Bulletin, 70, 213-220.
Fleiss, J. L., Cohen, J. and Everitt, B.S. (1969) Large sample standard errors of kappa and weighted kappa. Psychological Bulletin, 72, 332-327.
Zwick, R. (1988) Another look at interrater agreement. Psychological Bulletin, 103, 374-378.

## Examples

```
cohen <- matrix(c(
0.44, 0.07, 0.09,
0.05, 0.20, 0.05,
0.01, 0.03, 0.06),ncol=3,byrow=TRUE)
#cohen.weights weight differences
cohen.weights <- matrix(c(
0,1,3,
1,0,6,
3,6,0),ncol=3)
cohen.kappa(cohen,cohen.weights,n.obs=200)
#cohen reports . }492\mathrm{ and . }34
#another set of weights
#what if the weights are non-symmetric
wc <- matrix(c(
0,1,4,
1,0,6,
2,2,0),ncol=3,byrow=TRUE)
cohen.kappa(cohen,wc)
#Cohen reports kw = . 353
cohen.kappa(cohen,n.obs=200) #this uses the squared weights
fleiss.cohen <- 1 - cohen.weights/9
cohen.kappa(cohen,fleiss.cohen,n.obs=200)
#however, Fleiss, Cohen and Everitt weight similarities
fleiss <- matrix(c(
106, 10,4,
22,28, 10,
2, 12, 6),ncol=3,byrow=TRUE)
```

```
#Fleiss weights the similarities
weights <- matrix(c(
    1.0000, 0.0000, 0.4444,
    0.0000, 1.0000, 0.6666,
    0.4444, 0.6666, 1.0000),ncol=3)
    cohen.kappa(fleiss,weights,n.obs=200)
    #data may be a 2 column matrix
    #compare weighted and unweighted
    #also look at the ICC for this data set.
    twins <- matrix(c(
        1, 2,
        2, 3,
        3, 4,
        5, 6,
        6, 7), ncol=2,byrow=TRUE)
    cohen.kappa(twins)
#data may be explicitly categorical
x <- c("red","yellow","blue","red")
y <- c("red", "blue", "blue" ,"red")
xy.df <- data.frame(x,y)
ck <- cohen.kappa(xy.df)
ck
ck$agree
#finally, input can be a data.frame of ratings from more than two raters
ratings <- matrix(rep(1:5,4),ncol=4)
ratings[1,2] <- ratings[2,3] <- ratings[3,4] <- NA
ratings[2,1] <- ratings[3,2] <- ratings[4,3] <- 1
cohen.kappa(ratings)
```

comorbidity Convert base rates of two diagnoses and their comorbidity into phi,
Yule, and tetrachorics

## Description

In medicine and clinical psychology, diagnoses tend to be categorical (someone is depressed or not, someone has an anxiety disorder or not). Cooccurrence of both of these symptoms is called comorbidity. Diagnostic categories vary in their degree of comorbidity with other diagnostic categories. From the point of view of correlation, comorbidity is just a name applied to one cell in a four fold table. It is thus possible to analyze comorbidity rates by considering the probability of the separate diagnoses and the probability of the joint diagnosis. This gives the two by two table needed for a phi, Yule, or tetrachoric correlation.

## Usage

```
comorbidity(d1, d2, com, labels = NULL)
```


## Arguments

| d1 | Proportion of diagnostic category 1 |
| :--- | :--- |
| $d 2$ | Proportion of diganostic category 2 |
| com | Proportion of comorbidity (diagnostic category 1 and 2) |
| labels | Names of categories 1 and 2 |

## Value

twobytwo The two by two table implied by the input
phi Phi coefficient of the two by two table
Yule Yule coefficient of the two by two table
tetra Tetrachoric coefficient of the two by two table

Note
Requires the polycor package

## Author(s)

William Revelle

## See Also

phi, Yule

## Examples

```
if(require(polycor)) {comorbidity(.2,.15,.1,c("Anxiety","Depression")) }
```

```
cor.plot Create an image plot for a correlation or factor matrix
```


## Description

Correlation matrices may be shown graphically by using the image function to emphasize structure. This is a particularly useful tool for showing the structure of small correlation matrices with a clear structure. Meant for the pedagogical value of the graphic for teaching or discussing factor analysis and other multivariate techniques.

## Usage

cor.plot ( $\mathrm{r}, \mathrm{colors}=\mathrm{FALSE}, \mathrm{n}=10$, main=NULL, $\mathrm{zlim=c}(0,1)$, show.legend=TRUE, labels=NULL,.

## Arguments

$r \quad$ A correlation matrix or the output of factor. pa, factor.minres or omega.
colors Defaults to FALSE (grey), but colors=TRUE will use topo.colors
n
The number of levels of shading to use. Defaults to 10
main A title. Defaults to "correlation plot"
zlim The range of values to color - defaults to 0 to 1
show. legend A legend (key) to the colors is shown on the right hand side
labels if NULL, use column and row names, otherwise use labels
. . . Other parameters for axis (e.g., cex.axis to change the font size)

## Details

When teaching about factor analysis or cluster analysis, it is useful to graphically display the structure of correlation matrices. This is a simple graphical display using the image function.
The difference of mat.plot with a regular image plot is that the primary diagonal goes from the top left to the lower right.
The zlim parameter defaults to 0 to 1 . This means that negative correlations are treated as zero. This is advantageous when showing general factor structures, because it makes the 0 white.

The default shows a legend for the color coding on the right hand side of the figure.
Inspired, in part, by a paper by S. Dray (2008) on the number of components problem.

## Author(s)

William Revelle

## References

Dray, Stephane (2008) On the number of principal components: A test of dimensionality based on measurements of similarity between matrices. Computational Statistics <br>\& Data Analysis. 52, 4, 2228-2237.

## See Also

fa, mat.sort

## Examples

```
data(bifactor)
cor.plot(Thurstone,TRUE, main="9 cognitive variables from Thurstone")
cor.plot(mat.sort(Thurstone), TRUE, main="9 cognitive variables from Thurstone (sorted by fac
simp <- sim.circ(24)
cor.plot(cor(simp), colors=TRUE, zlim=c(-1,1),main="24 variables in a circumplex")
```

```
corr.test
```

Find the correlations, sample sizes, and probability values between elements of a matrix or data.frame.

## Description

Although the cor function finds the correlations for a matrix, it does not report probability values. corr.test uses cor to find the correlations for either complete or pairwise data and reports the sample sizes and probability values as well.

## Usage

```
corr.test(x, y = NULL, use = "pairwise",method="pearson")
```


## Arguments

$$
\begin{array}{ll}
\mathrm{x} & \text { A matrix or dataframe } \\
\mathrm{y} & \text { A second matrix or dataframe with the same number of rows as } \mathrm{x} \\
\text { use } & \begin{array}{l}
\text { use="pairwise" is the default value and will do pairwise deletion of cases. use="complete" } \\
\text { will select just complete cases. }
\end{array} \\
\text { method } & \begin{array}{l}
\text { method="pearson" is the default value. The alternatives to be passed to cor are } \\
\text { "spearman" and "kendall" }
\end{array}
\end{array}
$$

## Details

corr.test uses the cor function to find the correlations, and then applies a $t$-test to the individual correlations using the formula

$$
t=\frac{r * \sqrt{( } n-2)}{\sqrt{\left(1-r^{2}\right)}}
$$

## Value

| $r$ | The matrix of correlations |
| :--- | :--- |
| $n$ | Number of cases per correlation |
| $t$ | value of $t$-test for each correlation |
| $p$ | two tailed probability of $t$ for each correlation |

## See Also

cor.test for tests of a single correlation, Hmisc::rcorr for an equivalant function, r.test to test the difference between correlations, and cortest.mat to test for equality of two correlation matrices.

## Examples

```
data(sat.act)
corr.test(sat.act)
```

```
correct.cor Find dis-attenuated correlations given correlations and reliabilities
```


## Description

Given a raw correlation matrix and a vector of reliabilities, report the disattenuated correlations above the diagonal.

## Usage

correct. $\operatorname{cor}(x, y)$

## Arguments

| $x$ | A raw correlation matrix |
| :--- | :--- |
| $y$ | Vector of reliabilities |

## Details

Disattenuated correlations may be thought of as correlations between the latent variables measured by a set of observed variables. That is, what would the correlation be between two (unreliable) variables be if both variables were measured perfectly reliably.

This function is mainly used if importing correlations and reliabilities from somewhere else. If the raw data are available, use score.items, or cluster.loadings or cluster.cor.
Examples of the output of this function are seen in cluster.loadings and cluster.cor

## Value

Raw correlations below the diagonal, reliabilities on the diagonal, disattenuated above the diagonal.

## Author(s)

Maintainer: William Revelle [revelle@northwestern.edu](mailto:revelle@northwestern.edu)

## References

http://personality-project.org/revelle/syllabi/405.syllabus.html

## See Also

## Examples

```
# attitude from the datasets package
#example 1 is a rather clunky way of doing things
a1 <- attitude[,c(1:3)]
a2 <- attitude[,c(4:7)]
x1 <- rowSums(a1) #find the sum of the first 3 attitudes
x2 <- rowSums(a2) #find the sum of the last 4 attitudes
alpha1 <- alpha(a1)
alpha2 <- alpha(a2)
x <- matrix(c(x1, x2),ncol=2)
x.cor <- cor(x)
alpha <- c(alphal$total$raw_alpha,alpha2$total$raw_alpha)
round(correct.cor(x.cor, alpha), 2)
#
#much better - although uses standardized alpha
clusters <- matrix(c(rep (1, 3),rep (0,7),rep (1,4)),ncol=2)
cluster.loadings(clusters, cor(attitude))
# or
clusters <- matrix(c(rep (1, 3),rep (0,7),rep(1,4)),ncol=2)
cluster.cor(clusters,cor(attitude))
#
#best
scores <- score.items(matrix(c(rep (1, 3),rep (0,7),rep (1, 4)), ncol=2), attitude)
scores$corrected
```


## cortest.bartlett Bartlett's test that a correlation matrix is an identity matrix

## Description

Bartlett (1951) proposed that $-\ln (\operatorname{det}(\mathrm{R}) *(\mathrm{~N}-1-(2 \mathrm{p}+5) / 6)$ was distributed as chi square if R were an identity matrix. A useful test that residuals correlations are all zero.

## Usage

cortest.bartlett(R, $n=$ NULL)

## Arguments

$R \quad A$ correlation matrix. (If $R$ is not square, correlations are found and a warning is issued.
$\mathrm{n} \quad$ Sample size (if not specified, 100 is assumed.

## Details

More useful for pedagogical purposes than actual applications. The Bartlett test is asymptotically chi square distributed.

## Value

chisq Assymptotically chisquare
p.value Of chi square
$\mathrm{df} \quad$ The degrees of freedom

## Author(s)

William Revelle

## References

Bartlett, M. S., (1951), The Effect of Standardization on a chi square Approximation in Factor Analysis, Biometrika, 38, 337-344.

## See Also

```
cortest.mat, cortest.normal, cortest.jennrich
```


## Examples

```
set.seed(42)
x <- matrix(rnorm(1000),ncol=10)
r <- cor(x)
cortest.bartlett(r) #random data don't differ from an identity matrix
data(bfi)
cortest.bartlett(bfi) #not an identity matrix
```

cortest.mat Chi square tests of whether a single matrix is an identity matrix, or a pair of matrices are equal.

## Description

Steiger (1980) pointed out that the sum of the squared elements of a correlation matrix, or the Fisher z score equivalents, is distributed as chi square under the null hypothesis that the values are zero (i.e., elements of the identity matrix). This is particularly useful for examining whether correlations in a single matrix differ from zero or for comparing two matrices. Jennrich (1970) also examined tests of differences between matrices.

## Usage

```
cortest.normal(R1, R2 = NULL, n1 = NULL, n2 = NULL, fisher = TRUE)
cortest(R1,R2=NULL,n1=NULL,n2 = NULL, fisher = TRUE) #same as cortest.normal
cortest.mat(R1,R2=NULL,n1=NULL,n2 = NULL)
cortest.jennrich(R1,R2,n1=NULL, n2=NULL)
```


## Arguments

| R1 | A correlation matrix. (If R1 is not rectangular, the correlations are found). |
| :--- | :--- |
| R2 | A correlation matrix. If R2 is not rectangular, the correlations are found. If R2 |
| is NULL, then the test is just whether R1 is an identity matrix. |  |
| n1 | Sample size of R1 |
| n2 | Sample size of R2 |
| fisher | Fisher z transform the correlations? |

## Details

There are several ways to test if a matrix is the identity matrix. The most well known is the chi square test of Bartlett (1951) and Box (1949). A very straightforward test, discussed by Steiger (1980) is to find the sum of the squared correlations or the sum of the squared Fisher transformed correlations. Under the null hypothesis that all the correlations are equal, this sum is distributed as chi square.
Yet another test, is the Jennrich(1970) test of the equality of two matrices.

## Value

| chi2 | The chi square statistic |
| :--- | :--- |
| $d f$ | Degrees of freedom for the Chi Square |
| prob | The probability of observing the Chi Square under the null hypothesis. |

## Note

Both the cortest.jennrich and cortest.normal are probably overly stringent. The ChiSquare values for pairs of random samples from the same population are larger than would be expected. This is a good test for rejecting the null of no differences.

## Author(s)

William Revelle

## References

Steiger, James H. (1980) Testing pattern hypotheses on correlation matrices: alternative statistics and some empirical results. Multivariate Behavioral Research, 15, 335-352.

## See Also

```
cortest.bartlett
```


## Examples

```
x <- matrix(rnorm(1000), ncol=10)
y <- matrix(rnorm(500), ncol=10)
cortest.normal(x) #just test if this matrix is an identity
cortest.normal(x,y) #do these two matrices differ?
cortest.mat(x)
cortest.mat(x,y) #twice the degrees of freedom as the Jennrich
cortest.jennrich(x,y) #
```

cosinor Functions for analysis of circadian or diurnal data

## Description

Circadian data are periodic with a phase of 24 hours. These functions find the best fitting phase angle (cosinor), the circular mean, circular correlation with circadian data, and the linear by circular correlation

## Usage

```
cosinor(angle,x=NULL, code=NULL, period=24,plot=FALSE,opti=FALSE)
circadian.mean(angle, hours=TRUE)
circadian.cor(angle, hours=TRUE)
circadian.linear.cor(angle,x,hours=TRUE)
```


## Arguments

| angle | A data frame or matrix of observed values with the time of day as the first value <br> (unless specified in code) angle can be specified either as hours or as radians) |
| :--- | :--- |
| code | A subject identification variable |
| period | Although time of day is assumed to have a 24 hour rhythm, other rhythms may <br> be fit. |
| plot | if TRUE, then plot the first variable (angle) |
| opti | opti=TRUE: iterative optimization (slow) or opti=FALSE: linear fitting (fast) |
| hours | If TRUE, measures are in 24 hours to the day, otherwise, radians |
| $x$ | A set of external variables to correlate with the phase angles |

## Details

When data represent angles (such as the hours of peak alertness or peak tension during the day), we need to apply circular statistics rather than the more normal linear statistics (see Jammalamadaka (2006) for a very clear set of examples of circular statistics). The generalization of the mean to circular data is to convert each angle into a vector, average the x and y coordinates, and convert the result back to an angle. The generalization of Pearson correlation to circular statistics is straight forward and is implemented in cor.circular in the circular package and in circadian. cor here.

Just as the Pearson $r$ is a ratio of covariance to the square root of the product of two variances, so is the circular correlation. The circular covariance of two circular vectors is defined as the average product of the sines of the deviations from the circular mean. The variance is thus the average squared sine of the angular deviations from the circular mean. Circular statistics are used for data that vary over a period (e.g., one day) or over directions (e.g., wind direction or bird flight). Jammalamadaka and Lund (2006) give a very good example of the use of circular statistics in calculating wind speed and direction.
The code from CircStats and circular was adapted to allow for analysis of data from various studies of mood over the day.
The cosinor function will either iteratively fit cosines of the angle to the observed data (opti=TRUE) or use the circular by linear regression to estimate the best fitting phase angle. If cos.t <- $\cos (t i m e)$ and $\sin . t=\sin ($ time $)$ (expressed in hours), then beta.c and beta.s may be found by regression and the phase is $\operatorname{sign}($ beta.c $) * \operatorname{acos}\left(\right.$ beta.c $/ \sqrt{( }$ beta.c.c ${ }^{2}+$ beta.s $\left.\left.{ }^{2}\right)\right) * 12 / p i$
Simulations (see examples) suggest that with incomplete times, perhaps the optimization procedure yields slightly better fits with the correct phase than does the linear model, but the differences are very small. In the presence of noisey data, these advantages seem to reverse. The recommendation thus seems to be to use the linear model approach (the default).

Value
phase The phase angle that best fits the data
fit Value of the correlation of the fit
mean.angle A vector of mean angles
$R \quad$ A matrix of circular correlations or linear by circular correlations

## Author(s)

William Revelle

## References

See circular statistics Jammalamadaka, Sreenivasa and Lund, Ulric (2006),The effect of wind direction on ozone levels: a case study, Environmental and Ecological Statistics, 13, 287-298.

## See Also

See the circular and CircStats packages.

## Examples

```
time <- seq(1:24)
pure <- matrix(time,24,18)
pure <- cos((pure + col(pure)) *pi/12)
matplot(pure,type="l",main="Pure circadian arousal rhythms",xlab="time of day",ylab="Arousal
p <- cosinor(time,pure)
#set.seed(42)
noisy <- pure + rnorm(24*18)
n <- cosinor(time, noisy)
```

```
#small.pure <- pure[c(6:18),]
small.pure <- pure[c(8,11,14,17,20,23),]
#small.noisy <- noisy[c(6:18),]
small.noisy <- noisy[c(8,11,14,17,20,23),]
matplot(small.noisy,type="l",main="Noisy circadian arousal rhythms",xlab="time of day",ylab=
#sp <- cosinor(time[c(6:18)],small.pure) #linear fit
sp <- cosinor(time[c(8,11,14,17,20,23)],small.pure)
spo <- cosinor(time[c(8,11,14,17,20,23)],small.pure,opti=TRUE) #iterative fit
sn <- cosinor(time[c(8,11,14,17,20,23)],small.noisy) #linear
sno <- cosinor(time[c(8,11,14,17,20,23)],small.noisy,opti=TRUE) #iterative
sum.df <- data.frame(pure=p,noisy = n, small=sp,small.noise = sn, small.opt=spo,small.noise.
round(sum.df,2)
round(circadian.cor(sum.df[,c(1,3,5,7,9,11)]),2) #compare alternatives
round(cor(sum.df[,c(2,4,6,8,10,12)]),2)
```

count.pairwise $\quad$ Count number of pairwise cases for a data set with missing (NA) data.

## Description

When doing cor( x , use= "pairwise"), it is nice to know the number of cases for each pairwise correlation. This is particularly useful when doing SAPA type analyses.

## Usage

count.pairwise(x, $y=$ NULL)

## Arguments

$x \quad$ An input matrix, typically a data matrix ready to be correlated.
Y An optional second input matrix

## Value

result $=$ matrix of counts of pairwise observations

## Author(s)

Maintainer: William Revelle [revelle@northwestern.edu](mailto:revelle@northwestern.edu)

## Examples

```
## Not run:
x <- matrix(rnorm(1000),ncol=6)
y <- matrix(rnorm(500), ncol=3)
x[x<0] <- NA
y[y> 1] <- NA
count.pairwise(x)
```

```
count.pairwise(y)
count.pairwise(x,y)
## End(Not run)
```

cta $\quad$ Simulate the $C$ (ues) T(endency) A(ction) model of motivation

## Description

Dynamic motivational models such as the Dynamics of Action (Atkinson and Birch, 1970, Revelle, 1986) may be reparameterized as a simple pair of differential (matrix) equations (Revelle, 1986, 2008). This function simulates the dynamic aspects of the CTA.

## Usage

```
cta(n = 3, t = 5000, cues = NULL, act = NULL, inhibit = NULL, consume = NULL, ten
```


## Arguments

| $n$ | number of actions to simuate |
| :--- | :--- |
| $t$ | length of time to simulate |
| cues | a vector of cue strengths |
| act | matrix of associations between cues and action tendencies |
| inhibit | inhibition matrix |
| consume | Consummation matrix |
| ten | Initial values of action tendencies |
| type | show actions, tendencies, both, or state diagrams |
| fast | display every fast time (skips |
| compare | Compare? |

## Details

A very thorough discussion of the CTA model is available from Revelle (2008).

## Value

graphical output

| cues | echo back the cue input |
| :--- | :--- |
| inhibition | echo back the inhibitory matrix |
| time | time spent in each activity |
| frequency | Frequency of each activity |
| ten | final tension values |
| act | final action values |

## Author(s)

William Revelle

## References

Atkinson, John W. and Birch, David (1970) The dynamics of action. John Wiley, New York, N.Y.
Revelle, William (1986) Motivation and efficiency of cognitive performance in Brown, Donald R. and Veroff, Joe (ed). Frontiers of Motivational Psychology: Essays in honor of J. W. Atkinson. Springer.
Revelle, W. (2008) Cues, Tendencies and Actions. The Dynamics of Action revisted. http: //personality-project.org/revelle/publications/cta.pdf

## Examples

```
#not run
#cta() #default values, running over time
#cta(type="state") #default values, in a state space of tendency 1 versus tendency 2
```

cubits Galton's example of the relationship between height and 'cubit' or forearm length

## Description

Francis Galton introduced the 'co-relation' in 1888 with a paper discussing how to measure the relationship between two variables. His primary example was the relationship between height and forearm length. The data table (cubits) is taken from Galton (1888). Unfortunately, there seem to be some errors in the original data table in that the marginal totals do not match the table.
The data frame, heights, is converted from this table.

## Usage

data(cubits)

## Format

A data frame with 9 observations on the following 8 variables.
16.5 Cubit length of lowest category
16.75 a numeric vector
17.25 a numeric vector
17.75 a numeric vector
18.25 a numeric vector
18.75 a numeric vector
19.25 a numeric vector
19.75 a numeric vector

## Details

Sir Francis Galton (1888) published the first demonstration of the correlation coefficient. The regression (or reversion to mediocrity) of the height to the length of the left forearm (a cubit) was found to .8 . There seem to be some errors in the table as published in that the row sums do not agree with the actual row sums. These data are used to create a matrix using table2matrix for demonstrations of analysis and displays of the data.

## Source

Galton (1888)

## References

Galton, Francis (1888) Co-relations and their measurement. Proceedings of the Royal Society. London Series,45,135-145,

## See Also

```
table2matrix,table2df,ellipses, heights, peas,galton
```


## Examples

```
data(cubits)
cubits
heights <- table2df(cubits,labs = c("height","cubit"))
ellipses(heights,n=1,main="Galton's co-relation data set")
ellipses(jitter(heights$cubit,3),jitter(heights$height,3),pch=".",main="Galton's co-relation
```

```
describe Basic descriptive statistics useful for psychometrics
```


## Description

There are many summary statistics available in R; this function provides the ones most useful for scale construction and item analysis in classic psychometrics. Range is most useful for the first pass in a data set, to check for coding errors.

## Usage

describe(x, na.rm = TRUE, interp=FALSE, skew = TRUE, ranges = TRUE,trim=.1)

## Arguments

x
na.rm The default is to delete missing data. na.rm=FALSE will delete the case.
interp Should the median be standard or interpolated
skew $\quad$ Should the skew and kurtosis be calculated?
ranges $\quad$ Should the range be calculated?
trim trim=. 1 - trim means by dropping the top and bottom trim fraction

## Details

In basic data analysis it is vital to get basic descriptive statistics. Procedures such as summary and hmisc::describe do so. The describe function in the psych package is meant to produce the most frequently requested stats in psychometric and psychology studies, and to produce them in an easy to read data.frame. The results from describe can be used in graphics functions (e.g., error.crosses).

The range statistics (min, max, range) are most useful for data checking to detect coding errors, and should be found in early analyses of the data.

Although describe will work on data frames as well as matrices, it is important to realize that for data frames, descriptive statistics will be reported only for those variables where this makes sense (i.e., not for alphanumeric data). Variables that are categorical or logical are converted to numeric and then described. These variables are marked with an * in the row name.

In a typical study, one might read the data in from the clipboard (read.clipboard), show the splom plot of the correlations (pairs.panels), and then describe the data.
na.rm=FALSE is equivalent to describe(na.omit(x))

## Value

A data.frame of the relevant statistics:
item name
item number
number of valid cases
mean
standard deviation
trimmed mean (with trim defaulting to .1)
median (standard or interpolated
mad: median absolute deviation (from the median)
minimum
maximum
skew
kurtosis
standard error

## Note

Describe uses either the mean or colMeans functions depending upon whether the data are a data.frame or a matrix. The mean function supplies means for the columns of a data.frame, but the overall mean for a matrix. Mean will throw a warning for non-numeric data, but colMeans stops with non-numeric data. Thus, the describe function uses either mean (for data frames) or colMeans (for matrices). This is true for skew and kurtosi as well.

## Author(s)

```
http://personality-project.org/revelle.html
```

Maintainer: William Revelle [revelle@northwestern.edu](mailto:revelle@northwestern.edu)

## See Also

```
describe.by, skew, kurtosi interp.median, pairs.panels, read.clipboard,
error.crosses
```


## Examples

```
data(sat.act)
describe(sat.act)
describe(sat.act,skew=FALSE)
```

```
describe.by Basic summary statistics by group
```


## Description

Report basic summary statistics by a grouping variable. Useful if the grouping variable is some experimental variable and data are to be aggregated for plotting. Partly a wrapper for by and describe

## Usage

describe.by(x, group, mat=FALSE, ...)

## Arguments

x
group a grouping variable or a list of grouping variables
mat provide a matrix output rather than a list
... parameters to be passed to describe

## Details

To get descriptive statistics for several different grouping variables, make sure that group is a list. In the case of matrix output with multiple grouping variables, the grouping variable values are added to the output.

## Value

A data.frame of the relevant statistics broken down by group:
item name
item number
number of valid cases

## mean

standard deviation
median
mad: median absolute deviation (from the median)
minimum
maximum
skew
standard error

## Author(s)

William Revelle

## See Also

```
describe
```


## Examples

```
data(sat.act)
describe.by(sat.act,sat.act$gender) #just one grouping variable
#describe.by(sat.act,list(sat.act$gender,sat.act$education)) #two grouping variables
des.mat <- describe.by(sat.act$age,sat.act$education,mat=TRUE) #matrix (data.frame) output
des.mat <- describe.by(sat.act$age,list(sat.act$education,sat.act$gender),mat=TRUE) #matrix
```


## Description

Path models are used to describe structural equation models or cluster analytic output. These functions provide the primitives for drawing path models. Used as a substitute for some of the functionality of Rgraphviz.

## Usage

```
diagram()
dia.rect(x, y = NULL, labels = NULL, cex = 1, xlim = c(0, 1), ylim = c(0, 1), ..
        dia.ellipse(x, y = NULL, labels = NULL, cex=1,e.size=.05, xlim=c(0,1), ylim=c(0,1)
        dia.triangle(x, y = NULL, labels =NULL, cex = 1, xlim=c(0,1),ylim=c(0,1),...)
    dia.ellipse1(x,y,e.size=.05,xlim=c (0,1),ylim=c (0,1),...)
    dia.shape(x, y = NULL, labels = NULL, cex = 1, e.size=.05, xlim=c(0,1), ylim=c(0,1)
    dia.arrow(from,to,labels=NULL,scale=1,cex=1,...)
    dia.curve(from,to,labels=NULL,scale=1,...)
    dia.curved.arrow(from,to,labels=NULL,scale=1,...)
    dia.self(location,labels=NULL,scale=.8,side=2,...)
```


## Arguments

X
$y \quad y$ coordinate of a rectangle or ellipse
e.size The size of the ellipse (scaled by the number of variables
labels Text to insert in rectangle, ellipse, or arrow
cex adjust the text size
scale modifies size of rectangle and ellipse as well as the curvature of curves. (For curvature, positive numbers are concave down and to the left
from arrows and curves go from
to arrows and curves go to
location where is the rectangle?
shape Which shape to draw
xlim default ranges
ylim default ranges
side Which side of boxes should errors appear
... Most graphic parameters may be passed here

## Details

These functions are the graphic primitives used by fa.diagram, structure.diagram, omega.diagram, and ICLUST. diagram.
They create rectangles, ellipses or triangles surrounding text, connect them to straight or curved arrows, and can draw an arrow from and to the same rectangle.
Each shape (ellipse, rectangle or triangle) has a left, right, top and bottom and center coordinate that may be used to connect the arrows.
Curves are double-headed arrows.
These functions were developed to get around the infelicities associated with trying to install Rgraphviz and graphviz.
Better documentation will be added as these functions get improved. Currently the functions are just a work around for Rgraphviz.

## Value

Graphic output

## Author(s)

William Revelle

## See Also

The diagram functions that use the dia functions: fa.diagram, structure.diagram, omega.diagram, and ICLUST.diagram.

## Examples

```
#first, show the primitives
xlim=c (-2,10)
ylim=c (0,10)
plot(NA,xlim=xlim,ylim=ylim,main="Demonstration of diagram functions",axes=FALSE,xlab="",yla
ul <- dia.rect(1,9,labels="upper left",xlim=xlim,ylim=ylim)
ml <- dia.rect(1,6,"middle left",xlim=xlim,ylim=ylim)
ll <- dia.rect(1,3,labels="lower left",xlim=xlim,ylim=ylim)
bl <- dia.rect(1,1,"bottom left",xlim=xlim,ylim=ylim)
lr <- dia.ellipse(7,3,"lower right",xlim=xlim,ylim=ylim,e.size=.07)
ur <- dia.ellipse(7,9,"upper right",xlim=xlim,ylim=ylim,e.size=.07)
mr <- dia.ellipse(7,6,"middle right",xlim=xlim,ylim=ylim,e.size=.07)
lm <- dia.triangle(4,1,"Lower Middle",xlim=xlim,ylim=ylim)
br <- dia.rect(9,1,"bottom right",xlim=xlim,ylim=ylim)
dia.curve(from=ul$left,to=bl$left,"double headed",scale=-1)
dia.arrow(from=lr,to=ul,labels="right to left")
dia.arrow(from=ul,to=ur,labels="left to right")
dia.curved.arrow(from=lr,to=ll,labels ="right to left")
dia.curved.arrow(to=ur,from=ul,labels ="left to right")
dia.curve(ll$top,ul$bottom,"right") #for rectangles, specify where to point
```

```
dia.curve(ll$top,ul$bottom,"left",scale=-1) #for rectangles, specify where to point
dia.curve(mr,ur,"up") #but for ellipses, you may just point to it.
dia.curve(mr,lr,"down")
dia.curve (mr,ur,"up")
dia.curved.arrow(mr,ur,"up") #but for ellipses, you may just point to it.
dia.curved.arrow(mr,lr,"down") #but for ellipses, you may just point to it.
dia.curved.arrow(ur$right,mr$right,"3")
dia.curve(ml,mr,"across")
dia.curve(ur,lr,"top down")
dia.curved.arrow(br$top,lr$bottom,"up")
dia.curved.arrow(bl,br,"left to right")
dia.curved.arrow(br,bl,"right to left",scale=-1)
dia.arrow(bl,ll$bottom)
dia.curved.arrow(ml,ll$right)
dia.curved.arrow(mr,lr$top)
#now, put them together in a factor analysis diagram
v9 <- sim.hierarchical()
f3 <- fa(v9,3,rotate="cluster")
fa.diagram(f3, error=TRUE,side=3)
```

draw.tetra $\quad$| Draw a correlation ellipse and two normal curves to demonstrate |
| :--- |
| tetrachoric correlation |

## Description

A graphic of a correlation ellipse divided into 4 regions based upon x and y cutpoints on two normal distributions. This is also an example of using the layout function.

## Usage

draw.tetra(r, t1, t2,shade=TRUE)

## Arguments

| $r$ | the underlying Pearson correlation defines the shape of the ellipse |
| :--- | :--- |
| $t 1$ | $X$ is cut at tau |
| $t 2$ | $Y$ is cut at Tau |
| shade | shade the diagram (default is TRUE) |

## Details

A graphic demonstration of the tetrachoric correlation. Used for teaching purposes. The default values are for a correlation of .5 with cuts at 1 and 1 . Any other values are possible. The code is also a demonstration of how to use the layout function for complex graphics using base graphics.

## Author(s)

William Revelle

## See Also

```
tetrachoric,irt.fa
```


## Examples

```
draw.tetra(.5,1,1)
draw.tetra(.8,2,1)
```

eigen.loadings Convert eigen vectors and eigen values to the more normal (for psy-
chologists) component loadings

## Description

The default procedures for principal component returns values not immediately equivalent to the loadings from a factor analysis. eigen.loadings translates them into the more typical metric of eigen vectors multiplied by the squareroot of the eigenvalues. This lets us find pseudo factor loadings if we have used princomp or eigen.
If we use principal to do our principal components analysis, then we do not need this routine.

## Usage

eigen.loadings(x)

## Arguments

x
the output from eigen or a list of class princomp derived from princomp

## Value

A matrix of Principal Component loadings more typical for what is expected in psychometrics. That is, they are scaled by the square root of the eigenvalues.

## Note

Useful for SAPA analyses

## Author(s)

```
< revelle@northwestern.edu >
http://personality-project.org/revelle.html
```


## Examples

```
x <- eigen(Harman74.cor$cov)
x$vectors[1:8,1:4] #as they appear from eigen
y <- princomp(covmat=Harman74.cor$cov)
y$loadings[1:8,1:4] #as they appear from princomp
eigen.loadings(x)[1:8,1:4] # rescaled by the eigen values
```

ellipses

Plot data and 1 and 2 sigma correlation ellipses

## Description

For teaching correlation, it is useful to draw ellipses around the mean to reflect the correlation. This variation of the ellipse function from John Fox's car package does so. Input may be either two vectors or a matrix or data.frame. In the latter cases, if the number of variables $>2$, then the ellipses are done in the pairs. panels function. Ellipses may be added to existing plots. The minkowski function is included as a generalized ellipse.

## Usage

```
ellipses(x, y = NULL, add = FALSE, smooth=TRUE, lm=FALSE,data=TRUE, n = 2,span=2/3,
minkowski(r=2,add=FALSE,main=NULL,xl=1,yl=1)
```


## Arguments

| x | a vector,matrix, or data.frame |
| :---: | :---: |
| Y | Optional second vector |
| add | Should a new plot be created, or should it be added to? |
| smooth | smooth $=$ TRUE $->$ draw a loess fit |
| 1 m | $1 \mathrm{~m}=$ TRUE $->$ draw the linear fit |
| data | data=TRUE implies draw the data points |
| n | Should 1 or 2 ellipses be drawn |
| span | averaging window parameter for the lowess fit |
| iter | iteration parameter for lowess |
| col | color of ellipses (default is red |
| xlab | label for the x axis |
| ylab | label for the y axis |
| . . | Other parameters for plotting |
| r | $\mathrm{r}=1$ draws a city block, $\mathrm{r}=2$ is a Euclidean circle, $\mathrm{r}>2$ tends towards a square |
| main | title to use when drawing Minkowski circles |
| xl | stretch the x axis |
| yl | stretch the y axis |

## Details

Ellipse dimensions are calculated from the correlation between the x and y variables and are scaled as $\operatorname{sqrt}(1+r)$ and $\operatorname{sqrt}(1-r)$.

## Value

A single plot (for 2 vectors or data frames with fewer than 3 variables. Otherwise a call is made to pairs.panels.

## Note

Adapted from John Fox's ellipse and data.ellipse functions.

## Author(s)

William Revelle

## References

Galton, Francis (1888), Co-relations and their measurement. Proceedings of the Royal Society. London Series, 45, 135-145.

## See Also

```
pairs.panels
```


## Examples

```
data(galton)
ellipses(galton,lm=TRUE)
ellipses(galton$parent,galton$child,xlab="Mid Parent Height",ylab="Child Height") #input are
data(sat.act)
ellipses(sat.act) #shows the pairs.panels ellipses
minkowski(2,main="Minkowski circles")
minkowski(1,TRUE)
minkowski(4,TRUE)
```

epi.bfi 13 personality scales from the Eysenck Personality Inventory and Big
5 inventory

## Description

A small data set of 5 scales from the Eysenck Personality Inventory, 5 from a Big 5 inventory, a Beck Depression Inventory, and State and Trait Anxiety measures. Used for demonstrations of correlations, regressions, graphic displays.

## Usage

```
data(epi.bfi)
```


## Format

A data frame with 231 observations on the following 13 variables.

```
epiE EPI Extraversion
epiS EPI Sociability (a subset of Extraversion items
epi Imp EPI Impulsivity (a subset of Extraversion items
epilie EPI Lie scale
epiNeur EPI neuroticism
bfagree Big 5 inventory (from the IPIP) measure of Agreeableness
bfcon Big 5 Conscientiousness
bfext Big 5 Extraversion
bfneur Big 5 Neuroticism
bfopen Big 5 Openness
bdi Beck Depression scale
traitanx Trait Anxiety
stateanx State Anxiety
```


## Details

Self report personality scales tend to measure the "Giant 2" of Extraversion and Neuroticism or the "Big 5" of Extraversion, Neuroticism, Agreeableness, Conscientiousness, and Openness. Here is a small data set from Northwestern University undergraduates with scores on the Eysenck Personality Inventory (EPI) and a Big 5 inventory taken from the International Personality Item Pool.

## Source

Data were collected at the Personality, Motivation, and Cognition Lab (PMCLab) at Northwestern by William Revelle)

## References

http://personality-project.org/pmc.html

## Examples

```
data(epi.bfi)
pairs.panels(epi.bfi[,1:5])
describe(epi.bfi)
```


## Description

One of the many functions in R to plot means and confidence intervals. Can be done using barplots if desired. Can also be combined with such functions as boxplot to summarize distributions. Means and standard errors are calculated from the raw data using describe. Alternatively, plots of means +/- one standard deviation may be drawn.

## Usage

error.bars(x,stats=NULL, ylab = "Dependent Variable",xlab="Independent Variable",

## Arguments

| x | A data frame or matrix of raw data |
| :--- | :--- |
| stats | Alternatively, a data.frame of descriptive stats from (e.g., describe) |
| ylab | y label |
| xlab | x label |
| main | title for figure <br> ylim <br> alpha |
| if specified, the limits for the plot, otherwise based upon the data <br> alpha level of confidence interval - defaults to 95\% confidence interval <br> if TRUE, draw one standard deviation instead of standard errors at the alpha <br> level |  |
| labels | X axis label |
| pos | where to place text: below, left, above, right |
| arrow.len | How long should the top of the error bars be? |
| arrow. col | What color should the error bars be? |
| add | add=FALSE, new plot, add=TRUE, just points and error bars <br> bars |
| bithin | bars=TRUE will draw a bar graph if you really want to do that |
| should the error variance of a variable be corrected by 1-SMC? |  |

## Details

Drawing the mean $+/-$ a confidence interval is a frequently used function when reporting experimental results. By default, the confidence interval is 1.96 standard errors.
If within=TRUE, the error bars are corrected for the correlation with the other variables by reducing the variance by a factor of ( $1-\mathrm{smc}$ ). This allows for comparisons between variables.

The error bars are normally calculated from the data using the describe function. If, alternatively, a matrix of statistics is provided with column headings of values, means, and se, then those values will be used for the plot (using the stats option).
If sd is TRUE, then the error bars will represent one standard deviation from the mean rather than be a function of alpha and the standard errors.

## Value

Graphic output showing the means $+x$
These confidence regions are based upon normal theory and do not take into account any skew in the variables. More accurate confidence intervals could be found by resampling.

## Author(s)

William Revelle

## See Also

error.crosses for two way error bars, error.bars.by for error bars for different groups
In addition, as pointed out by Jim Lemon on the R-help news group, error bars or confidence intervals may be drawn using

| function <br> bar.err | package <br> (agricolae) |
| :--- | :--- |
| plotCI | (gplots) |
| xYplot | (Hmisc) |
| dispersion | (plotrix) |
| plotCI | (plotrix) |

For advice why not to draw bar graphs with error bars, see http://biostat.mc.vanderbilt. edu/wiki/Main/DynamitePlots

## Examples

```
x <- replicate(20,rnorm(50))
boxplot(x, notch=TRUE,main="Notched boxplot with error bars")
error.bars(x, add=TRUE)
abline(h=0)
error.bars(attitude, alpha=.5,main="50 percent confidence limits") #another example
error.bars(attitude,bar=TRUE) #show the use of bar graphs
#combine with a strip chart and boxplot
stripchart(attitude,vertical=TRUE,method="jitter",jitter=.1,pch=19,main="Stripchart with 95
boxplot(attitude, add=TRUE)
error.bars(attitude, add=TRUE, arrow.len=. 2)
```

```
#use statistics from somewhere else
my.stats <- data.frame(values=c (1,4,8),means=c (10, 12,18), se=c (2, 3,5))
error.bars(stats=my.stats,type="b",main="data with confidence intervals")
#use describe and then plot the results
ds <- describe(attitude)
error.bars(stats=ds)
```

error.bars.by Plot means and confidence intervals for multiple groups

## Description

One of the many functions in R to plot means and confidence intervals. Meant mainly for demonstration purposes for showing the probabilty of replication from multiple samples. Can also be combined with such functions as boxplot to summarize distributions. Means and standard errors for each group are calculated using describe.by.

## Usage

error.bars.by (x,group,by.var=FALSE,x.cat=TRUE,ylab =NULL, xlab=NULL, main=NULL,ylim=

## Arguments

| x | A data frame or matrix |
| :--- | :--- |
| group | A grouping variable |
| by.var | A different line for each group (default) or each variable |
| x.cat | Is the grouping variable categorical (TRUE) or continuous (FALSE |
| ylab | y label |
| xlab | x label |
| main | title for figure |
| ylim | if specified, the limits for the plot, otherwise based upon the data |
| alpha | alpha level of confidence interval. Default is 1- alpha =95\% confidence interval |
| sd | sd=TRUE will plot Standard Deviations instead of standard errors |
| labels | X axis label |
| pos | where to place text: below, left, above, right |
| arrow.len | How long should the top of the error bars be? |
| add | add=FALSE, new plot, add=TRUE, just points and error bars |
| bars | Draw a barplot with error bars rather than a simple plot of the means <br> within |
| Should the s.e. be corrected by the correlation with the other variables? |  |
| colors | groups will be plotted in different colors (mod n.groups) |
| lty | line type may be specified in the case of not plotting by variables <br> other parameters to pass to the plot function, e.g., typ="b" to draw lines, lty="dashed" |
| m | to draw dashed lines |

## Details

Drawing the mean $+/$ - a confidence interval is a frequently used function when reporting experimental results. By default, the confidence interval is 1.96 standard errors.

This function was originally just a wrapper for error.bars but has been written to allow groups to be organized either as the x axis or as separate lines.

If desired, a barplot with error bars can be shown. Many find this type of plot to be uninformative (e.g., http://biostat.mc.vanderbilt.edu/DynamitePlots )

## Value

Graphic output showing the means $+\mathrm{x} \%$ confidence intervals for each group. For $\mathrm{ci}=1.96$, and normal data, this will be the $95 \%$ confidence region. For ci=1, the $68 \%$ confidence region.

These confidence regions are based upon normal theory and do not take into account any skew in the variables. More accurate confidence intervals could be found by resampling.

## See Also

See Also as error.crosses, error.bars

## Examples

```
data(sat.act)
error.bars.by(sat.act[1:4],sat.act$gender)
error.bars.by(sat.act[5:6],sat.act$gender,bars=TRUE,labels=c("male","female"),main="SAT V ar
error.bars.by(sat.act[5:6],sat.act$education,bars=TRUE,xlab="Education",main="95 percent cor
error.bars.by(sat.act[5:6],sat.act$education,TRUE, xlab="Education") #plot SAT V and SAT Q
```


## Description

Given two vectors of data, plot the means and show standard errors in both X and Y directions.

## Usage

error.crosses $(x, y, l a b e l s=N U L L, m a i n=N U L L, x l i m=N U L L, y l i m=N U L L, x l a b=N U L L, y l a b=N U L L, p c$

## Arguments

X
Y
labels
main
xlim
ylim
$\mathrm{xlab} \quad$ label for x axis - grouping variable 1
Ylab label for y axis - grouping variable 2
pos Labels are located where with respect to the mean?
offset Labels are then offset from this location
arrow.len Arrow length
alpha alpha level of error bars
sd if sd is TRUE, then draw means +/-1 sd)
... Other parameters for plot

## Details

For an example of two way error bars describing the effects of mood manipulations upon positive and negative affect, see http://personality-project.org/revelle/publications/ happy-sad-appendix/FIG.A-6.pdf
The second example shows how error crosses can be done for multiple variables where the grouping variable is found dynamically.

## Author(s)

William Revelle
[revelle@northwestern.edu](mailto:revelle@northwestern.edu)

## See Also

To draw error bars for single variables error.bars, or by groups error.bars.by, or to find descriptive statistics describe or descriptive statistics by a grouping variable describe.by

## Examples

```
desc <- describe(attitude)
x <- desc[1,]
y <- desc[2,]
plot(x$mean,y$mean,xlab=rownames(x),ylab=rownames(y)) #in graphics window
error.crosses(x,y) #in graphics window
#now for a bit more complicated plotting
desc <- describe.by(attitude,(attitude[,7]>41)) #select a high and low group
g1 <- desc$'FALSE'
```

```
g2 <- desc$'TRUE'
plot(g1$mean,g2$mean,xlab = "Low Advance",ylab="High Advance",xlim=c(30,80),ylim=c(50,80))
error.crosses (g1,g2,labels=rownames (g1),pos=rep (1, 7) )
title("Attitudes grouped by high and low scores on Advance")
```

| fa | MinRes (minimum residual) Factor analysis as well as Factor Analysis |
| :--- | :--- |
| by Principal Axis, Weighted Least Squares or Maximum Likelihood |  |

## Description

Among the many ways to do latent variable factor analysis, one of the better is to use Ordinary Least Squares to find the minimum residual (minres) solution. This produces solutions very similar to maximum likelihood even for badly behaved matrices. A variation on minres is to do weighted least squares. Perhaps the most conventional technique is principal axes. An eigen value decomposition of a correlation matrix is done and then the communalities for each variable are estimated by the first n factors. These communalities are entered onto the diagonal and the procedure is repeated until the $\operatorname{sum}(\operatorname{diag}(r))$ does not vary. Yet another estimate procedure is maximum likelihood. For well behaved matrices, maximum likelihood factor analysis (either in the fa or in the factanal fuction) is probably preferred.

## Usage

```
fa(r,nfactors=1,n.obs = NA, rotate="oblimin", scores=FALSE, residuals=FALSE, SMC=TF
factor.pa(r, nfactors=1, residuals = FALSE, rotate = "varimax",n.obs = NA,
scores = FALSE,SMC=TRUE, missing=FALSE,impute="median",min.err = 0.001, digits = 2,
factor.minres(r, nfactors=1, residuals = FALSE, rotate = "varimax",n.obs = NA,
scores = FALSE,SMC=TRUE, missing=FALSE,impute="median",min.err = 0.001, digits = 2,
factor.wls(r,nfactors=1,residuals=FALSE,rotate="varimax",n.obs = NA,
scores=FALSE,SMC=TRUE,missing=FALSE,impute="median", min.err = .001,digits=2,max.it
```


## Arguments

$r$
nfactors
n.obs
rotate

A correlation matrix or a raw data matrix. If raw data, the correlation matrix will be found using pairwise deletion.
Number of factors to extract, default is 1
Number of observations used to find the correlation matrix if using a correlation matrix. Used for finding the goodness of fit statistics.
"none", "varimax", "quartimax", "bentlerT", and "geominT" are orthogonal rotations. "promax", "oblimin", "simplimax", "bentlerQ, and "geominQ" or "cluster" are possible rotations or transformations of the solution. The default is to do a oblimin transformation, although prior versions defaulted to varimax.

| residuals | Should the residual matrix be shown |
| :---: | :---: |
| scores | If TRUE, estimate factor scores |
| SMC | Use squared multiple correlations (SMC=TRUE) or use 1 as initial communality estimate. Try using 1 if imaginary eigen values are reported. |
| covar | if covar is TRUE, factor the covariance matrix, otherwise factor the correlation matrix |
| missing | if scores are TRUE, and missing=TRUE, then impute missing values using either the median or the mean |
| impute | "median" or "mean" values are used to replace missing values |
| min.err | Iterate until the change in communalities is less than min.err |
| digits | How many digits of output should be returned- deprecated - now specified in the print function |
| max.iter | Maximum number of iterations for convergence |
| symmetric | symmetric=TRUE forces symmetry by just looking at the lower off diagonal values |
| warnings | warnings=TRUE => warn if number of factors is too many |
| fm | factoring method $f m=$ "minres" will do a minimum residual (OLS), $\mathrm{fm}=$ "wls" will do a weighted least squares (WLS) solution, $\mathrm{fm}=$ "gls" does a generalized weighted least squares (GLS), fm="pa" will do the principal factor solution, $\mathrm{fm}=\mathrm{ml}$ " will do a maximum likelihood factor analysis |
| alpha | alpha level for the confidence intervals for RMSEA |
|  | additional parameters, specifically, keys may be passed if using the target rotation, or delta if using geominQ, or whether to normalize if using Varimax |

## Details

Factor analysis is an attempt to approximate a correlation or covariance matrix with one of lesser rank. The basic model is that ${ }_{n} R_{n} \approx_{n} F_{k k} F_{n}^{\prime}+U^{2}$ where k is much less than n . There are many ways to do factor analysis, and maximum likelihood procedures are probably the most preferred (see factanal ). The existence of uniquenesses is what distinguishes factor analysis from principal components analysis (e.g., principal). If variables are thought to represent a "true" or latent part then factor analysis provides an estimate of the correlations with the latent factor(s) representing the data. If variables are thought to be measured without error, then principal components provides the most parsimonious description of the data.
The fa function will do factor analyses using one of four different algorithms: minimum residual (minres), principal axes, weighted least squares, or maximum likelihood.
Principal axes factor analysis has a long history in exploratory analysis and is a straightforward procedure. Successive eigen value decompositions are done on a correlation matrix with the diagonal replaced with diag ( FF ') until sum( $\operatorname{diag}\left(\mathrm{FF}^{\prime}\right)$ ) does not change (very much). The current limit of max.iter $=50$ seems to work for most problems, but the Holzinger-Harmon 24 variable problem needs about 203 iterations to converge for a 5 factor solution.
Principal axes may be used in cases when maximum likelihood solutions fail to converge.
A problem in factor analysis is to find the best estimate of the original communalities. Using the Squared Multiple Correlation (SMC) for each variable will underestimate the communalities, using

1s will over estimate. By default, the SMC estimate is used. In either case, iterative techniques will tend to converge on a stable sollution. If, however, a solution fails to be achieved, it is useful to try again using ones (SMC =FALSE).
The algorithm does not attempt to find the best (as defined by a maximum likelihood criterion) solution, but rather one that converges rapidly using successive eigen value decompositions. The maximum likelihood criterion of fit and the associated chi square value are reported, and will be worse than that found using maximum likelihood procedures.

The minimum residual (minres) solution is an unweighted least squares solution that takes a slightly different approach. It uses the opt im function and adjusts the diagonal elements of the correlation matrix to mimimize the squared residual when the factor model is the eigen value decomposition of the reduced matrix. MINRES and PA will both work when ML will not, for they can be used when the matrix is singular. At least on a number of test cases, the MINRES solution is slightly more similar to the ML solution than is the PA solution. To a great extent, the minres and wls solutions follow ideas in the factanal function.

The weighted least squares (wls) solution weights the residual matrix by $1 /$ diagonal of the inverse of the correlation matrix. This has the effect of weighting items with low communalities more than those with high commumnalities. I
The generalized least squares (gls) solution weights the residual matrix by the inverse of the correlation matrix. This has the effect of weighting those variables with low communalities even more than those with high communalities.

The maximum likelihood solution takes yet another approach and finds those communality values that minimize the chi square goodness of fit test. The $\mathrm{fm}=\mathrm{mml}$ " option provides a maximum likelihood solution following the procedures used in factanal but does not provide all the extra features of that function.

Test cases comparing the output to SPSS suggest that the PA algorithm matches what SPSS calls uls, and that the wls solutions are equivalent in their fits. The wls and gls solutions have slightly larger eigen values, but slightly worse fits of the off diagonal residuals than do the minres or maximum likelihood solutions.
Although for items, it is typical to find factor scores by scoring the salient items (using, e.g.,score.items factor scores can be estimated by regression. There are multiple approaches that are possible (see Grice, 2001) and the one taken here is Thurstone's least squares regression where the weights are found by $\left.W=R^{( }-1\right) S$ where R is the correlation matrix of the variables ans S is the structure matrix.

For dichotomous items, it is recommended to analyze the tetrachoric correlations rather than the Pearson correlations.
Of the various rotation/transformation options, varimax, Varimax, quartimax, bentlerT and geominT do orthogonal rotations. Promax transforms obliquely with a target matix equal to the varimax solution. oblimin, quartimin, simplimax, bentlerQ, and geominQ are oblique transformations. Most of these are just calls to the GPArotation package. The "cluster" option does a targeted rotation to a structure defined by the cluster representation of a varimax solution. With the optional "keys" parameter, the "target" option will rotate to a target supplied as a keys matrix. (See target .rot.)
There are two varimax rotation functions. One, Varimax, in the GPArotation package does not by default apply Kaiser normalization. The other, varimax, in the stats package, does. It appears that the two rotation functions produce slightly different results even when normalization is set. For consistency with the other rotation functions, Varimax is probably preferred.

## Value

| values | Eigen values of the common factor solution |
| :--- | :--- |
| e.values | Eigen values of the original matrix |
| communality | Communality estimates for each item. These are merely the sum of squared <br> factor loadings for that item. |
| rotation | which rotation was requested? |
| n.obs | number of observations specified or found |
| loadings | An item by factor loading matrix of class "loadings" Suitable for use in other <br> programs (e.g., GPA rotation or factor2cluster. To show these by sorted order, |
| use print.psych with sort=TRUE |  |


| R2 | The multiple R square between the factors and factor score estimates, if they <br> were to be found. (From Grice, 2001). Derived from R2 is is the minimum <br> correlation between any two factor estimates = 2R2-1. |
| :--- | :--- |
| r.scores | The correlations of the factor score estimates, if they were to be found. |
| weights | The beta weights to find the factor score estimates |
| valid | The validity coffiecient of course coded (unit weighted) factor score estimates <br> (From Grice, 2001) |
| score.cor | The correlation matrix of course coded (unit weighted) factor score estimates, if <br> they were to be found, based upon the loadings matrix rather than the weights <br> matrix. |

## Note

Thanks to Erich Studerus for some very helpful suggestions about various rotation and factor scoring algorithms, and to Gumundur Arnkelsson for suggestions about factor scores for singular matrices.

## Author(s)

William Revelle

## References

Gorsuch, Richard, (1983) Factor Analysis. Lawrence Erlebaum Associates.
Grice, James W. (2001), Computing and evaluating factor scores. Psychological Methods, 6, 430450

Harman, Harry and Jones, Wayne (1966) Factor analysis by minimizing residuals (minres), Psychometrika, 31, 3, 351-368.
Revelle, William. (in prep) An introduction to psychometric theory with applications in R. Springer. Working draft available at http://personality-project.org/r/book/

## See Also

principal, VSS, ICLUST

## Examples

```
#using the Harman 24 mental tests, compare a principal factor with a principal components so
pc <- principal(Harman74.cor$cov,4,rotate="varimax")
pa <- fa(Harman74.cor$cov,4,fm="pa" ,rotate="varimax") #principal axis
uls <- fa(Harman74.cor$cov,4,rotate="varimax") #unweighted least squares is minres
wls <- fa(Harman74.cor$cov,4,fm="wls") #weighted least squares
#to show the loadings sorted by absolute value
print(uls,sort=TRUE)
#then compare with a maximum likelihood solution using factanal
mle <- factanal(covmat=Harman74.cor$cov,factors=4)
```

```
factor.congruence(list(mle,pa,pc,uls,wls))
#note that the order of factors and the sign of some of factors differ
#finally, compare the unrotated factor, ml, uls, and wls solutions
wls <- factor.wls(Harman74.cor$cov,4,rotate="none")
pa <- factor.pa(Harman74.cor$cov,4,rotate="none")
mle <- factanal(factors=4,covmat=Harman74.cor$cov,rotation="none")
uls <- factor.minres(Harman74.cor$cov,4,rotate="none")
factor.congruence(list(mle,pa,uls,wls))
#note that the order of factors and the sign of some of factors differ
```

```
#an example of where the ML and PA and MR models differ is found in Thurstone.33.
#compare the first two factors with the 3 factor solution
data(bifactor)
Thurstone.33 <- as.matrix(Thurstone.33)
mle2 <- factanal(covmat=Thurstone.33, factors=2, rotation="none")
mle3 <- factanal(covmat=Thurstone.33, factors=3 ,rotation="none")
pa2 <- factor.pa(Thurstone.33,2,rotate="none")
pa3 <- factor.pa(Thurstone.33,3,rotate="none")
mr2 <- fa(Thurstone.33,2,rotate="none")
mr3 <- fa(Thurstone.33,3,rotate="none")
factor.congruence(list(mle2,mle3,pa2,pa3,mr2,mr3))
```

fa.diagram Graph factor loading matrices

## Description

Factor analysis or principal components analysis results are typically interpreted in terms of the major loadings on each factor. These structures may be represented as a table of loadings or graphically, where all loadings with an absolute value $>$ some cut point are represented as an edge (path).

## Usage

```
fa.diagram(fa.results,Phi=NULL, sort=TRUE,labels=NULL, cut=.3,simple=TRUE,errors=FALS
        digits=1,e.size=.05,rsize=.15,side=2,main="Factor Analysis",cex=NULL, ...)
fa.graph(fa.results,out.file=NULL,labels=NULL, cut=.3, simple=TRUE,
    size=c(8,6), node.font=c("Helvetica", 14),
        edge.font=c("Helvetica", 10), rank.direction=c("RL","TB","LR","BT"), digits=1,n
```


## Arguments

fa.results
The output of factor analysis, principal components analysis, or ICLUST analysis. May also be a factor loading matrix from anywhere.

```
Phi Normally not specified (it is is found in the FA, pc, or ICLUST, solution), this
                may be given if the input is a loadings matrix.
out.file If it exists, a dot representation of the graph will be stored here (fa.graph)
labels Variable labels
cut Loadings with abs(loading) > cut will be shown
simple Only the biggest loading per item is shown
size graph size
sort sort the factor loadings before showing the diagram
errors include error estimates (as arrows)
e.size size of ellipses
rsize size of rectangles
side on which side should error arrows go?
cex modify font size
node.font what font should be used for nodes in fa.graph
edge.font what font should be used for edges in fa.graph
rank.direction
parameter passed to Rgraphviz- which way to draw the graph
digits Number of digits to show as an edgelable
main Graphic title
graphviz Should we try to use Rgraphviz for output?
... other parameters
```


## Details

Path diagram representations have become standard in confirmatory factor analysis, but are not yet common in exploratory factor analysis. Representing factor structures graphically helps some people understand the structure.
fa.diagram does not use Rgraphviz and is the preferred function.
In fa.graph, although a nice graph is drawn for the orthogonal factor case, the oblique factor drawing is acceptable, but is better if cleaned up outside of R or done using fa.diagram.
The normal input is taken from the output of either fa or ICLUST. It is also possible to just give a factor loading matrix as input. In this case, supplying a Phi matrix of factor correlations is also possible.
To specify the model for a structural equation confirmatory analysis of the results, use structure. diagram instead.

## Value

fa.diagram: A path diagram is drawn without using Rgraphviz. This is probably the more useful function.
fa.graph: A graph is drawn using rgraphviz. If an output file is specified, the graph instructions are also saved in the dot language.

## Note

fa.graph requires Rgraphviz. Because there are occasional difficulties installing Rgraphviz from Bioconductor in that some libraries are misplaced and need to be relinked, it is probably better to use fa.diagram.

## Author(s)

William Revelle

## See Also

omega.graph, ICLUST.graph, structure.diagram to convert the factor diagram to sem modeling code.

## Examples

```
test.simple <- fa(item.sim(16),2,rotate="oblimin")
#if(require(Rgraphviz)) {fa.graph(test.simple) }
fa.diagram(test.simple)
data(bifactor)
f3 <- fa(Thurstone,3,rotate="cluster")
fa.diagram(f3,cut=.4, digits=2)
f3l <- f3$loadings
fa.diagram(f3l,main="input from a matrix")
Phi <- f3$Phi
fa.diagram(f3l,Phi=Phi,main="Input from a matrix")
fa.diagram(ICLUST(Thurstone,2,title="Two cluster solution of Thurstone"),main="Input from IC
```

| fa.parallel | Scree plots of data or correlation matrix compared to random "paral- <br> lel" matrices |
| :--- | :--- |

## Description

One way to determine the number of factors or components in a data matrix or a correlation matrix is to examine the "scree" plot of the successive eigenvalues. Sharp breaks in the plot suggest the appropriate number of components or factors to extract. "Parallel" analyis is an alternative technique that compares the scree of factors of the observed data with that of a random data matrix of the same size as the original.

## Usage

fa.parallel(x, n.obs = NULL,fm="minres", fa="both", main = "Parallel Analysis Scree

## Arguments

X
n.obs n.obs=0 implies a data matrix/data.frame. Otherwise, how many cases were used to find the correlations.
fm What factor method to use. (minres, ml, uls, wls, gls, pa) See fa for details.
fa show the eigen values for a principal components ( $\mathrm{fa}=\mathrm{pc} \mathrm{pc}$ ) or a principal axis factor analysis (fa="fa") or both principal components and principal factors (fa="both")
main a title for the analysis
ntrials Number of simulated analyses to perform
error.bars $\quad$ Should error.bars be plotted (default $=$ FALSE)
smc
ylabel Label for the y axis - defaults to "eigen values of factors and components", can be made empty to show many graphs
show. legend the default is to have a legend. For multiple panel graphs, it is better to not show the legend

## Details

Cattell's "scree" test is one of most simple tests for the number of factors problem. Horn's (1965) "parallel" analysis is an equally compelling procedure. Other procedures for determining the most optimal number of factors include finding the Very Simple Structure (VSS) criterion (VSS) and Velicer's MAP procedure (included in VSS). fa.parallel plots the eigen values for a principal components and the factor solution (minres by default) and does the same for random matrices of the same size as the original data matrix. For raw data, the random matrices are 1) a matrix of univariate normal data and 2) random samples (randomized across rows) of the original data.
The means of (ntrials) random solutions are shown. Error bars are usually very small and are suppressed by default but can be shown if requested.

Alternative ways to estimate the number of factors problem are discussed in the Very Simple Structure (Revelle and Rocklin, 1979) documentation (VSS) and include Wayne Velicer's MAP algorithm (Veicer, 1976).
Parallel analysis for factors is actually harder than it seems, for the question is what are the appropriate communalities to use. If communalities are estimated by the Squared Multiple Correlation (SMC) smc, then the eigen values of the original data will reflect major as well as minor factors (see sim.minor to simulate such data). Random data will not, of course, have any structure and thus the number of factors will tend to be biased upwards by the presence of the minor factors.
By default, fa.parallel estimates the communalities based upon a one factor minres solution. Although this will underestimate the communalities, it does seem to lead to better solutions on simulated or real (e.g., the bfi or Harman74) data sets.
For comparability with other algorithms (e.g, the paran function in the paran package), setting $\mathrm{smc}=$ TRUE will use smcs as estimates of communalities. This will tend towards identifying more factors than the default option.

## Value

A plot of the eigen values for the original data, ntrials of resampling of the original data, and of a equivalent size matrix of random normal deviates. If the data are a correlation matrix, specify the number of observations.
Also returned (invisibly) are:
fa.values The eigen values of the factor model for the real data.
fa.sim The descriptive statistics of the simulated factor models.
pc.values The eigen values of a principal components of the real data.
pc.sim The descriptive statistics of the simulated principal components analysis.
nfact $\quad$ Number of factors with eigen values > eigen values of random data
ncomp $\quad$ Number of components with eigen values $>$ eigen values of random data
Printing the results will show the eigen values of the original data that are greater than simulated values.

## Author(s)

William Revelle

## References

Floyd, Frank J. and Widaman, Keith. F (1995) Factor analysis in the development and refinement of clinical assessment instruments. Psychological Assessment, 7(3):286-299, 1995.
Horn, John (1965) A rationale and test for the number of factors in factor analysis. Psychometrika, 30, 179-185.
Humphreys, Lloyd G. and Montanelli, Richard G. (1975), An investigation of the parallel analysis criterion for determining the number of common factors. Multivariate Behavioral Research, 10, 193-205.
Revelle, William and Rocklin, Tom (1979) Very simple structure - alternative procedure for estimating the optimal number of interpretable factors. Multivariate Behavioral Research, 14(4):403-414.
Velicer, Wayne. (1976) Determining the number of components from the matrix of partial correlations. Psychometrika, 41(3):321-327, 1976.

## See Also

```
fa,VSS,VSS.plot,VSS.parallel,sim.minor
```


## Examples

```
test.data <- Harman74.cor$cov
fa.parallel(test.data,n.obs=145)
set.seed(123)
minor <- sim.minor(24,4,400) #4 large and 12 minor factors
fa.parallel(minor$observed) #shows 4 factors -- compare with
fa.parallel(minor$observed,smc=TRUE) #which shows 8 factors
```

fa.sort Sort factor analysis or principal components analysis loadings

## Description

Although the print.psych function will sort factor analysis loadings, sometimes it is useful to do this outside of the print function. fa.sort takes the output from the fa or principal functions and sorts the loadings for each factor. Items are located in terms of their greatest loading.

## Usage

fa.sort(fa.results)

## Arguments

fa.results The output from a factor analysis or principal components analysis using fa or principal.

## Details

The fa.results\$loadings are replaced with sorted loadings.

## Value

A sorted factor analysis, principal components analysis, or omega loadings matrix.
These sorted values are used internally by the various diagram functions.

## Author(s)

William Revelle

## See Also

See Also as fa,print.psych, fa.diagram,

## Examples

```
test.simple <- fa(item.sim(16),2,rotate="oblimin")
fa.sort(test.simple)
```

factor. congruence Coefficient of factor congruence

## Description

Given two sets of factor loadings, report their degree of congruence (vector cosine).

## Usage

factor.congruence( $x, y=N U L L$, digits=2)

## Arguments

$x \quad$ A matrix of factor loadings or a list of matrices of factor loadings
$y \quad$ A second matrix of factor loadings (if $x$ is a list, then $y$ may be empty)
digits Round off to digits

## Details

Find the coefficient of factor congruence between two sets of factor loadings.
Factor congruences are the cosines of pairs of vectors defined by the loadings matrix and based at the origin. Thus, for loadings that differ only by a scaler (e.g. the size of the eigen value), the factor congruences will be 1 .
It is an interesting exercise to compare factor congruences with the correlations of factor loadings. Factor congruences are based upon the raw cross products, while correlations are based upon centered cross products. That is, correlations of factor loadings are cosines of the vectors based at the mean loading for each factor.

Input may either be matrices or factor analysis or principal components analyis output (which includes a loadings object), or a mixture of the two.
To compare more than two solutions, x may be a list of matrices, all of which will be compared.

## Value

A matrix of factor congruences.

## Author(s)

```
<revelle@northwestern.edu>
http://personality-project.org/revelle.html
```


## References

Gorsuch, Richard, (1983) Factor Analysis. Lawrence Erlebaum Associates.
Revelle, W. (In preparation) An Introduction to Psychometric Theory with applications in R (http : //personality-project.org/r/book/)

## See Also

```
principal,factor.pa
```


## Examples

```
#fa <- factanal(x,4,covmat=Harman74.cor$cov)
#pc <- principal(Harman74.cor$cov,4)
#pa <- factor.pa(Harman74.cov$cor,4)
#factor.congruence(fa,pc)
#
# Factor1 Factor2 Factor3 Factor4
#PC1 1.00 0.60 0.45 0.55
#PC2 0.44 0.49 1.00 0.56
#PC3 0.54 0.99 0.44 0.55
#PC4 0.47 0.52 0.48 0.99
# pa <- factor.pa(Harman74.cor$cov,4)
# factor.congruence(fa,pa)
# PA1 PA3 PA2 PA4
#Factor1 1.00 0.61 0.46 0.55
#Factor2 0.61 1.00 0.50 0.60
#Factor3 0.46 0.50 1.00 0.57
#Factor4 0.56 0.62 0.58 1.00
#compare with
#round(cor(fa$loading,pc$loading),2)
```

    factor.fit How well does the factor model fit a correlation matrix. Part of the
    VSS package
    
## Description

The basic factor or principal components model is that a correlation or covariance matrix may be reproduced by the product of a factor loading matrix times its transpose: F'F or P'P. One simple index of fit is the 1 - sum squared residuals/sum squared original correlations. This fit index is used by VSS, ICLUST, etc.

## Usage

```
factor.fit(r, f)
```


## Arguments

| r | a correlation matrix |
| :--- | :--- |
| f | A factor matrix of loadings. |

## Details

There are probably as many fit indices as there are psychometricians. This fit is a plausible estimate of the amount of reduction in a correlation matrix given a factor model. Note that it is sensitive to the size of the original correlations. That is, if the residuals are small but the original correlations are small, that is a bad fit.

## Value

fit

## Author(s)

William Revelle

## See Also

VSS, ICLUST

## Examples

```
## Not run:
#compare the fit of 4 to 3 factors for the Harman 24 variables
fa4 <- factanal(x,4,covmat=Harman74.cor$cov)
round(factor.fit(Harman74.cor$cov,fa4$loading),2)
#[1] 0.9
fa3 <- factanal(x,3,covmat=Harman74.cor$cov)
round(factor.fit(Harman74.cor$cov,fa3$loading),2)
#[1] 0.88
## End(Not run)
```

```
factor.model Find R=F F'+U2 is the basic factor model
```


## Description

The basic factor or principal components model is that a correlation or covariance matrix may be reproduced by the product of a factor loading matrix times its transpose. Find this reproduced matrix. Used by factor.fit, VSS, ICLUST, etc.

## Usage

factor.model(f, Phi=NULL, U2=TRUE)

## Arguments

$\mathrm{f} \quad$ A matrix of loadings.
Phi A matrix of factor correlations
U2 Should the diagonal be model by ff' (U2 = TRUE) or replaced with 1's (U2 = FALSE)

## Value

A correlation or covariance matrix.

## Author(s)

```
<revelle@northwestern.edu >
http://personality-project.org/revelle.html
```


## References

Gorsuch, Richard, (1983) Factor Analysis. Lawrence Erlebaum Associates.
Revelle, W. In preparation) An Introduction to Psychometric Theory with applications in R (http: //personality-project.org/r/book/)

## See Also

```
ICLUST.graph,ICLUST.cluster, cluster.fit ,VSS, omega
```


## Examples

```
f2 <- matrix(c(.9,.8,.7,rep (0,6),.6,.7,.8),ncol=2)
mod <- factor.model(f2)
round(mod, 2)
```

```
factor.residuals }\quad\mp@subsup{R}{}{*}=R-F\mp@subsup{F}{}{\prime
```


## Description

The basic factor or principal components model is that a correlation or covariance matrix may be reproduced by the product of a factor loading matrix times its transpose. Find the residuals of the original minus the reproduced matrix. Used by factor.fit, VSS, ICLUST, etc.

## Usage

factor.residuals(r, f)

## Arguments

| r | A correlation matrix |
| :--- | :--- |
| f | A factor model matrix or a list of class loadings |

## Details

The basic factor equation is ${ }_{n} R_{n} \approx_{n} F_{k k} F_{n}^{\prime}+U^{2}$. Residuals are just $\mathrm{R}^{*}=\mathrm{R}-\mathrm{F} \mathrm{F}^{\mathrm{F}}$. The residuals should be (but in practice probably rarely are) examined to understand the adequacy of the factor analysis. When doing Factor analysis or Principal Components analysis, one usually continues to extract factors/components until the residuals do not differ from those expected from a random matrix.

## Value

rstar is the residual correlation matrix.

## Author(s)

Maintainer: William Revelle [revelle@northwestern.edu](mailto:revelle@northwestern.edu)

## See Also

```
factor.pa,principal, VSS,ICLUST
```


## Examples

```
fa2 <- factor.pa(Harman74.cor$cov,2,rotate=TRUE)
    fa2resid <- factor.residuals(Harman74.cor$cov,fa2)
    fa2resid[1:4,1:4] #residuals with two factors extracted
    fa4 <- factor.pa(Harman74.cor$cov,4,rotate=TRUE)
    fa4resid <- factor.residuals(Harman74.cor$cov,fa4)
    fa4resid[1:4,1:4] #residuals with 4 factors extracted
```

    factor.rotate "Hand" rotate a factor loading matrix
    
## Description

Given a factor or components matrix, it is sometimes useful to do arbitrary rotations of particular pairs of variables. This supplements the much more powerful rotation package GPArotation and is meant for specific requirements to do unusual rotations.

## Usage

factor.rotate(f, angle, coll=1, col2=2,plot=FALSE,...)

## Arguments

| f | original loading matrix or a data frame (can be output from a factor analysis <br> function |
| :--- | :--- |
| angle | angle (in degrees!) to rotate |
| $\operatorname{coll}$ | column in factor matrix defining the first variable |
| $\operatorname{col} 2$ | column in factor matrix defining the second variable |
| plot | plot the original (unrotated) and rotated factors |
| $\ldots$ | parameters to pass to factor.plot |

## Details

Partly meant as a demonstration of how rotation works, factor.rotate is useful for those cases that require specific rotations that are not available in more advanced packages such as GPArotation. If the plot option is set to TRUE, then the original axes are shown as dashed lines.

The rotation is in degrees counter clockwise.

## Value

the resulting rotated matrix of loadings.

## Note

For a complete rotation package, see GPArotation

## Author(s)

Maintainer: William Revelle <revelle@northwestern.edu >

## References

http://personality-project.org/r/book

## Examples

```
#using the Harman 24 mental tests, rotate the 2nd and 3rd factors 45 degrees
f4<- fa(Harman74.cor$cov,4,rotate="TRUE")
f4r45 <- factor.rotate(f4,45,2,3)
f4r90 <- factor.rotate(f4r45,45,2,3)
print(factor.congruence(f4,f4r45), digits=3) #poor congruence with original
print (factor.congruence(f4,f4r90), digits=3) #factor 2 and 3 have been exchanged and 3 flippe
#a graphic example
data(Harman23.cor)
f2 <- fa(Harman23.cor$cov,2, rotate="none")
op <- par(mfrow=c(1,2))
cluster.plot(f2,xlim=c(-1,1),ylim=c(-1,1),title="Unrotated ")
f2r <- factor.rotate(f2,-33,plot=TRUE,xlim=c(-1,1),ylim=c(-1,1),title="rotated - 33 degrees")
```

```
op <- par(mfrow=c (1,1))
```

factor.stats Find various goodness of fit statistics for factor analysis and principal components

## Description

Chi square and other goodness of fit statistics are found based upon the fit of a factor or components model to a correlation matrix. Although these statistics are normally associated with a maximum likelihood solution, they can be found for minimal residual (OLS), principal axis, or principal component solutions as well. Primarily called from within these functions, factor.stats can be used by itself. Measures of factorial adequacy and validity follow the paper by Grice, 2001.

## Usage

```
factor.stats(r, f, phi=NULL, n.obs = NA,alpha=.1)
```

factor.scores (x,f)

## Arguments

$r \quad$ A correlation matrix or a data frame of raw data
$x \quad$ A data frame or matrix of raw data
f A factor analysis loadings matrix or the output from a factor or principal components analysis
phi A factor intercorrelation matrix if the factor solution was oblique
n. obs The number of observations for the correlation matrix. If not specified, and a correlation matrix is used, chi square will not be reported. Not needed if the input is a data matrix.
alpha alpha level of confidence intervals for RMSEA

## Details

Combines the goodness of fit tests used in factor.pa, factor.minres, and principal into one function. If the matrix is singular, will smooth the correlation matrix before finding the fit functions. Now will find the RMSEA (root mean square error of approximation) and the alpha confidence intervals similar to a SEM function. Also reports the root mean square residual.

Value
fit How well does the factor model reproduce the correlation matrix. (See VSS, ICLUST, and principal for this fit statistic.
fit.off how well are the off diagonal elements reproduced?

| dof | Degrees of Freedom for this model. This is the number of observed correlations minus the number of independent parameters. Let $\mathrm{n}=$ Number of items, $\mathrm{nf}=$ number of factors then $d o f=n *(n-1) / 2-n * n f+n f *(n f-1) / 2$ |
| :---: | :---: |
| objective | value of the function that is minimized by maximum likelihood procedures. This is reported for comparison purposes and as a way to estimate chi square goodness of fit. The objective function is $f=\log \left(\operatorname{trace}\left(\left(F F^{\prime}+U 2\right)^{-1} R\right)-\log \left(\left\|\left(F F^{\prime}+U 2\right)^{-1} R\right\|\right)-\text { n.items } .\right.$ |
| STATISTIC | If the number of observations is specified or found, this is a chi square based upon the objective function, f. Using the formula from fact anal(which seems to be Bartlett's test) : $\left.\chi^{2}=(\text { n.obs }-1-(2 * p+5) / 6-(2 * \text { factors }) / 3)\right) * f$ |
| PVAL | If n.obs $>0$, then what is the probability of observing a chisquare this large or larger? |
| Phi | If oblique rotations (using oblimin from the GPArotation package or promax) are requested, what is the interfactor correlation. |
| R2 | The multiple R square between the factors and factor score estimates, if they were to be found. (From Grice, 2001) |
| r.scores | The correlations of the factor score estimates, if they were to be found. |
| weights | The beta weights to find the factor score estimates |
| valid | The validity coffiecient of course coded (unit weighted) factor score estimates (From Grice, 2001) |
| score.cor | The correlation matrix of course coded (unit weighted) factor score estimates, if they were to be found, based upon the loadings matrix. |
| RMSEA | The Root Mean Square Error of Approximation and the alpha confidence intervals. Based upon the chi square non-centrality parameter. |
| rms | The empirically found square root of the squared residuals. This does not require sample size to be specified nor does it make assumptions about normality. |
| crms | While the rms uses the number of correlations to find the average, the crms uses the number of degrees of freedom. Thus, there is a subtle penalty for having too complex a model. |

## Author(s)

William Revelle

## References

Grice, James W.,2001, Computing and evaluating factor scores, Psychological Methods, 6,4, 430450.

## See Also

factor.pa for principal axis factor analysis, factor.minres for minimum residual factor analysis, and principal for principal components.

## Examples

```
v9 <- sim.hierarchical()
f3 <- factor.minres(v9,3)
factor.stats(v9,f3,n.obs=500)
f3o <- factor.pa(v9,3,rotate="Promax")
factor.stats(v9,f3o,n.obs=500)
```

factor2cluster Extract cluster definitions from factor loadings

## Description

Given a factor or principal components loading matrix, assign each item to a cluster corresponding to the largest (signed) factor loading for that item. Essentially, this is a Very Simple Structure approach to cluster definition that corresponds to what most people actually do: highlight the largest loading for each item and ignore the rest.

## Usage

factor2cluster(loads, cut $=0$ )

## Arguments

loads either a matrix of loadings, or the result of a factor analysis/principal components analyis with a loading component
cut $\quad$ Extract items with absolute loadings $>$ cut

## Details

A factor/principal components analysis loading matrix is converted to a cluster $(-1,0,1)$ definition matrix where each item is assigned to one and only one cluster. This is a fast way to extract items that will be unit weighted to form cluster composites. Use this function in combination with cluster.cor to find the corrleations of these composite scores.
A typical use in the SAPA project is to form item composites by clustering or factoring (see ICLUST, principal), extract the clusters from these results (factor2cluster), and then form the composite correlation matrix using cluster. cor. The variables in this reduced matrix may then be used in multiple R procedures using mat.regress.

The input may be a matrix of item loadings, or the output from a factor analysis which includes a loadings matrix.

## Value

a matrix of $-1,0,1$ cluster definitions for each item.

## Author(s)

```
http://personality-project.org/revelle.html
```

Maintainer: William Revelle < revelle@northwestern.edu >

## References

```
http://personality-project.org/r/r.vss.html
```


## See Also

```
cluster.cor, factor2cluster, factor.pa, principal, ICLUST
```


## Examples

| \#\# Not run: |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| factor2cluster(f) |  |  |  |  |
| \#\# End (Not run) |  |  |  |  |
| \# | Factor1 | Factor2 | Factor3 | Factor4 |
| \#VisualPerception | 0 | 1 | 0 | 0 |
| \#Cubes | 0 | 1 | 0 | 0 |
| \#PaperFormBoard | 0 | 1 | 0 | 0 |
| \#Flags | 0 | 1 | 0 | 0 |
| \#GeneralInformation | 1 | 0 | 0 | 0 |
| \#PargraphComprehension | 1 | 0 | 0 | 0 |
| \#SentenceCompletion | 1 | 0 | 0 | 0 |
| \#WordClassification | 1 | 0 | 0 | 0 |
| \#WordMeaning | 1 | 0 | 0 | 0 |
| \#Addition | 0 | 0 | 1 | 0 |
| \#Code | 0 | 0 | 1 | 0 |
| \#CountingDots | 0 | 0 | 1 | 0 |
| \#StraightCurvedCapitals | 0 | 0 | 1 | 0 |
| \#WordRecognition | 0 | 0 | 0 | 1 |
| \#NumberRecognition | 0 | 0 | 0 | 1 |
| \#FigureRecognition | 0 | 0 | 0 | 1 |
| \#ObjectNumber | 0 | 0 | 0 | 1 |
| \#NumberFigure | 0 | 0 | 0 | 1 |
| \#FigureWord | 0 | 0 | 0 | 1 |
| \#Deduction | 0 | 1 | 0 | 0 |
| \#NumericalPuzzles | 0 | 0 | 1 | 0 |
| \#ProblemReasoning | 0 | 1 | 0 | 0 |
| \#SeriesCompletion | 0 | 1 | 0 | 0 |
| \#ArithmeticProblems | 0 | 0 | 1 | 0 |

## fisherz Fisher r to $z$ and $z$ to $r$ and confidence intervals

## Description

convert a correlation to a z score or z to r using the Fisher transformation or find the confidence intervals for a specified correlation

## Usage

```
fisherz(rho)
fisherz2r(z)
r.con(rho,n,p=.95,twotailed=TRUE)
r2t(rho,n)
```


## Arguments

| rho | a Pearson $r$ |
| :--- | :--- |
| $z$ | A Fisher $z$ |
| $n$ | Sample size for confidence intervals |
| $p$ | Confidence interval |
| twotailed | Treat $p$ as twotailed $p$ |

## Value

$z$ value corresponding to $r$ (fisherz) $\backslash r$ corresponding to $z$ (fisherz $2 r$ ) $\backslash$ lower and upper $p$ confidence intervals (r.con) $\backslash t$ with $n-2 d f(r 2 t)$

## Author(s)

Maintainer: William Revelle <revelle@northwestern.edu >

## Examples

```
cors <- seq(-.9,.9,.1)
zs <- fisherz(cors)
rs <- fisherz2r(zs)
round(zs,2)
    n <- 30
    r<- seq(0,.9,.1)
    rc <- matrix(r.con(r,n),ncol=2)
t <- r*sqrt(n-2)/sqrt(1-r^2)
    p <- (1-pt (t,n-2))/2
    r.rc <- r.rc <- data.frame(r=r,z=fisherz(r), lower=rc[,1],upper=rc[, 2],t=t,p=p)
    round(r.rc,2)
```


## galton Galton's Mid parent child height data

## Description

Two of the earliest examples of the correlation coefficient were Francis Galton's data sets on the relationship between mid parent and child height and the similarity of parent generation peas with child peas. This is the data set for the Galton height.

## Usage

data(galton)

## Format

A data frame with 928 observations on the following 2 variables.
parent Mid Parent heights (in inches)
child Child Height

## Details

Female heights were adjusted by 1.08 to compensate for sex differences. (This was done in the original data set)

## Source

This is just the galton data set from UsingR, slightly rearranged.

## References

Stigler, S. M. (1999). Statistics on the Table: The History of Statistical Concepts and Methods. Harvard University Press. Galton, F. (1869). Hereditary Genius: An Inquiry into its Laws and Consequences. London: Macmillan.
Wachsmuth, A.W., Wilkinson L., Dallal G.E. (2003). Galton's bend: A previously undiscovered nonlinearity in Galton's family stature regression data. The American Statistician, 57, 190-1922.

## See Also

The other Galton data sets: heights, peas,cubits

## Examples

```
data(galton)
describe(galton)
pairs.panels(galton,main="Galton's Parent child heights") #show the scatter plot and the l
pairs.panels(galton,lm=TRUE,main="Galton's Parent child heights") #but this makes the regres
pairs.panels(galton,lm=TRUE,xlim=c (62,74),ylim=c (62,74),main="Galton's Parent child heights"
```

geometric.mean $\quad$ Find the geometric mean of a vector or columns of a data.frame.

## Description

The geometric mean is the nth root of $n$ products or e to the mean $\log$ of x . Useful for describing non-normal, i.e., geometric distributions.

## Usage

geometric.mean (x, na.rm=TRUE)

## Arguments

| x | a vector or data.frame |
| :--- | :--- |
| $\mathrm{na} . \mathrm{rm}$ | remove NA values before processing |

## Details

Useful for teaching how to write functions, also useful for showing the different ways of estimating central tendency.

## Value

geometric mean(s) of $x$ or $x . d f$.

Note
Not particularly useful if there are elements that are $<=0$.

## Author(s)

William Revelle

## See Also

harmonic.mean, mean

## Examples

```
x <- seq(1,5)
x2 <- x^2
x2[2] <- NA
X <- data.frame(x,x2)
geometric.mean(x)
geometric.mean(x2)
geometric.mean(X)
geometric.mean(X,na.rm=FALSE)
```


## Description

The greatest lower bound solves the "educational testing problem". That is, what is the reliability of a test? (See guttman for a discussion of the problem). Although there are many estimates of a test reliability (Guttman, 1945) most underestimate the true reliability of a test.

For a given covariance matrix of items, C, the function finds the greatest lower bound to reliability of the total score using the csdp function from the Rcsdp package.

## Usage

glb.algebraic(Cov, LoBounds = NULL, UpBounds = NULL)

## Arguments

Cov
LoBounds

UpBounds A vector $u=(u 1, .$. , up) of length p with upper bounds to the diagonal elements xi. The default is $u=v$.

## Details

If C is a $\mathrm{p} * \mathrm{p}$-covariance matrix, $\mathrm{v}=\operatorname{diag}(\mathrm{C})$ its diagonal (i. e. the vector of variances $v_{i}=c_{i i}$ ), $\tilde{C}=C-\operatorname{Diag}(v)$ is the covariance matrix with 0 s substituted in the diagonal and $\mathrm{x}=$ the vector $x_{1}, \ldots, x_{n}$ the educational testing problem is (see e. g., Al-Homidan 2008)

$$
\sum_{i=1}^{p} x_{i} \rightarrow \min
$$

s.t.

$$
\tilde{C}+\operatorname{Diag}(x) \geq 0
$$

(i.e. positive semidefinite) and $x_{i} \leq v_{i}, i=1, \ldots, p$. This is the same as minimizing the trace of the symmetric matrix

$$
\tilde{C}+\operatorname{diag}(x)=\left(\begin{array}{llll}
x_{1} & c_{12} & \ldots & c_{1 p} \\
c_{12} & x_{2} & \ldots & c_{2 p} \\
\vdots & \vdots & \ddots & \vdots \\
c_{1 p} & c_{2 p} & \ldots & x_{p}
\end{array}\right)
$$

s. t. $\tilde{C}+\operatorname{Diag}(x)$ is positive semidefinite and $x_{i} \leq v_{i}$.

The greatest lower bound to reliability is

$$
\frac{\sum_{i j} c_{i j}+\sum_{i} x_{i}}{\sum_{i j} c_{i j}}
$$

Additionally, function glb.algebraic allows the user to change the upper bounds $x_{i} \leq v_{i}$ to $x_{i} \leq u_{i}$ and add lower bounds $l_{i} \leq x_{i}$.
The greatest lower bound to reliability is applicable for tests with non-homogeneous items. It gives a sharp lower bound to the reliability of the total test score.
Caution: Though glb.algebraic gives exact lower bounds for exact covariance matrices, the estimates from empirical matrices may be strongly biased upwards for small and medium sample sizes. glb.algebraic is wrapper for a call to function csdp of package Rcsdp (see its documentation).
If Cov is the covariance matrix of subtests/items with known lower bounds, rel, to their reliabilities (e. g. Cronbachs $\alpha$ ), LoBounds can be used to improve the lower bound to reliability by setting LoBounds $<-$ rel ${ }^{*} \operatorname{diag}(\mathrm{Cov})$.
Changing UpBounds can be used to relax constraints $x_{i} \leq v_{i}$ or to fix $x_{i}$-values by setting LoBounds[i] $<-z$; UpBounds[i] <- z.

## Value

| glb | The algebraic greatest lower bound |
| :--- | :--- |
| solution | The vector $x$ of the solution of the semidefinite program. These are the elements <br> on the diagonal of $C$. |
| status | Status of the solution. See documentation of csdp in package Rcsdp. If status is <br> 2 or greater or equal than 4, no glb and solution is returned. If status is not 0, a <br> warning message is generated. |
| Call | The calling string |

## Author(s)

Andreas Moltner
Center of Excellence for Assessment in Medicine/Baden-Wurttemberg
University of Heidelberg

William Revelle
Department of Psychology
Northwestern University Evanston, Illiniois
http://personality-project.org/revelle.html

## References

Al-Homidan S (2008). Semidefinite programming for the educational testing problem. Central European Journal of Operations Research, 16:239-249.

Bentler PM (1972) A lower-bound method for the dimension-free measurement of internal consistency. Soc Sci Res 1:343-357.

Fletcher R (1981) A nonlinear programming problem in statistics (educational testing). SIAM J Sci Stat Comput 2:257-267.

Shapiro A, ten Berge JMF (2000). The asymptotic bias of minimum trace factor analysis, with applications to the greatest lower bound to reliability. Psychometrika, 65:413-425.
ten Berge, Socan G (2004). The greatest bound to reliability of a test and the hypothesis of unidimensionality. Psychometrika, 69:613-625.

## See Also

For an alternative estimate of the greatest lower bound, see glb.fa. For multiple estimates of reliablity, see guttman

## Examples

```
Cv<-matrix(c(215, 64, 33, 22,
            64, 97, 57, 25,
    33, 57,103, 36,
    22, 25, 36, 77), ncol=4)
Cv # covariance matrix of a test with 4 subtests
Cr<-cov2cor(Cv) # Correlation matrix of tests
if(require(Rcsdp)) {glb.algebraic(Cv)} # glb of total score
if(require(Rcsdp)) {glb.algebraic(Cr) } # glb of sum of standardized scores
w<-c(1,2,2,1) # glb of weighted total score
# glb.algebraic(diag(w) %*% Cv %*% diag(w))
alphas <- c(0.8,0,0,0) # Internal consistency of first test is known
if(require(Rcsdp)) {glb.algebraic(Cv,LoBounds=alphas*diag(Cv))}
    # Fix all diagonal elements to 1 but the first:
if(require(Rcsdp)) {lb<-glb.algebraic(Cr, LoBounds=c (0, 1, 1, 1), UpBounds=c (1, 1, 1, 1))
lb$solution[1] # should be the same as the squared mult. corr.
smc(Cr)[1]
} else {print ('I am sorry, you need to have the package Rcsdp installed to use glb.algebrai
```


## Description

Eight alternative estimates of test reliability include the six discussed by Guttman (1945), four discussed by ten Berge and Zergers (1978) $\left(\mu_{0} \ldots \mu_{3}\right)$ as well as $\beta$ (the worst split half, Revelle, 1979), the glb (greatest lowest bound) discussed by Bentler and Woodward (1980), and $\omega_{h}$ and $\omega_{t}$ (McDonald, 1999; Zinbarg et al., 2005).

## Usage

```
guttman(r,key=NULL)
tenberge(r)
glb(r,key=NULL)
glb.fa(r,key=NULL)
```


## Arguments

$$
\begin{array}{ll}
r & \text { A correlation matrix or raw data matrix. } \\
\text { key } & \text { a vector of }-1,0,1 \text { to select or reverse items }
\end{array}
$$

## Details

Surprisingly, 105 years after Spearman (1904) introduced the concept of reliability to psychologists, there are still multiple approaches for measuring it. Although very popular, Cronbach's $\alpha$ (1951) underestimates the reliability of a test and over estimates the first factor saturation. The guttman function includes the six estimates discussed by Guttman (1945), four of ten Berge and Zergers (1978), as well as Revelle's $\beta$ (1979) using ICLUST. The companion function, omega calculates omega hierarchical $\left(\omega_{h}\right)$ and omega total $\left(\omega_{t}\right)$.
Guttman's first estimate $\lambda_{1}$ assumes that all the variance of an item is error:

$$
\lambda_{1}=1-\frac{\operatorname{tr}\left(\vec{V}_{x}\right)}{V_{x}}=\frac{V_{x}-\operatorname{tr}\left(\vec{V}_{x}\right)}{V_{x}}
$$

This is a clear underestimate.
The second bound, $\lambda_{2}$, replaces the diagonal with a function of the square root of the sums of squares of the off diagonal elements. Let $C_{2}=\overrightarrow{1}\left(\vec{V}-\operatorname{diag}(\vec{V})^{2} \overrightarrow{1}^{\prime}\right.$, then

$$
\lambda_{2}=\lambda_{1}+\frac{\sqrt{\frac{n}{n-1} C_{2}}}{V_{x}}=\frac{V_{x}-\operatorname{tr}\left(\vec{V}_{x}\right)+\sqrt{\frac{n}{n-1} C_{2}}}{V_{x}}
$$

Effectively, this is replacing the diagonal with $n$ * the square root of the average squared off diagonal element.
Guttman's 3rd lower bound, $\lambda_{3}$, also modifies $\lambda_{1}$ and estimates the true variance of each item as the average covariance between items and is, of course, the same as Cronbach's $\alpha$.

$$
\lambda_{3}=\lambda_{1}+\frac{\frac{V_{X}-\operatorname{tr}\left(\vec{V}_{X}\right)}{n(n-1)}}{V_{X}}=\frac{n \lambda_{1}}{n-1}=\frac{n}{n-1}\left(1-\frac{\operatorname{tr}(\vec{V})_{x}}{V_{x}}\right)=\frac{n}{n-1} \frac{V_{x}-\operatorname{tr}\left(\vec{V}_{x}\right)}{V_{x}}=\alpha
$$

This is just replacing the diagonal elements with the average off diagonal elements. $\lambda_{2} \geq \lambda_{3}$ with $\lambda_{2}>\lambda_{3}$ if the covariances are not identical.
$\lambda_{3}$ and $\lambda_{2}$ are both corrections to $\lambda_{1}$ and this correction may be generalized as an infinite set of successive improvements. (Ten Berge and Zegers, 1978)

$$
\mu_{r}=\frac{1}{V_{x}}\left(p_{o}+\left(p_{1}+\left(p_{2}+\ldots\left(p_{r-1}+\left(p_{r}\right)^{1 / 2}\right)^{1 / 2} \ldots\right)^{1 / 2}\right)^{1 / 2}\right), r=0,1,2, \ldots
$$

where

$$
p_{h}=\sum_{i \neq j} \sigma_{i j}^{2 h}, h=0,1,2, \ldots r-1
$$

and

$$
p_{h}=\frac{n}{n-1} \sigma_{i j}^{2 h}, h=r
$$

tenberge and Zegers (1978). Clearly $\mu_{0}=\lambda_{3}=\alpha$ and $\mu_{1}=\lambda_{2} . \mu_{r} \geq \mu_{r-1} \geq \ldots \mu_{1} \geq \mu_{0}$, although the series does not improve much after the first two steps.

Guttman's fourth lower bound, $\lambda_{4}$ was originally proposed as any spit half reliability but has been interpreted as the greatest split half reliability. If $\vec{X}$ is split into two parts, $\vec{X}_{a}$ and $\vec{X}_{b}$, with correlation $r_{a b}$ then

$$
\lambda_{4}=2\left(1-\frac{V_{X_{a}}+V_{X_{b}}}{V_{X}}\right)=\frac{4 r_{a b}}{V_{x}}=\frac{4 r_{a b}}{V_{X_{a}}+V_{X_{b}}+2 r_{a b} V_{X_{a}} V_{X_{b}}}
$$

which is just the normal split half reliability, but in this case, of the most similar splits.
$\lambda_{5}$, Guttman's fifth lower bound, replaces the diagonal values with twice the square root of the maximum (across items) of the sums of squared interitem covariances

$$
\lambda_{5}=\lambda_{1}+\frac{2 \sqrt{\overline{C_{2}}}}{V_{X}}
$$

Although superior to $\lambda_{1}, \lambda_{5}$ underestimates the correction to the diagonal. A better estimate would be analogous to the correction used in $\lambda_{3}$ :

$$
\lambda_{5+}=\lambda_{1}+\frac{n}{n-1} \frac{2 \sqrt{\bar{C}_{2}}}{V_{X}}
$$

Guttman's final bound considers the amount of variance in each item that can be accounted for the linear regression of all of the other items (the squared multiple correlation or smc), or more precisely, the variance of the errors, $e_{j}^{2}$, and is

$$
\lambda_{6}=1-\frac{\sum e_{j}^{2}}{V_{x}}=1-\frac{\sum\left(1-r_{s m c}^{2}\right)}{V_{x}}
$$

Guttman's $\lambda_{4}$ is the greatest split half reliability. This is found here by combining the output from three different approaches, and seems to work for all test cases yet tried. Lambda 4 is reported as the max of these three algorithms.
The algorithms are
a) Do an ICLUST of the reversed correlation matrix. ICLUST normally forms the most distinct clusters. By reversing the correlations, it will tend to find the most related clusters. Truly a weird approach but tends to work.
b) Alternatively, a kmeans clustering of the correlations (with the diagonal replaced with 0 to make pseudo distances) can produce 2 similar clusters.
c) Clusters identified by assigning items to two clusters based upon their order on the first principal factor. (Highest to cluster 1, next 2 to cluster 2, etc.)

These three procedures will produce keys vectors for assigning items to the two splits. The maximum split half reliability is found by taking the maximum of these three approaches. This is not elegant but is fast.

There are three greatest lower bound functions. One, glb finds the greatest split half reliability, $\lambda_{4}$. This considers the test as set of items and examines how best to partition the items into splits. The other two, glb.fa and glb.algebraic, are alternative ways of weighting the diagonal of the matrix.
glb.fa estimates the communalities of the variables from a factor model where the number of factors is the number with positive eigen values. Then reliability is found by

$$
g l b=1-\frac{\sum e_{j}^{2}}{V_{x}}=1-\frac{\sum\left(1-h^{2}\right)}{V_{x}}
$$

This estimate will differ slightly from that found by glb. al gebraic, written by Andreas Moeltner which uses calls to csdp in the Rcsdp package. His algorithm, which more closely matches the description of the glb by Jackson and Woodhouse, seems to have a positive bias (i.e., will over estimate the reliability of some items; they are said to be $=1$ ) for small sample sizes. More exploration of these two algorithms is underway.

Compared to glb.algebraic, glb.fa seems to have less (positive) bias for smallish sample sizes $(\mathrm{n}<500)$ but larger for large $(>1000)$ sample sizes. This interacts with the number of variables so that equal bias sample size differs as a function of the number of variables. The differences are, however small. As samples sizes grow, glb. algebraic seems to converge on the population value while glb.fa has a positive bias.

## Value

| beta | The normal beta estimate of cluster similarity from ICLUST. This is an estimate <br> of the general factor saturation. |
| :--- | :--- |
| tenberge $\$ m u 1$ | tenBerge mu 1 is functionally alpha |
| tenberge $\$ m u 2$ one of the sequence of estimates mu1 ... mu3 |  |
| beta.factor | For experimental purposes, what is the split half based upon the two factor so- <br> lution? |
| glb.IC | Greatest split half based upon ICLUST of reversed correlations |
| gl.b. Km | Greatest split half based upon a kmeans clustering. |
| glb.Fa | Greatest split half based upon the items assigned by factor analysis. |
| glb.max | max of the above estimates |
| gl.b | glb found from factor analysis |
| keys | scoring keys from each of the alternative methods of forming best splits |

## Author(s)

William Revelle

## References

Cronbach, L.J. (1951) Coefficient alpha and the internal strucuture of tests. Psychometrika, 16, 297-334.

Guttman, L. (1945). A basis for analyzing test-retest reliability. Psychometrika, 10 (4), 255-282.
Revelle, W. (1979). Hierarchical cluster-analysis and the internal structure of tests. Multivariate Behavioral Research, 14 (1), 57-74.

Revelle, W. and Zinbarg, R. E. (2009) Coefficients alpha, beta, omega and the glb: comments on Sijtsma. Psychometrika, 2009.

Ten Berge, J. M. F., \& Zegers, F. E. (1978). A series of lower bounds to the reliability of a test. Psychometrika, 43 (4), 575-579.

Zinbarg, R. E., Revelle, W., Yovel, I., \& Li, W. (2005). Cronbach's $\alpha$, Revelle's $\beta$, and McDonald's $\omega_{h}$ ): Their relations with each other and two alternative conceptualizations of reliability. Psychometrika, 70 (1), 123-133.

## See Also

```
alpha, omega, ICLUST, glb.algebraic
```


## Examples

```
data(attitude)
glb(attitude)
glb.fa(attitude)
if(require(Rcsdp)) {glb.algebraic(cor(attitude)) }
guttman(attitude)
```

| Harman | Two data sets from Harman (1967). 9 cognitive variables from |
| :--- | :--- |
|  | Holzinger and 8 emotional variables from Burt |

## Description

Two classic data sets reported by Harman (1967) are 9 psychological (cognitive) variables taken from Holzinger and 8 emotional variables taken from Burt. Both of these are used for tests and demonstrations of various factoring algortithms.

## Usage

data (Harman)

## Details

- Harman.Holzinger: $9 \times 9$ correlation matrix of ability tests, $\mathrm{N}=696$.
- Harman.Burt: a $8 \times 8$ correlation matrix of "emotional" items. $\mathrm{N}=172$

Harman.Holzinger. The nine psychological variables from Harman (1967, p 244) are taken from unpublished class notes of K.J. Holzinger with 696 participants. This is a subset of 12 tests with 4 factors. It is yet another nice example of a bifactor solution. Bentler (2007) uses this data set to discuss reliablity analysis. The data show a clear bifactor structure and are a nice example of the various estimates of reliability included in the omega function. Should not be confused with the Holzinger or Holzinger. 9 data sets in bifactor.
Harman.Burt. Eight "emotional" variables are taken from Harman (1967, p 164) who in turn adapted them from Burt (1939). They are said be from 172 normal children aged nine to twelve. As pointed out by Harman, this correlation matrix is singular and has squared multiple correlations $>1$. Because of this problem, it is a nice test case for various factoring algorithms. (For instance, omega will issue warning messages for $\mathrm{fm}=$ "minres" or $\mathrm{fm}=$ "pa" but will fail for $\mathrm{fm}=\mathrm{ml}$ ".)
The Burt data set probably has a typo in the original correlation matrix. Changing the SorrowTenderness correlation from .87 to .81 makes the correlation positive definite.
As pointed out by Jan DeLeeuw, the Burt data set is a subset of 8 variables from the original 11 reported by Burt in 1915. That matrix has the same problem. See burt.
Other example data sets that are useful demonstrations of factor analysis are the seven bifactor examples in bifactor and the 24 ability measures in Harman 74 . cor

## Source

Harman (1967 p 164 and p 244.)

## References

Harman, Harry Horace (1967), Modern factor analysis. Chicago, University of Chicago Press.
P.Bentler. Covariance structure models for maximal reliability of unit-weighted composites. In Handbook of latent variable and related models, pages 1-17. North Holland, 2007.
Burt, C.General and Specific Factors underlying the Primary Emotions. Reports of the British Association for the Advancement of Science, 85th meeting, held in Manchester, September 7-11, 1915. London, John Murray, 1916, p. 694-696 (retrieved from the web at http://www.biodiversitylibrary.org/item/95822\#790)

## See Also

See also the original burt data set

## Examples

```
data(Harman)
cor.plot(Harman.Holzinger)
cor.plot(Harman.Burt)
smc(Harman.Burt) #note how this produces impossible results
```

```
harmonic.mean Find the harmonic mean of a vector, matrix, or columns of a data.frame
```


## Description

The harmonic mean is merely the reciprocal of the arithmetic mean of the reciprocals.

## Usage

harmonic.mean(x, na.rm=TRUE)

## Arguments

| x | a vector, matrix, or data.frame |
| :--- | :--- |
| na.rm | na.rm=TRUE remove NA values before processing |

## Details

Included as an example for teaching about functions. As well as for a discussion of how to estimate central tendencies.

Value
The harmonic mean(s)

## Note

Included as a simple demonstration of how to write a function

## Examples

```
x <- seq(1,5)
x2 <- x^2
x2[2] <- NA
X <- data.frame(x,x2)
harmonic.mean(x)
harmonic.mean(x2)
harmonic.mean(X)
harmonic.mean(X,FALSE)
```


## Description

A quick way to show the first and last $n$ lines of a data.frame, matrix, or a text object. Just a pretty call to head and tail

## Usage

headtail(x,hlength=4,tlength=4, digits=2)

## Arguments

$x \quad$ A matrix or data frame or free text
hlength The number of lines at the beginning to show
tlength The number of lines at the end to show
digits Round off the data to digits

## Value

The first hlength and last tlength lines of a matrix or data frame with an ellipsis in between. If the input is neither a matrix nor data frame, the output will be the first hlength and last tlength lines.

## See Also

```
head and tail
```


## Examples

```
x <- matrix(sample(10,1000,TRUE),ncol=5)
headtail(x, 4, 8)
```

```
heights
```

A data.frame of the Galton (1888) height and cubit data set.

## Description

Francis Galton introduced the 'co-relation' in 1888 with a paper discussing how to measure the relationship between two variables. His primary example was the relationship between height and forearm length. The data table (cubits) is taken from Galton (1888). Unfortunately, there seem to be some errors in the original data table in that the marginal totals do not match the table.
The data frame, heights, is converted from this table using table2df.

## Usage

```
data(heights)
```


## Format

A data frame with 348 observations on the following 2 variables.
height Height in inches
cubit Forearm length in inches

## Details

Sir Francis Galton (1888) published the first demonstration of the correlation coefficient. The regression (or reversion to mediocrity) of the height to the length of the left forearm (a cubit) was found to .8. The original table cubits is taken from Galton (1888). There seem to be some errors in the table as published in that the row sums do not agree with the actual row sums. These data are used to create a matrix using table2matrix for demonstrations of analysis and displays of the data.

## Source

Galton (1888)

## References

Galton, Francis (1888) Co-relations and their measurement. Proceedings of the Royal Society. London Series,45,135-145,

## See Also

table2matrix, table2df, cubits, ellipses, galton

## Examples

```
data(heights)
ellipses(heights,n=1,main="Galton's co-relation data set")
```


## Description

The Intraclass correlation is used as a measure of association when studying the reliability of raters. Shrout and Fleiss (1979) outline 6 different estimates, that depend upon the particular experimental design. All are implemented and given confidence limits.

## Usage

ICC ( x , missing=TRUE, digits=2, alpha=.05)

## Arguments

$x \quad a \operatorname{matrix}$ or dataframe of ratings
missing if TRUE, remove missing data - work on complete cases only
digits Round the output to digits
alpha The alpha level for significance for finding the confidence intervals

## Details

Shrout and Fleiss (1979) consider six cases of reliability of ratings done by k raters on n targets.
ICC1: Each target is rated by a different judge and the judges are selected at random. (This is a one-way ANOVA fixed effects model and is found by (MSB-MSW)/(MSB+ (nr-1)*MSW))

ICC2: A random sample of $k$ judges rate each target. The measure is one of absolute agreement in the ratings. Found as (MSB- MSE)/(MSB + (nr-1)*MSE + nr*(MSJ-MSE)/nc)

ICC3: A fixed set of $k$ judges rate each target. There is no generalization to a larger population of judges. (MSB - MSE)/(MSB+ (nr-1)*MSE)

Then, for each of these cases, is reliability to be estimated for a single rating or for the average of k ratings? (The 1 rating case is equivalent to the average intercorrelation, the k rating case to the Spearman Brown adjusted reliability.)
ICC1 is sensitive to differences in means between raters and is a measure of absolute agreement.
ICC2 and ICC3 remove mean differences between judges, but are sensitive to interactions of raters by judges. The difference between ICC2 and ICC3 is whether raters are seen as fixed or random effects.

ICC1k, ICC2k, ICC3K reflect the means of $k$ raters.
The intraclass correlation is used if raters are all of the same "class". That is, there is no logical way of distinguishing them. Examples include correlations between pairs of twins, correlations between raters. If the variables are logically distinguishable (e.g., different items on a test), then the more typical coefficient is based upon the inter-class correlation (e.g., a Pearson r) and a statistic such as alpha or omega might be used.

## Value

| results | A matrix of 6 rows and 8 columns, including the ICCs, F test, p values, and <br> confidence limits |
| :--- | :--- |
| summary | The anova summary table |
| stats | The anova statistics |
| MSW | Mean Square Within based upon the anova |

Note
The results for the Lower and Upper Bounds for ICC $(2, k)$ do not match those of SPSS 9 or 10, but do match the definitions of Shrout and Fleiss. SPSS seems to have been using the formula in McGraw and Wong, but not the errata on p 390. They seem to have fixed it in more recent releases (15).

## Author(s)

William Revelle

## References

Shrout, Patrick E. and Fleiss, Joseph L. Intraclass correlations: uses in assessing rater reliability. Psychological Bulletin, 1979, 86, 420-3428.
McGraw, Kenneth O. and Wong, S. P. (1996), Forming inferences about some intraclass correlation coefficients. Psychological Methods, 1, 30-46. + errata on page 390.

Revelle, W. (in prep) An introduction to psychometric theory with applications in R. Springer. (working draft available at http://personality-project.org/r/book/

## Examples


iclust iclust: Item Cluster Analysis - Hierarchical cluster analysis using psychometric principles

## Description

A common data reduction technique is to cluster cases (subjects). Less common, but particularly useful in psychological research, is to cluster items (variables). This may be thought of as an alternative to factor analysis, based upon a much simpler model. The cluster model is that the correlations between variables reflect that each item loads on at most one cluster, and that items that load on those clusters correlate as a function of their respective loadings on that cluster and items that define different clusters correlate as a function of their respective cluster loadings and the intercluster correlations. Essentially, the cluster model is a Very Simple Structure factor model of complexity one (see VSS).

This function applies the iclust algorithm to hierarchically cluster items to form composite scales. Clusters are combined if coefficients alpha and beta will increase in the new cluster.
Alpha, the mean split half correlation, and beta, the worst split half correlation, are estimates of the reliability and general factor saturation of the test. (See also the omega function to estimate McDonald's coeffients $\omega_{h}$ and $\omega_{t}$ )

## Usage

```
iclust(r.mat, nclusters=0, alpha=3, beta=1, beta.size=4, alpha.size=3,
correct=TRUE, correct.cluster=TRUE, reverse=TRUE, beta.min=.5, output=1, digits=2,la
n.iterations = 0,title="iclust",plot=TRUE,weighted=TRUE,cor.gen=TRUE,SMC=TRUE)
ICLUST(r.mat, nclusters=0, alpha=3, beta=1, beta.size=4, alpha.size=3,
correct=TRUE,correct.cluster=TRUE, reverse=TRUE, beta.min=.5, output=1, digits=2,la
n.iterations = 0,title="ICLUST",plot=TRUE,weighted=TRUE,cor.gen=TRUE,SMC=TRUE)
#iclust(r.mat) #use all defaults
#iclust(r.mat,nclusters =3) #use all defaults and if possible stop at 3 clusters
#ICLUST(r.mat, output =3) #long output shows clustering history
#ICLUST(r.mat, n.iterations =3) #clean up solution by item reassignment
```


## Arguments

| r.mat | A correlation matrix or data matrix/data.frame. (If r.mat is not square i.e, a correlation matrix, the data are correlated using pairwise deletion. |
| :---: | :---: |
| nclusters | Extract clusters until nclusters remain (default will extract until the other criteria are met or 1 cluster, whichever happens first). See the discussion below for alternative techniques for specifying the number of clusters. |
| alpha | Apply the increase in alpha criterion (0) never or for (1) the smaller, 2) the average, or 3 ) the greater of the separate alphas. (default $=3$ ) |
| beta | Apply the increase in beta criterion (0) never or for (1) the smaller, 2 ) the average, or 3 ) the greater of the separate betas. (default $=1$ ) |
| beta.size | Apply the beta criterion after clusters are of beta.size (default $=4$ ) |
| alpha.size | Apply the alpha criterion after clusters are of size alpha.size (default $=3$ ) |
| correct | Correct correlations for reliability (default = TRUE) |
| correct.cluster |  |
|  | Correct cluster -sub cluster correlations for reliability of the sub cluster, default is TRUE)) |
| reverse | Reverse negative keyed items (default = TRUE |
| beta.min | Stop clustering if the beta is not greater than beta.min (default $=.5$ ) |
| output | 1) short, 2) medium, 3 ) long output (default =1) |
| labels | vector of item content or labels. If NULL, then the colnames are used. If FALSE, then labels are V1 .. Vn |
| cut | sort cluster loadings $>$ absolute $($ cut $)($ default $=0)$ |
| .iteratio | iterate the solution n.iterations times to "purify" the clusters (default $=0$ ) |


| digits | Precision of digits of output (default =2) |
| :--- | :--- |
| title | Title for this run <br> plot |
| should ICLUST.rgraph be called automatically for plotting (requires Rgraphviz |  |
| default=TRUE) |  |
| wer.gen | Weight the intercluster correlation by the size of the two clusters (TRUE) or do <br> not weight them (FALSE) |
| SMC | When correlating clusters with subclusters, base the correlations on the general <br> factor (default) or general + group (cor.gen=FALSE) <br> When estimating cluster-item correlations, use the smcs as the estimate of an <br> item communality (SMC=TRUE) or use the maximum correlation (SMC=FALSE). |

## Details

Extensive documentation and justification of the algorithm is available in the original MBR 1979 http://personality-project.org/revelle/publications/iclust.pdf paper. Further discussion of the algorithm and sample output is available on the personality-project.org web page: http://personality-project.org/r/r.ICLUST.html
The results are best visualized using ICLUST.graph, the results of which can be saved as a dot file for the Graphviz program. http: / /www.graphviz.org/. With the installation of Rgraphviz, ICLUST will automatically provide cluster graphs.
A common problem in the social sciences is to construct scales or composites of items to measure constructs of theoretical interest and practical importance. This process frequently involves administering a battery of items from which those that meet certain criteria are selected. These criteria might be rational, empirical,or factorial. A similar problem is to analyze the adequacy of scales that already have been formed and to decide whether the putative constructs are measured properly. Both of these problems have been discussed in numerous texts, as well as in myriad articles. Proponents of various methods have argued for the importance of face validity, discriminant validity, construct validity, factorial homogeneity, and theoretical importance.

Revelle (1979) proposed that hierachical cluster analysis could be used to estimate a new coefficient (beta) that was an estimate of the general factor saturation of a test. More recently, Zinbarg, Revelle, Yovel and Li (2005) compared McDonald's Omega to Chronbach's alpha and Revelle's beta. They conclude that $\omega_{h}$ hierarchical is the best estimate. An algorithm for estimating omega is available as part of this package.
Revelle and Zinbarg (2009) discuss alpha, beta, and omega, as well as other estimates of reliability.
The original ICLUST program was written in FORTRAN to run on CDC and IBM mainframes and was then modified to run in PC-DOS. The R version of iclust is a completely new version written for the psych package. Please email me if you want help with this version of iclust or if you desire more features.

A requested feature (not yet available) is to specify certain items as forming a cluster. That is, to do confirmatory cluster analysis.
The program currently has three primary functions: cluster, loadings, and graphics.
In June, 2009, the option of weighted versus unweighted beta was introduced. Unweighted beta calculates beta based upon the correlation between two clusters, corrected for test length using the Spearman-Brown prophecy formala, while weighted beta finds the average interitem correlation between the items within two clusters and then finds beta from this. That is, for two clusters A and
$B$ of size N and M with between average correlation rb , weighted beta is $(\mathrm{N}+\mathrm{M})^{\wedge} 2 \mathrm{rb} /(\mathrm{Va}+\mathrm{Vb}+$ 2 Cab ). Raw (unweighted) beta is $2 \mathrm{rab} /(1+\mathrm{rab})$ where $\mathrm{rab}=\mathrm{Cab} / \mathrm{sqrt}(\mathrm{VaVb})$. Weighted beta seems a more appropriate estimate and is now the default. Unweighted beta is still available for consistency with prior versions.
Also modified in June, 2009 was the way of correcting for item overlap when calculating the clustersubcluster correlations for the graphic output. This does not affect the final cluster solution, but does produce slightly different path values. In addition, there are two ways to solve for the cluster subcluster correlation.

Given the covariance between two clusters, Cab with average $\mathrm{rab}=\mathrm{Cab} /(\mathrm{N} * \mathrm{M})$, and cluster variances Va and Vb with $\mathrm{Va}=\mathrm{N}+\mathrm{N}^{*}(\mathrm{~N}-1)^{*}$ ra then the correlation of cluster A with the combined cluster AB is either
a) $\left(\left(\mathrm{N}^{\wedge} 2\right) \mathrm{ra}+\mathrm{Cab}\right) / \mathrm{sqrt}(\mathrm{Vab} * \mathrm{Va})($ option cor.gen=TRUE) or b) $(\mathrm{Va}-\mathrm{N}+\mathrm{Nra}+\mathrm{Cab}) / \mathrm{sqrt}(\mathrm{Vab} * \operatorname{Va})$ (option cor.gen=FALSE)
The default is to use cor.gen=TRUE.
Although iclust will give what it thinks is the best solution in terms of the number of clusters to extract, the user will sometimes disagree. To get more clusters than the default solution, just set the nclusters parameter to the number desired. However, to get fewer than meet the alpha and beta criteria, it is sometimes necessary to set alpha $=0$ and beta $=0$ and then set the nclusters to the desired number.

Clustering 24 tests of mental ability
A sample output using the 24 variable problem by Harman can be represented both graphically and in terms of the cluster order. The default is to produce graphics using the Rgraphviz package (from BioConductor). Because this package is sometimes hard to install, there is an alternative option to write the output out using the dot language. This will create a graphic suitable for any viewing program that uses the dot language. ICLUST. graph produces the dot code for Graphviz. Somewhat lower resolution graphs with fewer options are available in the ICLUST.rgraph function which requires Rgraphviz. Dot code can be viewed directly in Graphviz or can be tweaked using commercial software packages (e.g., OmniGraffle)
Note that for this problem, with the default parameters, the data form one large cluster. (This is consistent with the Very Simple Structure (VSS) output as well, which shows a clear one factor solution for complexity 1 data.)
An alternative solution is to ask for a somewhat more stringent set of criteria and require an increase in the size of beta for all clusters greater than 3 variables. This produces a 4 cluster solution.
It is also possible to use the original parameter settings, but ask for a 4 cluster solution.
At least for the Harman 24 mental ability measures, it is interesting to compare the cluster pattern matrix with the oblique rotation solution from a factor analysis. The factor congruence of a four factor oblique pattern solution with the four cluster solution is $>.99$ for three of the four clusters and $>.97$ for the fourth cluster.

To see the graphic output go to http://personality-project.org/r/r.ICLUST.html or use ICLUST. rgraph (requires Rgraphviz).

## Value

$$
\text { title } \quad \text { Name of this run }
$$

$$
\begin{array}{ll}
\text { results } & \begin{array}{l}
\text { A list containing the step by step cluster history, including which pair was } \\
\text { grouped, what were the alpha and betas of the two groups and of the combined } \\
\text { group. } \\
\text { Note that the alpha values are "standardized alphas" based upon the correlation } \\
\text { matrix, rather than the raw alphas that will come from score. items } \\
\text { The print.psych and summary.psych functions will print out just the must im- } \\
\text { portant results. }
\end{array} \\
\text { corrected } & \text { The raw and corrected for alpha reliability cluster intercorrelations. } \\
\text { clusters } & \begin{array}{l}
\text { a matrix of }-1,0, \text { and } 1 \text { values to define cluster membership. }
\end{array} \\
\text { purified } & \text { A list of the cluster definitions and cluster loadings of the purified solution. } \\
\text { cluster.fit, } \begin{array}{l}
\text { To show just the most salient items, use the cutoff option in print. psych } \\
\text { structure.fit, pattern.fit } \\
\text { There are a number of ways to evaluate how well any factor or cluster matrix }
\end{array} \\
\text { reproduces the original matrix. Cluster fit considers how well the clusters fit if } \\
\text { only correlations with clusters are considered. Structure fit evaluates } \mathrm{R}=\mathrm{CC} \text { ' } \\
\text { while pattern fit evaluate } \mathrm{R}=\mathrm{C} \text { inverse (phi) C' where } \mathrm{C} \text { is the cluster loading } \\
\text { matrix, and phi is the intercluster correlation matrix. }
\end{array}
$$

## Note

iclust draws graphical displays with or without using Rgraphiviz. Rgraphviz produces slightly better graphics and will also export in the dot language for further massaging of the graphic. If Rgraphviz is not installed, ICLUST.graph will issue a warning message but continue. If, however, Rgraphviz is not properly installed, ICLUST.graph will think it is available, try to produce a graph, and fail (ungracefully). The solution to this problem is to specify plot=FALSE and the create graphs using the dot language. See the last example. With the introduction of the diagram functions, iclust now draws using iclust.diagram which is not as pretty, but more stable.

## Author(s)

William Revelle

## References

Revelle, W. Hierarchical Cluster Analysis and the Internal Structure of Tests. Multivariate Behavioral Research, 1979, 14, 57-74.

Revelle, W. and Zinbarg, R. E. (2009) Coefficients alpha, beta, omega and the glb: comments on Sijtsma. Psychometrika, 2009.
http://personality-project.org/revelle/publications/iclust.pdf
See also more extensive documentation at http://personality-project.org/r/r.ICLUST. html and
Revelle, W. (in prep) An introduction to psychometric theory with applications in R. To be published by Springer. (working draft available at http://personality-project.org/r/book/

## See Also

ICLUST.graph,ICLUST.cluster, cluster.fit, VSS, omega

## Examples

```
test.data <- Harman74.cor$cov
ic.out <- ICLUST(test.data)
summary(ic.out)
ic.out <- ICLUST(test.data,nclusters =4) #use all defaults and stop at 4 clusters
ic.out1 <- ICLUST(test.data,beta=3,beta.size=3) #use more stringent criteria
print(ic.out1)
plot(ic.out) #this shows the spatial representation
ic.no.graph <- ICLUST(test.data,plot=FALSE)
dot.graph <- ICLUST.graph(ic.no.graph,out.file="test.ICLUST.graph.dot") #use a dot graphic
```


## Description

The guts of the ICLUST algorithm. Called by ICLUST See ICLUST for description.

## Usage

ICLUST.cluster(r.mat, ICLUST.options)

## Arguments

r.mat
A correlation matrix
ICLUST.options
A list of options (see ICLUST)

## Details

See ICLUST

## Value

A list of cluster statistics, described more fully in ICLUST

| comp1 | Description of 'comp1' |
| :--- | :--- |
| comp2 | Description of 'comp2' |

## Note

Although the main code for ICLUST is here in ICLUST.cluster, the more extensive documentation is for ICLUST.

## Author(s)

William Revelle

## References

Revelle, W. 1979, Hierarchical Cluster Analysis and the Internal Structure of Tests. Multivariate Behavioral Research, 14, 57-74. http: / /personality-project.org/revelle/publications/ iclust.pdf
See also more extensive documentation at http://personality-project.org/r/r.ICLUST. html

## See Also

ICLUST.graph,ICLUST, cluster.fit , VSS, omega

```
iclust.diagram Draw an ICLUST hierarchical cluster structure diagram
```


## Description

Given a cluster structure determined by ICLUST, create a graphic structural diagram using graphic functions in the psych package To create dot code to describe the ICLUST output with more precision, use ICLUST. graph. If Rgraphviz has been successfully installed, the alternative is to use ICLUST.rgraph.

## Usage

iclust.diagram(ic, labels = NULL, short = FALSE, digits = 2, cex = NULL, min.size

## Arguments

| ic | Output from ICLUST |
| :--- | :--- |
| labels | labels for variables (if not specified as rownames in the ICLUST output |
| short | if short=TRUE, variable names are replaced with Vn |
| digits | Round the path coefficients to digits accuracy |
| cex | The standard graphic control parameter for font size modifications. This can be <br> used to make the labels bigger or smaller than the default values. |
| min.size | Don't provide statistics for clusters less than min.size |
| e.size | size of the ellipses with the cluster statistics. |
| colors | postive and negative |
| main | The main graphic title |

## Details

iclust.diagram provides most of the power of ICLUST. rgraph without the difficulties involved in installing Rgraphviz. It is called automatically from ICLUST.

## Value

Graphical output summarizing the hierarchical cluster structure. The graph is drawn using the diagram functions (e.g., dia.curve, dia.arrow, dia.rect, dia.ellipse) created as a work around to Rgraphviz.

Note
Suggestions for improving the graphic output are welcome.

## Author(s)

William Revelle

## References

Revelle, W. Hierarchical Cluster Analysis and the Internal Structure of Tests. Multivariate Behavioral Research, 1979, 14, 57-74.

## See Also

```
ICLUST
```


## Examples

```
v9 <- sim.hierarchical()
v9c <- ICLUST(v9)
test.data <- Harman74.cor$cov
ic.out <- ICLUST(test.data)
```

```
ICLUST.graph create control code for ICLUST graphical output
```


## Description

Given a cluster structure determined by ICLUST, create dot code to describe the ICLUST output. To use the dot code, use either http://www.graphviz.org/ Graphviz or a commercial viewer (e.g., OmniGraffle).

## Usage

```
ICLUST.graph(ic.results, out.file,min.size=1, short = FALSE,labels=NULL,
size = c(8, 6), node.font = c("Helvetica", 14), edge.font = c("Helvetica", 12),
    rank.direction=c("RL","TB","LR","BT"), digits = 2, title = "ICLUST", ...)
```


## Arguments

| ic.results | output list from ICLUST |
| :--- | :--- |
| out.file | name of output file (defaults to console) <br> min.size |
| draw a smaller node (without all the information) for clusters < min.size - useful <br> for large problems |  |
| short | if short==TRUE, don't use variable names <br> labels <br> size |
| vector of text labels (contents) for the variables |  |
| node.font | size of output |
| edge.font to use for nodes in the graph |  |
| rank. direction | Font to use for the labels of the arrows (edges) |
| digits | LR or RL |
| title number of digits to show |  |
| l. | any title |
|  | other options to pass |

## Details

Will create (or overwrite) an output file and print out the dot code to show a cluster structure. This dot file may be imported directly into a dot viewer (e.g., http://www.graphviz.org/). The "dot" language is a powerful graphic description language that is particulary appropriate for viewing cluster output. Commercial graphics programs (e.g., OmniGraffle) can also read (and clean up) dot files.

ICLUST.graph takes the output from ICLUST results and processes it to provide a pretty picture of the results. Original variables shown as rectangles and ordered on the left hand side (if rank direction is RL) of the graph. Clusters are drawn as ellipses and include the alpha, beta, and size of the cluster. Edges show the cluster intercorrelations.
It is possible to trim the output to not show all cluster information. Clusters < min.size are shown as small ovals without alpha, beta, and size information.

## Value

Output is a set of dot commands written either to console or to the output file. These commands may then be used as input to any "dot" viewer, e.g., Graphviz.

## Author(s)

```
<revelle@northwestern.edu >
http://personality-project.org/revelle.html
```


## References

ICLUST: http://personality-project.org/r/r.iclust.html

## See Also

```
VSS.plot, ICLUST
```


## Examples

```
## Not run:
test.data <- Harman74.cor$cov
ic.out <- ICLUST(test.data)
out.file <- file.choose(new=TRUE) #create a new file to write the plot commands to
ICLUST.graph(ic.out,out.file)
now go to graphviz (outside of R) and open the out.file you created
print(ic.out,digits=2)
## End(Not run)
#test.data <- Harman74.cor$cov
#my.iclust <- ICLUST(test.data)
#ICLUST.graph(my.iclust)
#
#
#digraph ICLUST {
# rankdir=RL;
# size="8,8";
# node [fontname="Helvetica" fontsize=14 shape=box, width=2];
# edge [fontname="Helvetica" fontsize=12];
# label = "ICLUST";
# fontsize=20;
#V1 [label = VisualPerception];
#V2 [label = Cubes];
#V3 [label = PaperFormBoard];
#V4 [label = Flags];
#V5 [label = GeneralInformation];
#V6 [label = PargraphComprehension];
#V7 [label = SentenceCompletion];
#V8 [label = WordClassification];
#V9 [label = WordMeaning];
#V10 [label = Addition];
#V11 [label = Code];
#V12 [label = CountingDots];
#V13 [label = StraightCurvedCapitals];
#V14 [label = WordRecognition];
#V15 [label = NumberRecognition];
#V16 [label = FigureRecognition];
#V17 [label = ObjectNumber];
#V18 [label = NumberFigure];
#V19 [label = FigureWord];
#V20 [label = Deduction];
#V21 [label = NumericalPuzzles];
#V22 [label = ProblemReasoning];
#V23 [label = SeriesCompletion];
#V24 [label = ArithmeticProblems];
```

```
#node [shape=ellipse, width ="1"];
#C1-> V9 [ label = 0.78 ];
#C1-> V5 [ label = 0.78 ];
#C2-> V12 [ label = 0.66 ];
#C2-> V10 [ label = 0.66 ];
#C3-> V18 [ label = 0.53 ];
#C3-> V17 [ label = 0.53 ];
#C4-> V23 [ label = 0.59 ];
#C4-> V20 [ label = 0.59 ];
#C5-> V13 [ label = 0.61 ];
#C5-> V11 [ label = 0.61 ];
#C6-> V7 [ label = 0.78 ];
#C6-> V6 [ label = 0.78 ];
#C7-> V4 [ label = 0.55 ];
#C7-> V1 [ label = 0.55 ];
#C8-> V16 [ label = 0.5 ];
#C8-> V14 [ label = 0.49 ];
#C9-> C1 [ label = 0.86 ];
#C9-> C6 [ label = 0.86 ];
#C10-> C4 [ label = 0.71 ];
#C10-> V22 [ label = 0.62 ];
#C11-> V21 [ label = 0.56 ];
#C11-> V24 [ label = 0.58 ];
#C12-> C10 [ label = 0.76 ];
#C12-> C11 [ label = 0.67 ];
#C13-> C8 [ label = 0.61 ];
#C13-> V15 [ label = 0.49 ];
#C14-> C2 [ label = 0.74 ];
#C14-> C5 [ label = 0.72 ];
#C15-> V3 [ label = 0.48 ];
#C15-> C7 [ label = 0.65 ];
#C16-> V19 [ label = 0.48 ];
#C16-> C3 [ label = 0.64 ];
#C17-> V8 [ label = 0.62 ];
#C17-> C12 [ label = 0.8 ];
#C18-> C17 [ label = 0.82 ];
#C18-> C15 [ label = 0.68 ];
#C19-> C16 [ label = 0.66 ];
#C19-> C13 [ label = 0.65 ];
#C20-> C19 [ label = 0.72 ];
#C20-> C18 [ label = 0.83 ];
#C21-> C20 [ label = 0.87 ];
#C21-> C9 [ label = 0.76 ];
#C22-> 0 [ label = 0 ];
#C22-> 0 [ label = 0 ];
#C23-> 0 [ label = 0 ];
#C23-> 0 [ label = 0 ];
#C1 [label = "C1\n alpha= 0.84\n beta= 0.84\nN= 2"] ;
#C2 [label = "C2\n alpha= 0.74\n beta= 0.74\nN= 2"] ;
#C3 [label = "C3\n alpha= 0.62\n beta= 0.62\nN= 2"] ;
#C4 [label = "C4\n alpha= 0.67\n beta= 0.67\nN= 2"] ;
#C5 [label = "C5\n alpha= 0.7\n beta= 0.7\nN= 2"] ;
#C6 [label = "C6\n alpha= 0.84\n beta= 0.84\nN= 2"] ;
```

```
#C7 [label = "C7\n alpha= 0.64\n beta= 0.64\nN= 2"] ;
#C8 [label = "C8\n alpha= 0.58\n beta= 0.58\nN= 2"] ;
#C9 [label = "C9\n alpha= 0.9\n beta= 0.87\nN= 4"] ;
#C10 [label = "C10\n alpha= 0.74\n beta= 0.71\nN= 3"] ;
#C11 [label = "C11\n alpha= 0.62\n beta= 0.62\nN= 2"] ;
#C12 [label = "C12\n alpha= 0.79\n beta= 0.74\nN= 5"] ;
#C13 [label = "C13\n alpha= 0.64\n beta= 0.59\nN=3"] ;
#C14 [label = "C14\n alpha= 0.79\n beta= 0.74\nN= 4"] ;
#C15 [label = "C15\n alpha= 0.66\n beta= 0.58\nN= 3"] ;
#C16 [label = "C16\n alpha= 0.65\n beta= 0.57\nN= 3"] ;
#C17 [label = "C17\n alpha= 0.81\n beta= 0.71\nN= 6"] ;
#C18 [label = "C18\n alpha= 0.84\n beta= 0.75\nN= 9"] ;
#C19 [label = "C19\n alpha= 0.74\n beta= 0.65\nN= 6"] ;
#C20 [label = "C20\n alpha= 0.87\n beta= 0.74\nN= 15"] ;
#C21 [label = "C21\n alpha= 0.9\n beta= 0.77\nN= 19"] ;
#C22 [label = "C22\n alpha= 0\n beta= 0\nN= 0"] ;
#C23 [label = "C23\n alpha= 0\n beta= 0\nN= 0"] ;
#{ rank=same;
#V1;V2;V3;V4;V5;V6;V7;V8;V9;V10;V11;V12;V13;V14;V15;V16;V17;V18;V19;V20;V21;V22;V23;V24;}}
#
#copy the above output to Graphviz and draw it
#see \url{http://personality-project.org/r/r.ICLUST.html} for an example.
```

ICLUST.rgraph Draw an ICLUST graph using the Rgraphviz package

## Description

Given a cluster structure determined by ICLUST, create a rgraphic directly using Rgraphviz. To create dot code to describe the ICLUST output with more precision, use ICLUST.graph. As an option, dot code is also generated and saved in a file. To use the dot code, use either http: //www.graphviz.org/ Graphviz or a commercial viewer (e.g., OmniGraffle).

## Usage

ICLUST.rgraph(ic.results, out.file $=$ NULL, min.size $=1$, short $=$ FALSE, labels $=\mathrm{NL}$

## Arguments

ic.results output list from ICLUST
out.file File name to save optional dot code.
min.size draw a smaller node (without all the information) for clusters < min.size - useful for large problems
short if short==TRUE, don't use variable names
labels vector of text labels (contents) for the variables
size size of output

```
node.font Font to use for nodes in the graph
edge.font Font to use for the labels of the arrows (edges)
rank.direction
    LR or TB or RL
digits number of digits to show
title any title
label.font The variable labels can be a different size than the other nodes. This is particu-
        larly helpful if the number of variables is large or the labels are long.
. . . other options to pass
```


## Details

Will create (or overwrite) an output file and print out the dot code to show a cluster structure. This dot file may be imported directly into a dot viewer (e.g., http://www.graphviz.org/). The "dot" language is a powerful graphic description language that is particulary appropriate for viewing cluster output. Commercial graphics programs (e.g., OmniGraffle) can also read (and clean up) dot files.

ICLUST.rgraph takes the output from ICLUST results and processes it to provide a pretty picture of the results. Original variables shown as rectangles and ordered on the left hand side (if rank direction is RL) of the graph. Clusters are drawn as ellipses and include the alpha, beta, and size of the cluster. Edges show the cluster intercorrelations.
It is possible to trim the output to not show all cluster information. Clusters $<$ min.size are shown as small ovals without alpha, beta, and size information.

## Value

Output is a set of dot commands written either to console or to the output file. These commands may then be used as input to any "dot" viewer, e.g., Graphviz.
ICLUST.rgraph is a version of ICLUST. graph that uses Rgraphviz to draw on the screen as well.
Additional output is drawn to main graphics screen.

## Note

Requires Rgraphviz

## Author(s)

```
<revelle@northwestern.edu >
http://personality-project.org/revelle.html
```


## References

ICLUST: http://personality-project.org/r/r.iclust.html

## See Also

VSS.plot, ICLUST

## Examples

```
test.data <- Harman74.cor$cov
ic.out <- ICLUST(test.data) #uses iclust.diagram instead
```

```
ICLUST.sort Sort items by absolute size of cluster loadings
```


## Description

Given a cluster analysis or factor analysis loadings matrix, sort the items by the (absolute) size of each column of loadings. Used as part of ICLUST and SAPA analyses.

## Usage

ICLUST.sort(ic.load, cut $=0$, labels $=$ NULL,keys=FALSE)

## Arguments

ic.load The output from a factor or principal components analysis, or from ICLUST, or a matrix of loadings.
cut Do not include items in clusters with absolute loadings less than cut
labels labels for each item.
keys should cluster keys be returned? Useful if clusters scales are to be scored.

## Details

When interpreting cluster or factor analysis outputs, is is useful to group the items in terms of which items have their biggest loading on each factor/cluster and then to sort the items by size of the absolute factor loading.
A stable cluster solution will be one in which the output of these cluster definitions does not vary when clusters are formed from the clusters so defined.

With the keys=TRUE option, the resulting cluster keys may be used to score the original data or the correlation matrix to form clusters from the factors.

Value
sorted A data.frame of item numbers, item contents, and item x factor loadings.
cluster A matrix of $-1,0,1 \mathrm{~s}$ defining each item by the factor/cluster with the row wise largest absolute loading.

## Note

Although part of the ICLUST set of programs, this is also more useful for factor or principal components analysis.

## Author(s)

William Revelle

## References

```
http://personality-project.org/r/r.ICLUST.html
```


## See Also

ICLUST.graph,ICLUST.cluster, cluster.fit , VSS, factor2cluster

```
income US family income from US census 2008
```


## Description

US census data on family income from 2008

## Usage

data(income)

## Format

A data frame with 44 observations on the following 4 variables.
value lower boundary of the income group
count Number of families within that income group
mean Mean of the category
prop proportion of families

## Details

The distribution of income is a nice example of a log normal distribution. It is also an interesting example of the power of graphics. It is quite clear when graphing the data that income statistics are bunched to the nearest 5 K . That is, there is a clear sawtooth pattern in the data.
The all.income set is interpolates intervening values for $100-150 \mathrm{~K}, 150-200 \mathrm{~K}$ and $200-250 \mathrm{~K}$

## Source

US Census: Table HINC-06. Income Distribution to $\$ 250,000$ or More for Households: 2008
http://www.census.gov/hhes/www/cpstables/032009/hhinc/new06_000.htm

## Examples

```
data(income)
with(income[1:40,], plot(mean,prop, main="US family income for 2008",xlab="income", ylab="Pr
with (income[1:40,], points(lowess(mean,prop,f=.3),typ="l"))
describe(income)
with(all.income, plot(mean,prop, main="US family income for 2008",xlab="income", ylab="Propo
with (all.income[1:50,], points(lowess(mean,prop,f=.25),typ="l"))
#curve(100000* dlnorm(x, 10.8, .8), x = c(0,250000),ylab="Proportion")
```

interp.median Find the interpolated sample median, quartiles, or specific quantiles for a vector, matrix, or data frame

## Description

For data with a limited number of response categories (e.g., attitude items), it is useful treat each response category as range with width, $w$ and linearly interpolate the median, quartiles, or any quantile value within the median response.

## Usage

```
interp.median(x, w = 1,na.rm=TRUE)
interp.quantiles(x, q = .5, w = 1,na.rm=TRUE)
interp.quartiles(x,w=1,na.rm=TRUE)
interp.boxplot(x,w=1,na.rm=TRUE)
interp.values(x,w=1,na.rm=TRUE)
interp.qplot.by(y,x,w=1,na.rm=TRUE,xlab="group",ylab="dependent",ylim=NULL,arrow.le
```


## Arguments

| x | input vector |
| :--- | :--- |
| q | quantile to estimate $(0<\mathrm{q}<1$ |
| w | category width |
| y | input vector for interp.qplot.by |
| na.rm | should missing values be removed |
| xlab | x label |
| ylab | Y label |
| ylim | limits for the y axis |
| arrow.len | length of arrow in interp.qplot.by |
| typ | plot type in interp.qplot.by |
| add | add the plot or not |
| . . | additional parameters to plotting function |

## Details

If the total number of responses is N , with median, M , and the number of responses at the median value, $\mathrm{Nm}>1$, and $\mathrm{Nb}=$ the number of responses less than the median, then with the assumption that the responses are distributed uniformly within the category, the interpolated median is M -.5 w $+\mathrm{w}^{*}(\mathrm{~N} / 2-\mathrm{Nb}) / \mathrm{Nm}$.

The generalization to 1st, 2nd and 3rd quartiles as well as the general quantiles is straightforward.
A somewhat different generalization allows for graphic presentation of the difference between interpolated and non-interpolated points. This uses the interp.values function.

If the input is a matrix or data frame, quantiles are reported for each variable.

## Value

| im | interpolated median(quantile) |
| :--- | :--- |
| v | interpolated values for all data points |

## See Also

median

## Examples

```
interp.median(c(1,2,3,3,3)) # compare with median = 3
interp.median(c (1, 2, 2,5))
interp.quantiles(c(1, 2, 2,5),.25)
x <- sample(10,100,TRUE)
interp.quartiles(x)
#
x <- c(1, 1,2,2,2,3,3,3,3,4,5,1,1,1,2,2,3,3,3,3,4,5,1,1,1,2,2,3,3,3,3,4,2)
y<- c(1,2,3,3,3,3,4,4,4,5,5,1,2,3,3,3,3,4,4,5,5,5,1,5,3,3,3,3,4,4,4,5,5)
x <- x[order(x)] #sort the data by ascending order to make it clearer
y <- y[order (y)]
xv <- interp.values(x)
yv <- interp.values(y)
barplot(x,space=0,xlab="ordinal position",ylab="value")
lines(1:length(x)-.5,xv)
points(c(length(x)/4,length(x)/2, 3*length(x)/4),interp.quartiles(x))
barplot(y,space=0,xlab="ordinal position",ylab="value")
lines(1:length(y) - .5,yv)
points(c(length(y)/4,length(y)/2,3*length(y)/4),interp.quartiles(y))
data(galton)
interp.median(galton)
interp.qplot.by(galton$child,galton$parent,ylab="child height"
,xlab="Mid parent height")
```

iqitems 14 multiple choice IQ items

## Description

14 multiple choice ability items were included as part of the Synthetic Aperture Personality Assessment (SAPA) web based personality assessment project. The data from 1000 subjects are included here as a demonstration set for scoring multiple choice inventories and doing basic item statistics.

## Usage

data(iqitems)

## Format

A data frame with 1000 observations on the following 14 variables.
iq1 In the following number series, what number comes next?
iq8 Please mark the word that does not match the other words:
iq10 If you rearrange the letters ATNHIDLA, you will have the name of a:
iq15 If Jerks are Perks and some Perks are Lerks, then some Jerks are definitely Lerks. This statement is:
iq20 How many total legs do two ducks and three dogs have?
iq44 Matrix reasoning 2
iq47 Matrix reasoning 5
iq2 In the following number series, what number comes next? 124712
iq11 The opposite of a 'stubborn' person is a', 'person.
iq16 Zach is taller than Matt and Richard is shorter than Zach. Which of the following statements would be most accurate?
iq32 If the day before yesterday is three days after Saturday then what day is today?
iq37 In the following alphanumeric series, what letter comes next? Q, S, N, P, L
iq43 Matrix Reasoning 1
iq49 Matrix Reasoning 9

## Details

14 items were sampled from 54 items given as part of the SAPA project (Revelle, Wilt and Rosenthal, 2009) to develop online measures of ability.

## Source

http://personality-project.org

## References

Revelle, William, Wilt, Joshua, and Rosenthal, Allen (2009) Personality and Cognition: The PersonalityCognition Link. In Gruszka, Alexandra and Matthews, Gerald and Szymura, Blazej (Eds.) Handbook of Individual Differences in Cognition: Attention, Memory and Executive Control, Springer.

## Examples

```
data(iqitems)
iq.keys <- c(4,4,3,1,4,3,2,3,1,4,1,3,4,3)
score.multiple.choice(iq.keys,iqitems)
```


## irt.1p Item Response Theory estimate of theta (ability) using a Rasch (like)

 model
## Description

Item Response Theory models individual responses to items by estimating individual ability (theta) and item difficulty (diff) parameters. This is an early and crude attempt to capture this modeling procedure. A better procedure is to use irt.fa.

## Usage

```
irt.person.rasch(diff, items)
irt.0p(items)
irt.1p(delta,items)
irt.2p(delta,beta,items)
```


## Arguments

diff A vector of item difficulties -probably taken from irt.item.diff.rasch
items A matrix of 0,1 items nrows $=$ number of subjects, ncols $=$ number of items
delta delta is the same as diff and is the item difficulty parameter
beta beta is the item discrimination parameter found in irt.discrim

## Details

A very preliminary IRT estimation procedure. Given scores xij for ith individual on jth item Classical Test Theory ignores item difficulty and defines ability as expected score : abilityi = theta(i) $=x(i$.$) A zero parameter model rescales these mean scores from 0$ to 1 to a quasi logistic scale ranging from - 4 to 4 This is merely a non-linear transform of the raw data to reflect a logistic mapping.

Basic 1 parameter (Rasch) model considers item difficulties (delta $j$ ): $p$ (correct on item $j$ for the ith subject Itheta i, deltaj $)=1 /(1+\exp ($ deltaj - thetai $))$ If we have estimates of item difficulty (delta), then we can find theta i by optimization

Two parameter model adds item sensitivity (beta j ): p (correct on item j for subject i lthetai, deltaj, betaj $)=1 /(1+\exp ($ betaj $*($ deltaj- theta $i)))$ Estimate delta, beta, and theta to maximize fit of model to data.
The procedure used here is to first find the item difficulties assuming theta $=0$ Then find theta given those deltas Then find beta given delta and theta.

This is not an "official" way to do IRT, but is useful for basic item development.

## Value

a data.frame with estimated ability (theta) and quality of fit. (for irt.person.rasch)
a data.frame with the raw means, theta0, and the number of items completed

## Note

Not recommended for serious use. This code is under development. Much better functions are in the ltm and eRm packages. Similar analyses can be done using irt.fa

## Author(s)

William Revelle

## See Also

sim.irt, sim.rasch, logistic, irt.fa, tetrachoric, irt.item.diff.rasch

irt.fa | Item Response Analysis by factor analysis of tetrachoric/polychoric |
| :--- |
| correlations |

## Description

Although factor analysis and Item Response Theory seem to be very different models of binary data, they can provide equivalent parameter estimates of item difficulty and item discrimination. Tetrachoric correlations of a data set of dichotomous items may be factor analysed using a minimum residual or maximum likelihood factor analysis and the result loadings transformed to item discrimination parameters. The tau parameter from the tetrachoric correlation combined with the item factor loading may be used to estimate item difficulties. Similar analyses can be done for discrete item responses using the polychoric correlation.

## Usage

irt.fa(x,...)

## Arguments

x
A data matrix of dichotomous or discrete items, or the result of tetrachoric or polychoric
... Additional parameters to pass to the factor analysis function

## Details

The tetrachoric correlation matrix of dichotomous items may be factored using a (e.g.) minimum residual factor analyis function fa and the resulting loadings, $\lambda_{i}$ are transformed to discriminations by $\alpha=\frac{\lambda_{i}}{\sqrt{1-\lambda_{i}^{2}}}$.

The difficulty parameter, $\delta$ may be found from the $\tau$ parameter of the tetrachoric or polychoric function.
$\delta_{i}=\frac{\tau_{i}}{\sqrt{1-\lambda_{i}^{2}}}$
Similar analyses may be done with discrete item responses using polychoric correlations and distinct estimates of item difficulty (location) for each item response.

The results may be shown graphically using plot. For plotting there are three options: type $=$ "ICC" will plot the item characteristic respone function. type = "IIC" will plot the item information function, and type $=$ "test" will plot the test information function.

The normal input is just the raw data. If, however, the correlation matrix has already been found using tetrachoric or polychoric, then that result can be processed directly.

## Value

coefficients
A data frame of Item difficulty, discrimination, tau and the factor loadings
stats A list of statistics for the factor analyis
rho The tetrachoric/polychoric correlation matrix

## Note

Still under development. Comments welcome

## Author(s)

William Revelle

## References

McDonald, Roderick P. (1999) Test theory: A unified treatment. L. Erlbaum Associates.
Revelle, William. (in prep) An introduction to psychometric theory with applications in R. Springer. Working draft available at http://personality-project.org/r/book/

## See Also

fa, sim.irt, tetrachoric, polychoric as well as plot.psych for plotting the IRT item curves.

## Examples

```
set.seed(17)
d9 <- sim.irt(9,1000,-2.5,2.5,mod="normal") #dichotomous items
test <- irt.fa(d9$items)
test
op <- par(mfrow=c (3,1))
plot(test,type="ICC")
plot(test,type="IIC")
plot(test,type="test")
par(op)
set.seed(17)
items <- sim.congeneric(N=500,short=FALSE,categorical=TRUE) #500 responses to 4 discrete ite
d4 <- irt.fa(items$observed) #item response analysis of congeneric measures
op <- par(mfrow=c(2,2))
plot(d4,type="ICC")
par(op)
```

irt.item.diff.rasch

Simple function to estimate item difficulties using IRT concepts

## Description

Steps toward a very crude and preliminary IRT program. These two functions estimate item difficulty and discrimination parameters. A better procedure is to use irt.fa or the ltm package.

## Usage

irt.item.diff.rasch(items)
irt. discrim(item.diff,theta,items)

## Arguments

items a matrix of items
item.diff a vector of item difficulties (found by irt.item.diff)
theta ability estimate from irt.person.theta

## Details

Item Response Theory (aka "The new psychometrics") models individual responses to items with a logistic function and an individual (theta) and item difficulty (diff) parameter.
irt.item.diff.rasch finds item difficulties with the assumption of theta=0 for all subjects and that all items are equally discriminating.
irt.discrim takes those difficulties and theta estimates from irt.person.rasch to find item discrimination (beta) parameters.
A far better package with these features is the ltm package. The IRT functions in the psych-package are for pedagogical rather than production purposes. They are believed to be accurate, but are not guaranteed. They do seem to be slightly more robust to missing data structures associated with SAPA data sets than the 1 tm package.

The irt.fa function is also an alternative. This will find tetrachoric or polychoric correlations and then convert to IRT parameters using factor analysis (fa).

## Value

a vector of item difficulties or item discriminations.

## Note

Under development. Not recommended for public consumption. See irt. fa for a better option.

## Author(s)

William Revelle

## See Also

```
irt.fa,irt.person.rasch
```

logistic Logistic transform from $x$ to $p$ and logit transform from $p$ to $x$

## Description

The logistic function $(1 /(1+\exp (-x))$ and logit function $(\log (\mathrm{p} /(1-\mathrm{p}))$ are fundamental to Item Response Theory. Although just one line functions, they are included here for ease of demonstrations and in drawing IRT models. Also included is the logistic.grm for a graded response model.

## Usage

logistic ( $x, d=0, a=1, c=0, z=1$ )
logit(p)
$\operatorname{logistic.grm}(x, d=0, a=1.5, c=0, z=1, r=2, s=c(-1.5,-.5, .5,1.5)$ )

## Arguments

x
d Item difficulty or delta parameter
a The slope of the curve at $x=0$ is equivalent to the discrimination parameter in 2PL models or alpha parameter. Is either 1 in 1PL or 1.702 in 1PN approximations.

| c | Lower asymptote $=$ guessing parameter in 3PL models or gamma |
| :--- | :--- |
| z | The upper asymptote - in 4PL models |
| p | Probability to be converted to logit value |
| r | The response category for the graded response model |
| $s$ | The response thresholds |

## Details

These three functions are provided as simple helper functions for demonstrations of Item Response Theory. The one parameter logistic (1PL) model is also known as the Rasch model. It assumes items differ only in difficulty. 1PL, 2PL, 3PL and 4PL curves may be drawn by choosing the appropriate d (delta or item difficulty), a (discrimination or slope), c (gamma or guessing) and z (zeta or upper asymptote).
logit is just the inverse of logistic.
logistic.grm will create the responses for a graded response model for the rth category where cutpoints are in s .

## Value

p logistic returns the probability associated with $x$
$\mathrm{x} \quad$ logit returns the real number associated with p

## Author(s)

## William Revelle

## Examples

```
curve(logistic(x,a=1.702),-3,3,ylab="Probability of x",main="Logistic transform of x",xlab="
curve(pnorm(x), add=TRUE, lty="dashed")
curve(logistic(x), add=TRUE)
text(2,.8, expression(alpha ==1))
text(2,1.0,expression(alpha==1.7))
curve(logistic(x),-4,4,ylab="Probability of x", main = "Logistic transform of x in logit uni
curve(logistic(x,d=-1), add=TRUE)
curve(logistic(x,d=1), add=TRUE)
curve(logistic(x, c=.2), add=TRUE,lty="dashed")
text(1.3,.5,"d=1")
text (.3,.5,"d=0")
text(-1.5,.5,"d=-1")
text (-3,.3,"c=.2")
#demo of graded response model
    curve(logistic.grm(x,r=1), -4,4,ylim=c(0,1),main="Five level response scale",ylab="Probabili
    curve(logistic.grm(x,r=2),add=TRUE)
    curve(logistic.grm(x,r=3), add=TRUE)
    curve(logistic.grm(x,r=4), add=TRUE)
    curve(logistic.grm(x,r=5), add=TRUE)
    text(-2.,.5,1)
```

```
text(-1.,.4,2)
text (0,.4,3)
text(1.,.4,4)
    text (2.,.5,5)
```

make. keys Create a keys matrix for use by score.items or cluster.cor

## Description

When scoring items by forming composite scales either from the raw data using score.items or from the correlation matrix using cluster. cor, it is necessary to create a keys matrix. This is just a short cut for doing so. The keys matrix is a nvar x nscales matrix of $-1,0,1$ and defines the membership for each scale.

## Usage

make.keys(nvars, keys.list, key.labels = NULL, item.labels = NULL)

## Arguments

nvars $\quad$ Number of variables items to be scored
keys.list A list of the scoring keys,one element for each scale
key.labels Labels for the scales can be specified here, or in the key.list
item.labels Typically, just the colnames of the items data matrix.

## Details

There are two ways to create keys for the score. items function. One is to laboriously do it in a spreadsheet and then copy them into R. The other is to just specify them by item number in a list.

## Value

keys a nvars $x$ nkeys matrix of $-1,0$, or 1 s describing how to score each scale. nkeys is the length of the keys.list

## See Also

```
score.items,cluster.cor
```


## Examples

```
data(attitude)
    key.list <- list(all=c(1, 2, 3, 4, -5,6,7),
                    first=c (1, 2, 3),
                    last=c(4,5,6,7))
    keys <- make.keys(7,key.list,item.labels = colnames(attitude))
    keys
    scores <- score.items(keys,attitude, short=TRUE)
    scores
    data(bfi)
    keys.list <- list(agree=c(-1,2:5), conscientious=c(6:8,-9,-10), extraversion=c (-11, -12,13:15)
    keys <- make.keys(25,keys.list,item.labels=colnames(bfi)[1:25])
    scores <- score.items(keys,bfi[,1:25], short=TRUE)
    summary(scores)
```

```
mat.regress Multiple Regression from matrix input
```


## Description

Extract subsets of variables ( $x$ and $y$ ) from a correlation or data matrix matrix and find the multiple correlation and beta weights of the (x) set predicting each member of the (y) set.

## Usage

mat.regress (m, $x, y, n . o b s=N U L L)$

## Arguments

$m \quad a \operatorname{matrix}$ of correlations or, if not square of data
$x \quad$ either the column numbers of the $x$ set (e.g., $c(1,3,5)$ or the column names of the x set (e.g. c("Cubes","PaperFormBoard")

Y either the column numbers of the $y$ set (e.g., $c(2,4,6)$ or the column names of the y set (e.g., c("Flags","Addition")
n. obs If specified, then confidence intervals, etc. are calculated, not needed if raw data are given

## Details

Although it is more common to calculate multiple regression from raw data, it is, of course, possible to do so from a set of correlations. The input to the function is a square covariance or correlation matrix, as well as the column numbers (or names) of the x (predictor) and y (criterion) variables. The program will find the correlations if given raw data.
The output is a set of multiple correlations, one for each dependent variable in the $y$ set.

A typical use in the SAPA project is to form item composites by clustering or factoring (see ICLUST, principal), extract the clusters from these results (factor2cluster), and then form the composite correlation matrix using cluster. cor. The variables in this reduced matrix may then be used in multiple R procedures using mat.regress.
Although the overall matrix can have missing correlations, the correlations in the subset of the matrix used for prediction must exist.

If the number of observations is entered, then the conventional confidence intervals, statistical significance, and shrinkage estimates are reported.
If the input matrix is rectangular, correlations are found from the data.

## Value

beta the beta weights for each variable in X for each variable in Y
$R \quad$ The multiple R for each equation (the amount of change a unit in the predictor set leads to in the criterion set).
R2 The multiple R2 (\% variance acounted for) for each equation

## Author(s)

William Revelle

Maintainer: William Revelle <revelle@ northwestern.edu>

## See Also

cluster.cor, factor2cluster,principal,ICLUST

## Examples

```
test.data <- Harman74.cor$cov #24 mental variables
#choose 3 of them to regress against another 4 -- arbitrary choice of variables
summary(mat.regress(test.data,c(1,2,3),c(4,5,10,12),n.obs=213),digits=2) #choose the variabl
mat.regress(test.data,c("VisualPerception", "Cubes", "PaperFormBoard"),c("Flags","Addition"))
data(attitude)
mat.regress(attitude,c(1:3),c(4:7)) #standardized regression from raw data
```

mat.sort

Sort the elements of a correlation matrix to reflect factor loadings

## Description

To see the structure of a correlation matrix, it is helpful to organize the items so that the similar items are grouped together. One such grouping technique is factor analysis. mat.sort will sort the items by a factor model (if specified), or any other order, or by the loadings on the first factor (if unspecified)
matrix.addition

## Usage

mat. sort (m, $f=$ NULL)

## Arguments

$m \quad$ A correlation matrix
f A factor analysis output (i.e., one with a loadings matrix) or a matrix of weights

## Details

The factor analysis output is sorted by size of the largest factor loading for each variable and then the matrix items are organized by those loadings. The default is to sort by the loadings on the first factor. Alternatives allow for ordering based upon any vector or matrix.

## Value

A sorted correlation matrix, suitable for showing with cor.plot.

## Author(s)

William Revelle

## See Also

fa, cor.plot

## Examples

```
data(bifactor)
sorted <- mat.sort(Bechtoldt.1,fa(Bechtoldt.1,5))
cor.plot(sorted)
```

matrix.addition A function to add two vectors or matrices

## Description

It is sometimes convenient to add two vectors or matrices in an operation analogous to matrix multiplication. For matrices $n \mathrm{Xm}$ and mYp , the matrix sum of the $\mathrm{i}, \mathrm{jth}$ element of $\mathrm{nSp}=\operatorname{sum}$ (over $m)$ of $i X m+m Y j$.

## Usage

$x \%+\%$

## Arguments

$x$
a $n$ by $m$ matrix (or vector if $m=1$ )
$\mathrm{y} \quad$ a m by p matrix (or vector if $\mathrm{m}=1$ )

## Details

Used in such problems as Thurstonian scaling. Although not technically matrix addition, as pointed out by Krus, there are many applications where the sum or difference of two vectors or matrices is a useful operation. An alternative operation for vectors is outer ( $\mathrm{x}, \mathrm{y}, \mathrm{FUN}=\mathrm{F}+\mathrm{+}$ ) but this does not work for matrices.

## Value

a $n$ by $p$ matix of sums

## Author(s)

William Revelle

## References

Krus, D. J. (2001) Matrix addition. Journal of Visual Statistics, 1, (February, 2001).

## Examples

```
x <- seq}(1,4
z <- x %+% -t(x)
X
z
#compare with outer(x,-x,FUN="+")
x <- matrix(seq(1,6),ncol=2)
y <- matrix(seq(1,10),nrow=2)
z<- x %+% y
x
Y
z
#but compare this with outer(x ,y,FUN="+")
```

```
msq
```

75 mood items from the Motivational State Questionnaire for 3896 participants

## Description

Emotions may be described either as discrete emotions or in dimensional terms. The Motivational State Questionnaire (MSQ) was developed to study emotions in laboratory and field settings. The data can be well described in terms of a two dimensional solution of energy vs tiredness and tension versus calmness. Additional items include what time of day the data were collected and a few personality questionnaire scores.
$m s q$

## Usage

```
data(msq)
```


## Format

A data frame with 3896 observations on the following 92 variables.
MSQ_Time Time of day the data were collected
active a numeric vector
afraid a numeric vector
alert a numeric vector
alone a numeric vector
angry a numeric vector
aroused a numeric vector
ashamed a numeric vector
astonished a numeric vector
at-ease a numeric vector
at-rest a numeric vector
attentive a numeric vector
blue a numeric vector
bored a numeric vector
calm a numeric vector
clutched-up a numeric vector
confident a numeric vector
content a numeric vector
delighted a numeric vector
depressed a numeric vector
determined a numeric vector
distressed a numeric vector
drowsy a numeric vector
dull a numeric vector
elated a numeric vector
energetic a numeric vector
enthusiastic a numeric vector
excited a numeric vector
fearful a numeric vector
frustrated a numeric vector
full-of-pep a numeric vector
gloomy a numeric vector
grouchy a numeric vector
guilty a numeric vector
happy a numeric vector
hostile a numeric vector
inspired a numeric vector
intense a numeric vector
interested a numeric vector
irritable a numeric vector
jittery a numeric vector
kindly a numeric vector
lively a numeric vector
lonely a numeric vector
nervous a numeric vector
placid a numeric vector
pleased a numeric vector
proud a numeric vector
quiescent a numeric vector
quiet a numeric vector
relaxed a numeric vector
sad a numeric vector
satisfied a numeric vector
scared a numeric vector
scornful a numeric vector
serene a numeric vector
sleepy a numeric vector
sluggish a numeric vector
sociable a numeric vector
sorry a numeric vector
still a numeric vector
strong a numeric vector
surprised a numeric vector
tense a numeric vector
tired a numeric vector
unhappy a numeric vector
upset a numeric vector
vigorous a numeric vector
wakeful a numeric vector
warmhearted a numeric vector
wide-awake a numeric vector
anxious a numeric vector
idle a numeric vector
cheerful a numeric vector
inactive a numeric vector
tranquil a numeric vector
EA Thayer's Energetic Arousal Scale
TA Thayer's Tense Arousal Scale
PA Positive Affect scale
NegAff Negative Affect scale
Extraversion Extraversion from the Eysenck Personality Inventory
Neuroticism Neuroticism from the Eysenck Personality Inventory
Lie Lie from the EPI
Sociability The sociability subset of the Extraversion Scale
Impulsivity The impulsivity subset of the Extraversions Scale
MSQ_Round Rounded time of day
scale a factor with levels msq r original or revised msq
ID subject ID
exper Which study were the data collected: a factor with levels AGES BING BORN CART CITY COPE EMIT FAST Fern FILM FLAT Gray imps item knob MAPS mite pat-1 pat2 PATS post RAFT Rim. 1 Rim. 2 rob-1 rob-2 ROG1 ROG2 SALT sam-1 sam-2 SAVE/PATS sett swam swam-2 TIME VALE-1 VALE-2 VIEW
condition What was the experimental condition after the msq was given
TOD a numeric vector
TOD24 a numeric vector

## Details

The Motivational States Questionnaire (MSQ) is composed of 72 items, which represent the full affective range (Revelle \& Anderson, 1996). The MSQ consists of 20 items taken from the ActivationDeactivation Adjective Check List (Thayer, 1986), 18 from the Positive and Negative Affect Schedule (PANAS, Watson, Clark, \& Tellegen, 1988) along with the items used by Larsen and Diener (1992). The response format was a four-point scale that corresponds to Russell and Carroll's (1999) "ambiguous-likely-unipolar format" and that asks the respondents to indicate their current standing ("at this moment") with the following rating scale:
$0-1-2-3$
Not at all A little Moderately Very much
The original version of the MSQ included 70 items. Intermediate analyses (done with 1840 subjects) demonstrated a concentration of items in some sections of the two dimensional space, and a paucity of items in others. To begin correcting this, 3 items from redundantly measured sections
(alone, kindly, scornful) were removed, and 5 new ones (anxious, cheerful, idle, inactive, and tranquil) were added. Thus, the correlation matrix is missing the correlations between items 5, 42, and 55 and 72-76.

Procedure. The data were collected over nine years, as part of a series of studies examining the effects of personality and situational factors on motivational state and subsequent cognitive performance. In each of 38 studies, prior to any manipulation of motivational state, participants signed a consent form and filled out the MSQ. (The procedures of the individual studies are irrelevant to this data set and could not affect the responses to the MSQ, since this instrument was completed before any further instructions or tasks).

In addition to the MSQ, there are 5 scales from the Eysenck Personality Inventory.

## Source

Data collecte at the Personality, Motivation, and Cognition Laboratory, Northwestern University.

## References

William Revelle and Kristen Joan Anderson (1997) Personality, motivation and cognitive performance: Final report to the Army Research Institute on contract MDA 903-93-K-0008

Rafaeli, Eshkol and Revelle, William (2006), A premature consensus: Are happiness and sadness truly opposite affects? Motivation and Emotion, 30, 1, 1-12.

## Examples

```
data(msq)
```

describe (msq)
multi.hist

Multiple histograms with density and normal fits on one page

## Description

Given a matrix or data.frame, produce histograms for each variable in a "matrix" form. Include normal fits and density distributions for each plot.

The number of rows and columns may be specified, or calculated.
May be used for single variables.

## Usage

multi.hist(x,nrow=NULL, ncol=NULL, density=TRUE,main="Histogram, Density, and Normal

## Arguments

| x | matrix or data.frame |
| :--- | :--- |
| nrow | number of rows in the plot |
| ncol | number of columns in the plot |
| density | density=TRUE, show the normal fits and density distributions |
| main | title for each panel |

Author(s)
William Revelle

## Examples

\#multi.hist (attitude [-1])
neo NEO correlation matrix from the NEO_PI_R manual

## Description

The NEO.PI.R is a widely used personality test to assess 5 broad factors (Neuroticism, Extraversion, Openness, Agreeableness and Conscientiousness) with six facet scales for each factor. The correlation matrix of the facets is reported in the NEO.PI.R manual for 1000 subjects.

## Usage

```
data(neo)
```


## Format

A data frame of a $30 \times 30$ correlation matrix with the following 30 variables.
N1 Anxiety
N2 AngryHostility
N3 Depression
N4 Self-Consciousness
N5 Impulsiveness
N6 Vulnerability
E1 Warmth
E2 Gregariousness
E3 Assertiveness
E4 Activity
E5 Excitement-Seeking
E6 PositiveEmotions
01 Fantasy
02 Aesthetics
03 Feelings
O4 Ideas
05 Actions
O6 Values
A1 Trust
A2 Straightforwardness
A3 Altruism
A4 Compliance
A5 Modesty
A6 Tender-Mindedness
C1 Competence
C2 Order
C3 Dutifulness
C4 AchievementStriving
C5 Self-Discipline
C6 Deliberation

## Details

The past thirty years of personality research has led to a general consensus on the identification of major dimensions of personality. Variously known as the "Big 5" or the "Five Factor Model", the general solution represents 5 broad domains of personal and interpersonal experience. Neuroticism and Extraversion are thought to reflect sensitivity to negative and positive cues from the environment and the tendency to withdraw or approach. Openness is sometimes labeled as Intellect and reflects an interest in new ideas and experiences. Agreeableness and Conscientiousness reflect tendencies to get along with others and to want to get ahead.
The factor structure of the NEO suggests five correlated factors as well as two higher level factors. The NEO was constructed with 6 "facets" for each of the five broad factors.

## Source

Costa, Paul T. and McCrae, Robert R. (1992) (NEO PI-R) professional manual. Psychological Assessment Resources, Inc. Odessa, FL. (with permission of the author and the publisher)

## References

Digman, John M. (1990) Personality structure: Emergence of the five-factor model. Annual Review of Psychology. 41, 417-440.
John M. Digman (1997) Higher-order factors of the Big Five. Journal of Personality and Social Psychology, 73, 1246-1256.

McCrae, Robert R. and Costa, Paul T., Jr. (1999) A Five-Factor theory of personality. In Pervin, Lawrence A. and John, Oliver P. (eds) Handbook of personality: Theory and research (2nd ed.) 139-153. Guilford Press, New York. N.Y.
Revelle, William (1995), Personality processes, Annual Review of Psychology, 46, 295-328.
Joshua Wilt and William Revelle (2009) Extraversion and Emotional Reactivity. In Mark Leary and Rick H. Hoyle (eds). Handbook of Individual Differences in Social Behavior. Guilford Press, New York, N.Y.

## Examples

```
data(neo)
n5 <- factor.minres(neo,5)
neo.keys <- make.keys(30, list(N=c(1:6), E=c(7:12),O=C(13:18),A=c(19:24),C=c(25:30)))
n5p <- target.rot(n5,neo.keys) #show a targeted rotation for simple structure
n5p
```


## omega Calculate McDonald's omega estimates of general and total factor

 saturation
## Description

McDonald has proposed coefficient omega as an estimate of the general factor saturation of a test. One way to find omega is to do a factor analysis of the original data set, rotate the factors obliquely, do a Schmid Leiman transformation, and then find omega. This function estimates omega as suggested by McDonald by using hierarchical factor analysis (following Jensen). A related option is to define the model using omega and then perform a confirmatory factor analysis using the sem package. This is done by omegaSem and omegaFromSem.

## Usage

omega(m, nfactors=3, fm = "minres",key = NULL, flip=TRUE, digits=2,title="Omega",s] omegaSem(m,nfactors=3,fm="minres",key=NULL,flip=TRUE, digits=2,title="Omega",sl=TRL omegaFromSem (m,s,flip=TRUE)

## Arguments

nfactors $\quad$ Number of factors believed to be group factors
m
fm

A correlation matrix, or a data.frame/matrix of data, or (if Phi is specified, an oblique factor pattern matrix
factor method (the default is minres) $\mathrm{fm}=$ "pa" for principal axes, $\mathrm{fm}=$ "minres" for a minimum residual (OLS) solution, $\mathrm{fm}=\mathrm{pc} \mathrm{pc}$ for principal components, $\mathrm{fm}=\mathrm{ml}$ " for maximum likelihood.

| key | a vector of +/- 1s to specify the direction of scoring of items. The default is <br> to assume all items are positively keyed, but if some items are reversed scored, <br> then key should be specified. |
| :--- | :--- |
| flip | If flip is TRUE, then items are automatically flipped to have positive correlations <br> on the general factor. Items that have been reversed are shown with a - sign. <br> if specified, round the output to digits |
| digits | Title for this analysis |
| title | If plotting the results, should the Schmid Leiman solution be shown or should <br> the hierarchical solution be shown? (default sl=TRUE) |
| slabels | If plotting, what labels should be applied to the variables? If not specified, will <br> default to the column names. <br> plot=TRUE (default) calls omega.diagram, plot =FALSE does not. If Rgraphviz |
| n.obs | is available, then omega.graph may be used separately. <br> Number of observations - used for goodness of fit statistic <br> What rotation to apply? The default is oblimin, the alternatives include simpli- <br> max, Promax, cluster and target. target will rotate to an optional keys matrix |
| Phi | (See target . rot) |
| If specified, then omega is found from the pattern matrix (m) and the factor |  |

## Details

"Many scales are assumed by their developers and users to be primarily a measure of one latent variable. When it is also assumed that the scale conforms to the effect indicator model of measurement (as is almost always the case in psychological assessment), it is important to support such an interpretation with evidence regarding the internal structure of that scale. In particular, it is important to examine two related properties pertaining to the internal structure of such a scale. The first property relates to whether all the indicators forming the scale measure a latent variable in common.
The second internal structural property pertains to the proportion of variance in the scale scores (derived from summing or averaging the indicators) accounted for by this latent variable that is common to all the indicators (Cronbach, 1951; McDonald, 1999; Revelle, 1979). That is, if an effect indicator scale is primarily a measure of one latent variable common to all the indicators forming the scale, then that latent variable should account for the majority of the variance in the scale scores. Put differently, this variance ratio provides important information about the sampling fluctuations when estimating individuals' standing on a latent variable common to all the indicators arising from the sampling of indicators (i.e., when dealing with either Type 2 or Type 12 sampling, to use the terminology of Lord, 1956). That is, this variance proportion can be interpreted as the square of the correlation between the scale score and the latent variable common to all the indicators in the infinite universe of indicators of which the scale indicators are a subset. Put yet another way, this variance ratio is important both as reliability and a validity coefficient. This is a reliability
issue as the larger this variance ratio is, the more accurately one can predict an individual's relative standing on the latent variable common to all the scale's indicators based on his or her observed scale score. At the same time, this variance ratio also bears on the construct validity of the scale given that construct validity encompasses the internal structure of a scale." (Zinbarg, Yovel, Revelle, and McDonald, 2006).
McDonald has proposed coefficient omega (hierarchical $\left(\omega_{h}\right)$ as an estimate of the general factor saturation of a test. Zinbarg, Revelle, Yovel and Li (2005) http: / /personality-project. org/revelle/publications/zinbarg.revelle.pmet.05.pdf compare McDonald's $\omega_{h}$ to Cronbach's $\alpha$ and Revelle's $\beta$ They conclude that $\omega_{h}$ is the best estimate. (See also Zinbarg et al., 2006 and Revelle and Zinbarg (2009)).
One way to find $\omega_{h}$ is to do a factor analysis of the original data set, rotate the factors obliquely, factor that correlation matrix, do a Schmid-Leiman (schmid) transformation to find general factor loadings, and then find $\omega_{h}$. Here we present code to do that.
$\omega_{h}$ differs as a function of how the factors are estimated. Four options are available, the default does a minres factor solution, $\mathrm{fm}=$ " pa " does a principle axes factor analysis (factor.pa), fm="mle" uses the factanal function, and $\mathrm{fm}=$ " pc " does a principal components analysis (principal).
For ability items, it is typically the case that all items will have positive loadings on the general factor. However, for non-cognitive items it is frequently the case that some items are to be scored positively, and some negatively. Although probably better to specify which directions the items are to be scored by specifying a key vector, if flip =TRUE (the default), items will be reversed so that they have positive loadings on the general factor. The keys are reported so that scores can be found using the score. items function. Arbitrarily reversing items this way can overestimate the general factor. (See the example with a simulated circumplex).
$\beta$, an alternative to $\omega$, is defined as the worst split half reliability. It can be estimated by using ICLUST (a hierarchical clustering algorithm originally developed for main frames and written in Fortran and that is now part of the psych package. (For a very complimentary review of why the ICLUST algorithm is useful in scale construction, see Cooksey and Soutar, 2005).
The omega function uses exploratory factor analysis to estimate the $\omega_{h}$ coefficient. It is important to remember that "A recommendation that should be heeded, regardless of the method chosen to estimate $\omega_{h}$, is to always examine the pattern of the estimated general factor loadings prior to estimating $\omega_{h}$. Such an examination constitutes an informal test of the assumption that there is a latent variable common to all of the scale's indicators that can be conducted even in the context of EFA. If the loadings were salient for only a relatively small subset of the indicators, this would suggest that there is no true general factor underlying the covariance matrix. Just such an informal assumption test would have afforded a great deal of protection against the possibility of misinterpreting the misleading $\omega_{h}$ estimates occasionally produced in the simulations reported here." (Zinbarg et al., 2006, p 137).
A simple demonstration of the problem of an omega estimate reflecting just one of two group factors can be found in the last example.
Diagnostic statistics that reflect the quality of the omega solution include a comparison of the relative size of the $g$ factor eigen value to the other eigen values, the percent of the common variance for each item that is general factor variance ( p 2 ), the mean of p 2 , and the standard deviation of p 2 . Further diagnostics can be done by describing (describe) the \$schmid\$sl results.
Although omega_h is uniquely defined only for cases where 3 or more subfactors are extracted, it is sometimes desired to have a two factor solution. By default this is done by forcing the schmid extraction to treat the two subfactors as having equal loadings.

There are three possible options for this condition: setting the general factor loadings between the two lower order factors to be "equal" which will be the sqrt(oblique correlations between the factors) or to "first" or "second" in which case the general factor is equated with either the first or second group factor. A message is issued suggesting that the model is not really well defined. This solution discussed in Zinbarg et al., 2007. To do this in omega, add the option="first" or option="second" to the call.

Although obviously not meaningful for a 1 factor solution, it is of course possible to find the sum of the loadings on the first (and only) factor, square them, and compare them to the overall matrix variance. This is done, with appropriate complaints.

In addition to $\omega_{h}$, another of McDonald's coefficients is $\omega_{t}$. This is an estimate of the total reliability of a test.

McDonald's $\omega_{t}$, which is similar to Guttman's $\lambda_{6}$, guttman but uses the estimates of uniqueness ( $u^{2}$ from factor analysis to find $e_{j}^{2}$. This is based on a decomposition of the variance of a test score, $V_{x}$ into four parts: that due to a general factor, $\vec{g}$, that due to a set of group factors, $\vec{f}$, (factors common to some but not all of the items), specific factors, $\vec{s}$ unique to each item, and $\vec{e}$, random error. (Because specific variance can not be distinguished from random error unless the test is given at least twice, some combine these both into error).
Letting $\vec{x}=\overrightarrow{c g}+\overrightarrow{A f}+\overrightarrow{D s}+\vec{e}$ then the communality of item ${ }_{j}$, based upon general as well as group factors, $h_{j}^{2}=c_{j}^{2}+\sum f_{i j}^{2}$ and the unique variance for the item $u_{j}^{2}=\sigma_{j}^{2}\left(1-h_{j}^{2}\right)$ may be used to estimate the test reliability. That is, if $h_{j}^{2}$ is the communality of item ${ }_{j}$, based upon general as well as group factors, then for standardized items, $e_{j}^{2}=1-h_{j}^{2}$ and

$$
\omega_{t}=\frac{\overrightarrow{1} c \overrightarrow{c^{\prime}} \overrightarrow{1}+\overrightarrow{1} \overrightarrow{A A}^{\prime} \overrightarrow{1}^{\prime}}{V_{x}}=1-\frac{\sum\left(1-h_{j}^{2}\right)}{V_{x}}=1-\frac{\sum u^{2}}{V_{x}}
$$

Because $h_{j}^{2} \geq r_{s m c}^{2}, \omega_{t} \geq \lambda_{6}$.
It is important to distinguish here between the two $\omega$ coefficients of McDonald, 1978 and Equation 6.20a of McDonald, 1999, $\omega_{t}$ and $\omega_{h}$. While the former is based upon the sum of squared loadings on all the factors, the latter is based upon the sum of the squared loadings on the general factor.

$$
\omega_{h}=\frac{\overrightarrow{1} c c^{\prime} \overrightarrow{1}}{V_{x}}
$$

Another estimate reported is the omega for an infinite length test with a structure similar to the observed test. This is found by

$$
\omega_{l} i m i t=\frac{\overrightarrow{1} c \vec{c}^{\prime} \overrightarrow{1}}{\overrightarrow{1} c \vec{c}^{\prime} \overrightarrow{1}+\overrightarrow{1} \overrightarrow{A^{\prime}} \overrightarrow{1}^{\prime}}
$$

The input to omega may be a correlation matrix or a raw data matrix, or a factor pattern matrix with the factor intercorrelations (Phi) matrix.
omega is an exploratory factor analysis function that uses a Schmid-Leiman transformation. omegaSem first calls omega and then takes the Schmid-Leiman solution, converts this to a confirmatory sem model and then calls the sem package to conduct a confirmatory model. $\omega_{h}$ is then calculated from the CFA output. Although for well behaved problems, the efa and cfa solutions will be practically identical, the CFA solution will not always agree with the EFA solution. In particular, the estimated $R^{2}$ will sometimes exceed 1. (An example of this is the Harman 24 cognitive abilities problem.)

In addition, not all EFA solutions will produce workable CFA solutions. Model misspecifications will lead to very strange CFA estimates.
omegaFromSem takes the output from a sem model and uses it to find $\omega_{h}$. The estimate of factor indeterminacy, found by the multiple $R^{2}$ of the variables with the factors, will not match that found by the EFA model. In particular, the estimated $R^{2}$ will sometimes exceed 1. (An example of this is the Harman 24 cognitive abilities problem.)

## Value

omega hierarchical
The $\omega_{h}$ coefficient
omega.lim The limit of $\omega_{h}$ as the test becomes infinitly large
omega total The omega ${ }_{t}$ coefficient
alpha Cronbach's $\alpha$
schmid The Schmid Leiman transformed factor matrix and associated matrices
schmid\$sl The $g$ factor loadings as well as the residualized factors
schmid\$orthog
Varimax rotated solution of the original factors
schmid\$oblique
The oblimin or promax transformed factors
schmid\$phi the correlation matrix of the oblique factors
schmid\$gloading
The loadings on the higher order, g , factor of the oblimin factors
key A vector of -1 or 1 showing which direction the items were scored.
model a matrix suitable to be given to the sem function for structure equation models
various fit statistics
various fit statistics, see output

Note
Requires the GPArotation package.
The default rotation uses oblimin from the GPArotation package. Alternatives include the simplimax function, as well as Promax.

If the factor solution leads to an exactly orthogonal solution (probably only for demonstration data sets), then use the rotate="Promax" option to get a solution.
omegaSem requires the sem package. omegaFromSem uses the output from the sem package.

## Author(s)

http://personality-project.org/revelle.html
Maintainer: William Revelle < revelle@northwestern.edu >

## References

http://personality-project.org/r/r.omega.html

Revelle, W. and Zinbarg, R. E. (2009) Coefficients alpha, beta, omega and the glb: comments on Sijtsma. Psychometrika, 74, 1, 145-154.
Revelle, W. (1979). Hierarchical cluster analysis and the internal structure of tests. Multivariate Behavioral Research, 14, 57-74. (http://personality-project.org/revelle/ publications/iclust.pdf)
Zinbarg, R.E., Revelle, W., Yovel, I., \& Li. W. (2005). Cronbach's Alpha, Revelle's Beta, McDonald's Omega: Their relations with each and two alternative conceptualizations of reliability. Psychometrika. 70, 123-133. http://personality-project.org/revelle/publications/ zinbarg.revelle.pmet.05.pdf
Zinbarg, R., Yovel, I. \& Revelle, W. (2007). Estimating omega for structures containing two group factors: Perils and prospects. Applied Psychological Measurement. 31 (2), 135-157.
Zinbarg, R., Yovel, I., Revelle, W. \& McDonald, R. (2006). Estimating generalizability to a universe of indicators that all have one attribute in common: A comparison of estimators for omega. Applied Psychological Measurement, 30, 121-144. DOI: 10.1177/0146621605278814 http://apm.sagepub.com/cgi/reprint/30/2/121

## See Also

omega.graph ICLUST, ICLUST.graph, VSS, schmid, make.hierarchical

## Examples

```
## Not run:
    test.data <- Harman74.cor$cov
    if(!require(GPArotation)) {message("Omega requires GPA rotation" )} else {my.omega <- omega
    print(my.omega,digits=2)}
#create 9 variables with a hierarchical structure
v9 <- sim.hierarchical()
#with correlations of
round(v9,2)
#find omega
v9.omega <- omega(v9,digits=2)
v9.omega
#create 8 items with a two factor solution, showing the use of the flip option
sim2 <- item.sim(8)
omega(sim2) #an example of misidentification-- remember to look at the loadings matrices.
omega(sim2,2) #this shows that in fact there is no general factor
omega(sim2,2,option="first") #but, if we define one of the two group factors as a general fa
#apply omega to analyze 6 mental ability tests
data(ability.cov) #has a covariance matrix
omega(ability.cov$cov)
## End(Not run)
```

omega.graph Graph hierarchical factor structures

## Description

Hierarchical factor structures represent the correlations between variables in terms of a smaller set of correlated factors which themselves can be represented by a higher order factor.
Two alternative solutions to such structures are found by the omega function. The correlated factors solutions represents the effect of the higher level, general factor, through its effect on the correlated factors. The other representation makes use of the Schmid Leiman transformation to find the direct effect of the general factor upon the original variables as well as the effect of orthogonal residual group factors upon the items.
Graphic presentations of these two alternatives are helpful in understanding the structure. omega.graph and omega.diagram draw both such structures. Graphs are drawn directly onto the graphics window or expressed in "dot" commands for conversion to graphics using implementations of Graphviz (if using omega.graph).
Using Graphviz allows the user to clean up the Rgraphviz output. However, if Graphviz and Rgraphviz are not available, use omega.diagram.
See the other structural diagramming functions, fa.diagram and structure. diagram.
In addition

## Usage

omega.diagram(om.results,sl=TRUE, sort=TRUE, labels=NULL, cut=. 2 , gcut, simple=TRUE, errc
omega.graph(om.results, out.file $=$ NULL, sl $=$ TRUE, labels $=$ NULL, size $=c(8,6)$,

## Arguments

om.results The output from the omega function
out.file Optional output file for off line analysis using Graphviz
sl Orthogonal clusters using the Schmid-Leiman transform (sl=TRUE) or oblique clusters
labels variable labels
size size of graphics window
node. font What font to use for the items
edge.font What font to use for the edge labels
rank.direction
Defaults to left to right
digits Precision of labels
cex control font size
color.lines Use black for positive, red for negative
title Figure title

| main | main figure caption |
| :--- | :--- |
| $\ldots$ | Other options to pass into the graphics packages |
| e.size | the size to draw the ellipses for the factors. This is scaled by the number of |
| cut | variables. |
| gcut | Minimum path coefficient to draw |
| simple | Minimum general factor path to draw |
| sort | draw just one path per item |
| side | sort the solution before making the diagram |
| errors | on which side should errors be drawn? |
| rsize | show the error estimates |
|  | size of the rectangles |

## Details

While omega.graph requires the Rgraphviz package, omega.diagram does not. codeomega requires the GPArotation package.

## Value

| clust.graph | A graph object |
| :--- | :--- |
| sem | A matrix suitable to be run throughe the sem function in the sem package. |

## Note

omega.graph requires rgraphviz. - omega requires GPArotation

## Author(s)

http://personality-project.org/revelle.html
Maintainer: William Revelle < revelle@northwestern.edu >

## References

http://personality-project.org/r/r.omega.html

Revelle, W. (in preparation) An Introduction to Psychometric Theory with applications in R. http: //personality-project.org/r/book
Revelle, W. (1979). Hierarchical cluster analysis and the internal structure of tests. Multivariate Behavioral Research, 14, 57-74. (http://personality-project.org/revelle/ publications/iclust.pdf)
Zinbarg, R.E., Revelle, W., Yovel, I., \& Li. W. (2005). Cronbach's Alpha, Revelle's Beta, McDonald's Omega: Their relations with each and two alternative conceptualizations of reliability. Psychometrika. 70, 123-133. http://personality-project.org/revelle/publications/ zinbarg.revelle.pmet.05.pdf
Zinbarg, R., Yovel, I., Revelle, W. \& McDonald, R. (2006). Estimating generalizability to a universe of indicators that all have one attribute in common: A comparison of estimators for
omega. Applied Psychological Measurement, 30, 121-144. DOI: 10.1177/0146621605278814 http://apm.sagepub.com/cgi/reprint/30/2/121

## See Also

```
omega, make.hierarchical, ICLUST.rgraph
```


## Examples

```
#24 mental tests from Holzinger-Swineford-Harman
if(require(GPArotation) ) {om24 <- omega(Harman74.cor$cov,4) } #run omega
#
#example hierarchical structure from Jensen and Weng
if(require(GPArotation) ) {jen.omega <- omega(make.hierarchical())}
```

p.rep Find the probability of replication for an $F, t$, or $r$ and estimate effect
size

## Description

The probability of replication of an experimental or correlational finding as discussed by Peter Killeen (2005) is the probability of finding an effect in the same direction upon an exact replication. For articles submitted to Psychological Science, p.rep needs to be reported.
$\mathrm{F}, \mathrm{t}, \mathrm{p}$ and r are all estimates of the size of an effect. But $\mathrm{F}, \mathrm{t}$, and p also are also a function of the sample size. Effect size, d prime, may be expressed as differences between means compared to within cell standard deviations, or as a correlation coefficient. These functions convert $\mathrm{p}, \mathrm{F}$, and t to d prime and the r equivalent.

## Usage

```
p.rep(p = 0.05, n=NULL,twotailed = FALSE)
p.rep.f(F,df2,twotailed=FALSE)
p.rep.r(r,n,twotailed=TRUE)
p.rep.t(t,df,df2=NULL,twotailed=TRUE)
```


## Arguments

p
conventional probability of statistic (e.g., of F, t , or r )
F The F statistic
$\mathrm{df} \quad$ Degrees of freedom of the t -test, or of the first group if unequal sizes
df2 Degrees of freedom of the denominator of F or the second group in an unequal sizes $t$ test
$r \quad$ Correlation coefficient

| $n$ | Total sample size if using $r$ |
| :--- | :--- |
| $t$ | $t$-statistic if doing a $t$-test or testing significance of a regression slope |
| twotailed | Should a one or two tailed test be used? |

## Details

The conventional Null Hypothesis Significance Test (NHST) is the likelihood of observing the data given the null hypothesis of no effect. But this tells us nothing about the probability of the null hypothesis. Peter Killeen (2005) introduced the probability of replication as a more useful measure. The probability of replication is the probability that an exact replication study will find a result in the same direction as the original result.
p.rep is based upon a 1 tailed probability value of the observed statistic.

Other frequently called for statistics are estimates of the effect size, expressed either as Cohen's d, Hedges g, or the equivalent value of the correlation, r.
For p.rep.t, if the cell sizes are unequal, the effect size estimates are adjusted by the ratio of the mean cell size to the harmonic mean cell size (see Rownow et al., 2000).

## Value

| P.rep | Probability of replication |
| :--- | :--- |
| dprime | Effect size (Cohen's d) if more than just p is specified |
| prob | Probability of $\mathrm{F}, \mathrm{t}$, or r . Note that this can be either the one-tailed or two tailed <br> probability value. |
| r.equivalent | For t-tests, the r equivalent to the t (see Rosenthal and Rubin(2003), Rosnow, |
|  | Rosenthal, and Rubin, 2000)) |

## Note

The p.rep value is the one tailed probability value of obtaining a result in the same direction.

## References

Cummings, Geoff (2005) Understanding the average probability of replication: comment on Killeen 2005). Psychological Science, 16, 12, 1002-1004).

Killeen, Peter H. (2005) An alternative to Null-Hypothesis Significance Tests. Psychological Science, 16, 345-353

Rosenthal, R. and Rubin, Donald B.(2003), r-sub(equivalent): A Simple Effect Size Indicator. Psychological Methods, 8, 492-496.
Rosnow, Ralph L., Rosenthal, Robert and Rubin, Donald B. (2000) Contrasts and correlations in effect-size estimation, Psychological Science, 11. 446-453.

## Examples

```
p.rep(.05) #probability of replicating a result if the original study had a p = .05
p.rep.f(9.0,98) #probability of replicating a result with F = 9.0 with 98 df
p.rep.r(.4,50) #probability of replicating a result if r =.4 with n = 50
p.rep.t(3,98) #probability of replicating a result if t = 3 with df =98
p.rep.t(2.14,84,14) #effect of equal sample sizes (see Rosnow et al.)
```

```
paired.r
```

Test the difference between (un)paired correlations

## Description

Test the difference between two (paired or unpaired) correlations. Given 3 variables, $\mathrm{x}, \mathrm{y}, \mathrm{z}$, is the correlation between $x y$ different than that between $x z$ ? If $y$ and $z$ are independent, this is a simple t -test of the z transformed rs. But, if they are dependent, it is a bit more complicated.

## Usage

paired.r(xy, $x z, y z=N U L L, n, n 2=N U L L, t w o t a i l e d=T R U E)$

## Arguments

| $x y$ | $r(x y)$ |
| :--- | :--- |
| $x z$ | $r(x z)$ |
| $y z$ | $r(y z)$ |
| $n$ | Number of subjects for first group |
| n2 | Number of subjects in second group (if not equal to n) |
| twotailed | Calculate two or one tailed probability values |

## Details

To find the z of the difference between two independent correlations, first convert them to z scores using the Fisher r-z transform and then find the z of the difference between the two correlations. The default assumption is that the group sizes are the same, but the test can be done for different size groups by specifying n 2 .
If the correlations are not independent (i.e., they are from the same sample) then the correlation with the third variable $r(y z)$ must be specified. Find a $t$ statistic for the difference of thee two dependent correlations.

## Value

a list containing the calculated t or z values and the associated two (or one) tailed probability.
$t \quad t$ test of the difference between two dependent correlations
$p \quad$ probability of the $t$ or of the $z$
$z \quad z$ test of the difference between two independent correlations

## Author(s)

William Revelle

## See Also <br> ```p.rep.r,cor.test```

## Examples

```
paired.r(.5,.3, .4, 100) #dependent correlations
paired.r(.5,.3,NULL,100) #independent correlations same sample size
paired.r(.5,.3,NULL, 100, 64) # independent correlations, different sample sizes
```

```
pairs.panels SPLOM, histograms and correlations for a data matrix
```


## Description

Adapted from the help page for pairs, pairs.panels shows a scatter plot of matrices (SPLOM), with bivariate scatter plots below the diagonal, histograms on the diagonal, and the Pearson correlation above the diagonal. Useful for descriptive statistics of small data sets. If $1 m=T R U E$, linear regression fits are shown for both y by x and x by y . Correlation ellipses are also shown. Points may be given different colors depending upon some grouping variable.

## Usage

pairs.panels(x, smooth = TRUE, scale = FALSE, density=TRUE,ellipses=TRUE,digits =

## Arguments

| x | a data.frame or matrix |
| :--- | :--- |
| smooth | TRUE draws loess smooths |
| scale | TRUE scales the correlation font by the size of the absolute correlation. |
| density | TRUE shows the density plots as well as histograms |
| ellipses | TRUE draws correlation ellipses |
| lm | Plot the linear fit rather than the LOESS smoothed fits. |
| digits | the number of digits to show |
| pch | The plot character (defaults to 20 which is a '.,'). |
| jiggle | Should the points be jittered before plotting? <br> factor |
| factor for jittering (1-5) |  |
| hist.col | What color should the histogram on the diagonal be? <br> show.points |
| If FALSE, do not show the data points, just the data ellipses and smoothed func- <br> tions |  |
| ... | other options for pairs |

## Details

Shamelessly adapted from the pairs help page. Uses panel.cor, panel.cor.scale, and panel.hist, all taken from the help pages for pairs. Also adapts the ellipse function from John Fox's car package.
pairs.panels is most useful when the number of variables to plot is less than about 6-10. It is particularly useful for an initial overview of the data.
To show different groups with different colors, use a plot character (pch) between 21 and 25 and then set the background color to vary by group. (See the second example).
When plotting more than about 10 variables, it is useful to set the gap parameter to something less than 1 (e.g., 0). Alternatively, consider using cor. plot

Whe plotting more than about 100-200 cases, it is useful to use the show.points=FALSE option.

## Value

A scatter plot matrix (SPLOM) is drawn in the graphic window. The lower off diagonal draws scatter plots, the diagonal histograms, the upper off diagonal reports the Pearson correlation (with pairwise deletion).

If $1 m=T R U E$, then the scatter plots are drawn above and below the diagonal, each with a linear regression fit. Useful to show the difference between regression lines.

## See Also

```
pairs,cor.plot
```


## Examples

```
pairs.panels(attitude) #see the graphics window
data(iris)
pairs.panels(iris[1:4],bg=c("red","yellow","blue")[iris$Species],pch=21,main="Fisher Iris da
pairs.panels(iris[1:4],bg=c("red","yellow","blue")[iris$Species],pch=21,main="Fisher Iris da
#demonstrate not showing the data points
data(sat.act)
pairs.panels(sat.act,show.points=FALSE)
#show many variables with 0 gap between scatterplots
data(bfi)
pairs.panels(bfi,show.points=FALSE,gap=0)
```

```
partial.r Find the partial correlations for a set (x) of variables with set (y) re- moved.
```


## Description

A straightforward application of matrix algebra to remove the effect of the variables in the $y$ set from the $x$ set. Input may be either a data matrix or a correlation matrix. Variables in $x$ and $y$ are specified by location.

## Usage

partial.r(m, $x, y, \operatorname{digits}=2)$

## Arguments

$m \quad$ A data or correlation matrix
$x \quad$ The variable numbers associated with the X set.
$\mathrm{y} \quad$ The variable numbers associated with the Y set
digits $\quad$ Report correlations to digits of accuracy $($ default $=2)$

## Details

It is sometimes convenient to partial the effect of a number of variables (e.g., sex, age, education) out of the correlations of another set of variables. This could be done laboriously by finding the residuals of various multiple correlations, and then correlating these residuals. The matrix algebra alternative is to do it directly.

## Value

The matrix of partial correlations.

## Author(s)

William Revelle

## References

Revelle, W. (in prep) An introduction to psychometric theory with applications in R. To be published by Springer. (working draft available at http://personality-project.org/r/book/

## See Also

mat. regress for a similar application for regression

## Examples

```
jen <- make.hierarchical() #make up a correlation matrix
round(jen[1:5,1:5],2)
par.r <- partial.r(jen,c(1, 3,5),c(2,4))
par.r
```

```
peas Galton's Peas
```


## Description

Francis Galton introduced the correlation coefficient with an analysis of the similarities of the parent and child generation of 700 sweet peas.

## Usage

data(peas)

## Format

A data frame with 700 observations on the following 2 variables.
parent The mean diameter of the mother pea for 700 peas
child The mean diameter of the daughter pea for 700 sweet peas

## Details

Galton's introduction of the correlation coefficient was perhaps the most important contribution to the study of individual differences. This data set allows a graphical analysis of the data set. There are two different graphic examples. One shows the regression lines for both relationships, the other finds the correlation as well.

## Source

Stanton, Jeffrey M. (2001) Galton, Pearson, and the Peas: A brief history of linear regression for statistics intstructors, Journal of Statistics Education, 9. (retrieved from the web from http://www.amstat.org/publications/jse// reproduces the table from Galton, 1894, Table 2.
The data were generated from this table.

## References

Galton, Francis (1877) Typical laws of heredity. paper presented to the weekly evening meeting of the Royal Institution, London. Volume VIII (66) is the first reference to this data set. The data appear in
Galton, Francis (1894) Natural Inheritance (5th Edition), New York: MacMillan).

## See Also

The other Galton data sets: heights, galton,cubits

## Examples

```
data(peas)
pairs.panels(peas,lm=TRUE,xlim=c(14,22),ylim=c(14,22),main="Galton's Peas")
describe(peas)
pairs.panels(peas,main="Galton's Peas")
```

```
phi
Find the phi coefficient of correlation between two dichotomous variables
```


## Description

Given a $1 \times 4$ vector or a $2 \times 2$ matrix of frequencies, find the phi coefficient of correlation. Typical use is in the case of predicting a dichotomous criterion from a dichotomous predictor.

## Usage

phi(t, digits = 2)

## Arguments

$\begin{array}{ll}t & \text { a } 1 \times 4 \text { vector or a } 2 \times 2 \text { matrix } \\ \text { digits } & \text { round the result to digits }\end{array}$

## Details

In many prediction situations, a dichotomous predictor (accept/reject) is validated against a dichotomous criterion (success/failure). Although a polychoric correlation estimates the underlying Pearson correlation as if the predictor and criteria were continuous and bivariate normal variables, the phi coefficient is the Pearson applied to a matrix of 0's and 1s.
For a very useful discussion of various measures of association given a $2 \times 2$ table, and why one should probably prefer the Yule coefficient, see Warren (2008).
Given a two x two table of counts

| $a$ | $b$ | $a+b$ |
| :--- | :--- | :--- |
| $c$ | $d$ | $c+d$ |
| $a+c$ | $b+d$ | $a+b+c+d$ |

convert all counts to fractions of the total and then $\backslash P h i=a-(a+b) *(a+c) / s q r t((a+b)(c+d)(a+c)(b+d)$ )

## Value

phi coefficient of correlation

## Author(s)

William Revelle with modifications by Leo Gurtler

## References

Warrens, Matthijs (2008), On Association Coefficients for $2 \times 2$ Tables and Properties That Do Not Depend on the Marginal Distributions. Psychometrika, 73, 777-789.

## See Also

```
phi2poly,Yule, Yule2phi
```


## Examples

```
phi(c(30,20,20,30))
phi(c(40,10,10,40))
x <- matrix(c (40,5,20,20),ncol=2)
phi(x)
```


## Description

A not very interesting demo of what happens if bivariate continuous data are dichotomized. Bascially a demo of $r$, phi, and polychor.

## Usage

phi. demo( $n=1000, r=.6$, $\operatorname{cuts}=c(-2,-1,0,1,2))$

## Arguments

n
number of cases to simulate
$r \quad$ correlation between latent and observed
cuts form dichotomized variables at the value of cuts

## Details

A demonstration of the problem of different base rates on the phi correlation, and how these are partially solved by using the polychoric correlation. Not one of my more interesting demonstrations. See http://personality-project.org/r/simulating-personality.html and http://personality-project.org/r/r.datageneration.html for better demonstrations of data generation.

## Value

a matrix of correlations and a graphic plot. The items above the diagonal are the tetrachoric correlations, below the diagonal are raw correlations.

## Author(s)

William Revelle

## References

http://personality-project.org/r/simulating-personality.html and http: //personality-project.org/r/r.datageneration.html for better demonstrations of data generation.

## See Also

```
VSS.simulate,item.sim
```


## Examples

```
if(require(polycor)) {demo <- phi.demo() #compare the phi (lower off diagonal and polychoric
#show the result from poly.mat
round(demo$tetrachoric,2)
#show the result from phi2poly
#tetrachorics above the diagonal, phi below the diagonal
round(demo$phis,2) }
```

phi2poly Convert a phi coefficient to a polychoric correlation

## Description

Given a phi coefficient (a Pearson r calculated on two dichotomous variables), and the marginal frequencies (in percentages), what is the corresponding estimate of the polychoric correlation?

Given a two x two table of counts
a b
c d

The phi coefficient is $\left(a-(a+b)^{*}(a+c)\right) / \operatorname{sqrt}((a+b)(a+c)(b+d)(c+c))$.
This function reproduces the cell entries for specified marginals and then calls John Fox's polychor function.

## Usage

phi2poly(ph, cp, cc)

## Arguments

| ph | phi |
| :--- | :--- |
| cp | probability of the predictor - the so called selection ratio |
| cc | probability of the criterion - the so called success rate. |

## Details

Uses John Fox's polycor package, which in turn requires the mvtnorm package

## Value

a polychoric correlation

## Author(s)

William Revelle

## See Also

polychor.matrix, Yule2phi.matrix, phi2poly.matrix

## Examples

```
#phi2poly(.3,.5,.5)
#phi2poly(.3,.3,.7)
```

```
plot.psych Plotting functions for the psych package of class "psych"
```


## Description

Combines several plotting functions into one for objects of class "psych". This can be used to plot the results of fa, irt.fa, VSS, ICLUST, omega, factor.pa, or principal.

## Usage

```
plot.psych (x,labels=NULL,...)
plot.irt (x, xlab,ylab, main, D,type=c("ICC","IIC","test"), ...)
plot.poly(x,D,xlab,ylab,ylim,main,type=c("ICC", "IIC","test"), ...)
```


## Arguments

X
The object to plot
labels
Variable labels
xlab
ylab
Label for the x axis - defaults to Latent Trait
Label for the $y$ axis
main Main title for graph
type ICC plots items, IIC plots item information, test plots test information, defaults to IIC for plot.irt, to ICC for plot.poly

D The discrimination parameter
. . other calls to plot

## Details

Passes the appropriate values to plot. For plotting the results of irt.fa, there are three options: type = "ICC" (default) will plot the item characteristic respone function. type = "IIC" will plot the item information function, and type= "test" will plot the test information function.
These are calls to the generic plot function that are intercepted for objects of type "psych". More precise plotting control is available in the separate plot functions. plot may be used for psych objects returned from fa, irt.fa, ICLUST, omega, as well as principal

## Value

Graphic output for factor analysis, cluster analysis and item response analysis.

## Note

More precise plotting control is available in the separate plot functions.

## Author(s)

William Revelle

## See Also

```
VSS.plot and factor.plot, cluster.plot, fa, irt.fa,VSS,ICLUST,omega, factor.pa,
or principal
```


## Examples

```
test.data <- Harman74.cor$cov
f4 <- fa(test.data,4)
plot(f4)
#not run
#data(bfi)
#e.irt <- irt.fa(bfi[11:15]) #just the extraversion items
```

```
#plot(e.irt) #the information curves
#
#ic <- iclust(test.data,3) #shows hierarchical structure
#plot(ic) #plots loadings
#
```

polar Convert Cartesian factor loadings into polar coordinates

## Description

Factor and cluster analysis output typically presents item by factor correlations (loadings). Tables of factor loadings are frequently sorted by the size of loadings. This style of presentation tends to make it difficult to notice the pattern of loadings on other, secondary, dimensions. By converting to polar coordinates, it is easier to see the pattern of the secondary loadings.

## Usage

polar(f, sort = TRUE)

## Arguments

$$
\begin{array}{ll}
\text { f } & \text { A matrix of loadings or the output from a factor or cluster analysis program } \\
\text { sort } & \text { sort=TRUE: sort items by the angle of the items on the first pair of factors. }
\end{array}
$$

## Details

Although many uses of factor analysis/cluster analysis assume a simple structure where items have one and only one large loading, some domains such as personality or affect items have a more complex structure and some items have high loadings on two factors. (These items are said to have complexity 2, see VSS). By expressing the factor loadings in polar coordinates, this structure is more readily perceived.
For each pair of factors, item loadings are converted to an angle with the first factor, and a vector length corresponding to the amount of variance in the item shared with the two factors.
For a two dimensional structure, this will lead to a column of angles and a column of vector lengths. For $n$ factors, this leads to $n^{*}(n-1) / 2$ columns of angles and an equivalent number of vector lengths.

Value
polar A data frame of polar coordinates

## Author(s)

William Revelle

## References

Rafaeli, E. \& Revelle, W. (2006). A premature consensus: Are happiness and sadness truly opposite affects? Motivation and Emotion. \}
Hofstee, W. K. B., de Raad, B., \& Goldberg, L. R. (1992). Integration of the big five and circumplex approaches to trait structure. Journal of Personality and Social Psychology, 63, 146-163.

See Also

```
ICLUST, cluster.plot, circ.tests, factor.pa
```


## Examples

```
circ.data <- circ.sim(24,500)
circ.fa <- factor.pa(circ.data,2)
circ.polar <- round(polar(circ.fa),2)
circ.polar
#compare to the graphic
cluster.plot(circ.fa)
```

```
polychor.matrix Phi or Yule coefficient matrix to polychoric coefficient matrix
```


## Description

Given a matrix of phi or Yule correlation coefficients and a vector of marginals, use John Fox's polycor function to convert these to polychoric correlations.

Some older correlation matrices were reported as matrices of Phi or of Yule correlations. That is, correlations were found from the two by two table of counts:
a b
c d

Yule Q is $(\mathrm{ad}-\mathrm{bc}) /(\mathrm{ad}+\mathrm{bc})$.

With marginal frequencies of $a+b, c+d, a+c, b+d$.
Given a square matrix of such correlations, and the proportions for each variable that are in the a +b cells, it is possible to reconvert each correlation into a two by two table and then estimate the corresponding polychoric correlation (using John Fox's polychor function.

## Usage

Yule2poly.matrix(x, v)
phi2poly.matrix(x, v)
principal

Yule2phi.matrix(x, v)

## Arguments

x
a matrix of phi or Yule coefficients
v A vector of marginal frequencies

## Details

These functions call Yule2poly, Yule2phi or phi2poly for each cell of the matrix. See those functions for more details. See phi. demo for an example.

## Value

A matrix of correlations

## Author(s)

William Revelle

## Examples

```
if(require(polycor)) {demo <- phi.demo() #compare the phi (lower off diagonal and polychoric
#show the result from poly.mat
round(demo$tetrachoric,2)
#show the result from phi2poly
#tetrachorics above the diagonal, phi below the diagonal
round(demo$phis,2) }
```

principal Principal components analysis

## Description

Does an eigen value decomposition and returns eigen values, loadings, and degree of fit for a specified number of components. Basically it is just doing a principal components for n principal components. Can show the residual correlations as well. The quality of reduction in the squared correlations is reported by comparing residual correlations to original correlations. Unlike princomp, this returns a subset of just the best nfactors. The eigen vectors are rescaled by the sqrt of the eigen values to produce the component loadings more typical in factor analysis.

## Usage

principal(r, nfactors $=1$, residuals $=$ FALSE, rotate="varimax", n.obs=NA, scores=FALS

## Arguments

$\Upsilon$
a correlation matrix. If a raw data matrix is used, the correlations will be found using pairwise deletions for missing values.
nfactors Number of components to extract
residuals
FALSE, do not show residuals, TRUE, report residuals
rotate "none", "varimax", "quatimax", "promax", "oblimin", "simplimax", and "cluster" are possible rotations/transformations of the solution.
n. obs Number of observations used to find the correlation matrix if using a correlation matrix. Used for finding the goodness of fit statistics.
scores If TRUE, find component scores
missing if scores are TRUE, and missing=TRUE, then impute missing values using either the median or the mean
impute $\quad$ "median" or "mean" values are used to replace missing values

## Details

Useful for those cases where the correlation matrix is improper (perhaps because of SAPA techniques).
There are a number of data reduction techniques including principal components and factor analysis. Both PC and FA attempt to approximate a given correlation or covariance matrix of rank n with matrix of lower rank (p). ${ }_{n} R_{n} \approx_{n} F_{k k} F_{n}^{\prime}+U^{2}$ where k is much less than n . For principal components, the item uniqueness is assumed to be zero and all elements of the correlation matrix are fitted. That is, ${ }_{n} R_{n} \approx_{n} F_{k k} F_{n}^{\prime}$ The primary empirical difference between a components versus a factor model is the treatment of the variances for each item. Philosophically, components are weighted composites of observed variables while in the factor model, variables are weighted composites of the factors.
For a n x n correlation matrix, the n principal components completely reproduce the correlation matrix. However, if just the first k principal components are extracted, this is the best k dimensional approximation of the matrix.

It is important to recognize that rotated principal components are not principal components (the axes associated with the eigen value decomposition) but are merely components. To point this out, unrotated principal components are labelled as PCi , while rotated PCs are now labeled as RCi (for rotated components) and obliquely transformed components as TCi (for transformed components). (Thanks to Ulrike Gromping for this suggestion.)
Rotations and transformations are either part of psych (Promax and cluster), of base R (varimax), or of GPArotation (simplimax, quartimax, oblimin).
Some of the statistics reported are more appropriate for (maximum likelihood) factor analysis rather than principal components analysis, and are reported to allow comparisons with these other models.
Although for items, it is typical to find component scores by scoring the salient items (using, e.g., score.items) component scores can be estimated either by regression (which is done) or just multiplying the data by the components. The regression approach is done to be parallel with the factor analysis function fa. The regression weights are found from the inverse of the correlation matrix times the component loadings. This has the result that the component scores are standard scores (mean $=0, \mathrm{sd}=1$ ) of the standardized input. A comparison to the scores from princomp
shows this difference. princomp does not, by default, standardize the data matrix, nor are the components themselves standardized.

## Value

| values | Eigen Values of all components - useful for a scree plot |
| :---: | :---: |
| rotation | which rotation was requested? |
| n.obs | number of observations specified or found |
| communality | Communality estimates for each item. These are merely the sum of squared factor loadings for that item. |
| loadings | A standard loading matrix of class "loadings" |
| fit | Fit of the model to the correlation matrix |
| fit.off | how well are the off diagonal elements reproduced? |
| residual | Residual matrix - if requested |
| dof | Degrees of Freedom for this model. This is the number of observed correlations minus the number of independent parameters (number of items * number of factors $-\mathrm{nf} *(\mathrm{nf}-1) / 2$. That is, $\mathrm{dof}=\mathrm{niI} *(\mathrm{ni}-1) / 2-\mathrm{ni} * \mathrm{nf}+\mathrm{nf}^{*}(\mathrm{nf}-1) / 2$. |
| objective | value of the function that is minimized by maximum likelihood procedures. This is reported for comparison purposes and as a way to estimate chi square goodness of fit. The objective function is $f=\log \left(\operatorname{trace}\left(\left(F F^{\prime}+U 2\right)^{-1} R\right)-\log \left(\left\|\left(F F^{\prime}+U 2\right)^{-1} R\right\|\right)-\right.$ n.items. Because components do not minimize the off diagonal, this fit will be not as good as for factor analysis. |
| STATISTIC | If the number of observations is specified or found, this is a chi square based upon the objective function, f. Using the formula from factanal: $\left.\chi^{2}=(n . o b s-1-(2 * p+5) / 6-(2 * \text { factors }) / 3)\right) * f$ |
| PVAL | If n.obs $>0$, then what is the probability of observing a chisquare this large or larger? |
| phi | If oblique rotations (using oblimin from the GPArotation package) are requested, what is the interfactor correlation. |
| scores | If scores=TRUE, then estimates of the factor scores are reported |
| weights | The beta weights to find the principal components from the data |
| R2 | The multiple R square between the factors and factor score estimates, if they were to be found. (From Grice, 2001) For components, these are of course 1.0. |
| valid | The correlations of the component score estimates with the components, if they were to be found and unit weights were used. (So called course coding). |

## Author(s)

William Revelle

## References

Grice, James W. (2001), Computing and evaluating factor scores. Psychological Methods, 6, 430450

Revelle, W. An introduction to psychometric theory with applications in R (in prep) Springer. Draft chapters available at http://personality-project.org/r/book/

## See Also

VSS (to test for the number of components or factors to extract), VSS. scree and fa.parallel to show a scree plot and compare it with random resamplings of the data), factor2cluster (for course coding keys), fa (for factor analysis), factor. congruence (to compare solutions)

## Examples

```
#Four principal components of the Harmon 24 variable problem
#compare to a four factor principal axes solution using factor.congruence
pc <- principal(Harman74.cor$cov,4,rotate="varimax")
mr <- fa(Harman74.cor$cov,4,rotate="varimax") #minres factor analysis
pa <- fa(Harman74.cor$cov,4,rotate="varimax",fm="pa") # principal axis factor analysis
round(factor.congruence(list(pc,mr,pa)), 2)
```

```
print.psych Print and summary functions for the psych class
```


## Description

Give limited output (print) or somewhat more detailed (summary) for the factor.pa, omega, ICLUST, score.items, cluster.cor, cluster.loadings and the sim functions. In addition, will supply the factor correlations for output from a promax rotation applied to a factanal output.

## Usage

print.psych(x,digits=2, all=FALSE, cut=NULL, sort=FALSE, ...)
summary.psych(object, digits=2,items=FALSE, ...)

## Arguments

X
object Output from a psych function
items items=TRUE (default) does not print the item whole correlations
digits $\quad$ Number of digits to use in printing

| all | if all=TRUE, then the object is declassed and all output from the function is <br> printed |
| :--- | :--- |
| cut | Cluster loadings < cut will not be printed. For the factor analysis functions (fa <br> and factor.pa etc.), cut defaults to 0, for ICLUST to .3 , for omega to .2. |
| sort | Cluster loadings are in sorted order |
| $\ldots$ | More options to pass to summary and print |

## Details

Most of the psych functions produce too much output. print.psych and summary.psych use generic methods for printing just the highlights. To see what else is available, either ask for the structure ( $\operatorname{str}($ theobject $)$.

To get complete output, unclass(theobject) and then print it.
As an added feature, if the promax function is applied to a factanal loadings matrix, the normal output just provides the rotation matrix. print.psych will provide the factor correlations. (Following a suggestion by John Fox and Uli Keller to the R-help list). The alternative is to just use the Promax function directly on the factanal object.

## Value

Various psych functions produce copious output. This is a way to summarize the most important parts of the output of the score.items, cluster.scores, and ICLUST functions. See those ( score.items, cluster.cor, cluster.loadings, or ICLUST) for details on what is produced.

## Note

See score.items, cluster.cor, cluster.loadings, or ICLUSTfor details on what is printed.

## Author(s)

William Revelle

## Examples

```
data(bfi)
    keys.list <- list(agree=c(-1,2:5), conscientious=c(6:8,-9,-10), extraversion=c (-11,-12,13:15)
    keys <- make.keys(25,keys.list,item.labels=colnames(bfi[1:25]))
    scores <- score.items(keys,bfi[1:25], short=TRUE)
    scores
    summary(scores)
```

Promax Perform promax or targeted rotations and return the inter factor angles

## Description

promax is an oblique rotation function introduced by Hendrickson and White (1964) and implemented in the promax function in the stats package. Unfortunately, promax does not report the inter factor correlations. Promax does. target.rot does general target rotations to an arbitrary target matrix. The default target rotation is for an independent cluster solution.

## Usage

Promax (x, m = 4)
target.rot $(x$, keys=NULL)

## Arguments

X
m
keys

## A loadings matrix

the power to which to raise the varimax loadings (for Promax)
An arbitrary target matrix, can be composed of any weights, but probably $-1,0$, 1 weights. If missing, the target is the independent cluster structure determined by assigning every item to it's highest loaded factor.

## Details

Promax is a very direct adaptation of the stats::promax function. The addition is that it will return the interfactor correlations as well as the loadings and rotation matrix.

In addition, it will take output from either the factanal, fa or ealier (factor. pa, factor.minres or principal) functions and select just the loadings matrix for analysis.
The target.rot function is an adaptation of a function of Michael Browne's to do rotations to arbitrary target matrices. Suggested by Pat Shrout.

The default for target.rot is to rotate to an independent cluster structure (every items is assigned to a group with its highest loading.)
target.rot will not handle targets that have linear dependencies (e.g., a pure bifactor model where there is a g loading and a group factor for all variables).

## Value

loadings Oblique factor loadings
rotmat The rotation matrix applied to the original loadings to produce the promax soluion or the targeted matrix

Phi The interfactor correlation matrix

## Note

A direct adaptation of the stats:promax function following suggestions to the R-help list by Ulrich Keller and John Fox. Further modified to do targeted rotation similar to a function of Michael Browne.

## Author(s)

William Revelle

## References

Hendrickson, A. E. and White, P. O, 1964, British Journal of Statistical Psychology, 17, 65-70.

## See Also

promax, factor.pa, factor.minres, or principal

## Examples

```
jen <- sim.hierarchical()
f3 <- factor.minres(jen,3)
Promax(f3)
target.rot(f3)
m3 <- factanal(covmat=jen,factors=3)
Promax(m3) #example of taking the output from factanal
#compare this rotation with the solution from a targeted rotation aimed for an independent
target.rot(m3)
```

r.test Tests of significance for correlations

## Description

Tests the significance of a single correlation, the difference between two independent correlations, the difference between two dependent correlations sharing one variable (Williams's Test), or the difference between two dependent correlations with different variables (Steiger Tests).

## Usage

r.test $(\mathrm{n}, \mathrm{r} 12, r 34=$ NULL, $r 23=$ NULL, $r 13=$ NULL, $r 14=$ NULL, $r 24=$ NULL, $n 2=$ NUI

## Arguments

| n | Sample size of first group |
| :--- | :--- |
| r12 | Correlation to be tested |
| r34 | Test if this correlation is different from r12, if r23 is specified, but r13 is not, <br> then r34 becomes r13 |


| r23 | if $\mathrm{ra}=\mathrm{r}(12)$ and $\mathrm{rb}=\mathrm{r}(13)$ then test for differences of dependent correlations <br> given r 23 |
| :--- | :--- |
| r 13 | implies $\mathrm{ra}=\mathrm{r}(12)$ and $\mathrm{rb}=\mathrm{r}(34)$ test for difference of dependent correlations <br> r 14 <br> r 24 <br> n 2 |
| $\mathrm{ra}=\mathrm{r}(12)$ and $\mathrm{rb}=\mathrm{r}(34)$ <br> n 2 is specified in the case of two independent correlations. n 2 defaults to n if if <br> pooled <br> twot specified |  |
|  | use pooled estimates of correlations |
| should a twotailed or one tailed test be used |  |

## Details

Depending upon the input, one of four different tests of correlations is done. 1) For a sample size n , find the t value for a single correlation.
2) For sample sizes of $n$ and $n 2(n 2=n$ if not specified) find the $z$ of the difference between the $z$ transformed correlations divided by the standard error of the difference of two z scores.
3) For sample size $n$, and correlations $r a=r 12, r b=r 23$ and $r 13$ specified, test for the difference of two dependent correlations.
4) For sample size $n$, test for the difference between two dependent correlations involving different variables.

For clarity, correlations may be specified by value. If specified by location and if doing the test of dependent correlations, if three correlations are specified, they are assumed to be in the order r12, r13, r23.

## Value

| test | Label of test done |
| :--- | :--- |
| $z$ | $z$ value for tests 2 or 4 |
| $t$ | $t$ value for tests 1 and 3 |
| $p$ | probability value of $z$ or $t$ |

## Note

Steiger specifically rejects using the Hotelling T test to test the difference between correlated correlations. Instead, he recommends Williams' test. (See also Dunn and Clark, 1971). These tests follow Steiger's advice.

## Author(s)

William Revelle

## References

Olkin, I. and Finn, J. D. (1995). Correlations redux. Psychological Bulletin, 118(1):155-164.
Steiger, J.H. (1980), Tests for comparing elements of a correlation matrix, Psychological Bulletin, 87, 245-251.
Williams, E.J. (1959) Regression analysis. Wiley, New York, 1959.

## See Also

See also corr.test which tests all the elements of a correlation matrix, and cortest.mat to compare two matrices of correlations. r.test extends the tests in paired.r,r.con

## Examples

```
n <- 30
r<- seq(0,.9,.1)
rc <- matrix(r.con(r,n),ncol=2)
test <- r.test(n,r)
r.rc <- data.frame(r=r,z=fisherz(r),lower=rc[,1],upper=rc[,2],t=test$t,p=test$p)
round(r.rc,2)
r.test (50,r)
r.test(30,.4,.6) #test the difference between two independent correlations
r.test(103,.4,.5,.1) #Steiger case A
r.test (103,.5,.6,.7,.5,.5,.8) #steiger Case B
```

read.clipboard shortcut for reading from the clipboard

## Description

input from the keyboard is easy but a bit obscure, particularly for Mac users. This is just an easier mnemonic to do so. Also will do some processing to read lower triangular matrices and fill them out to square matrices.

## Usage

```
read.clipboard(header = TRUE, ...) #assumes headers and tab or space delimited
read.clipboard.csv(header=TRUE,sep=',',...) #assumes headers and comma delimited
read.clipboard.tab(header=TRUE,sep='\t',...) #assumes headers and tab delimited
read.clipboard.lower(diag=TRUE,names=NULL,...) #read in a matrix given the lower c
read.clipboard.upper(diag=TRUE, names=NULL,...)
read.clipboard.fwf(header=FALSE,widths=rep(1,10),...) #read in data using a fixed
```


## Arguments

| header | Does the first row have variable labels |
| :--- | :--- |
| sep | What is the designated separater between data fields? |
| diag | for upper or lower triangular matrices, is the diagonal specified or not <br> names |
| for read.clipboard.lower or upper, what colnames to use <br> widths | how wide are the columns in fixed width input. The default is to read 10 columns <br> of size 1. |
| $\ldots$ | Other parameters to pass to read |

## Details

A typical session of R might involve data stored in text files, generated on line, etc. Although it is easy to just read from a file (particularly if using file.locate() or file.choose(), copying from the file to the clipboard and then reading from the clipboard is also very convenient (and somewhat more intuitive to the naive user. This is particularly convenient when copying from a text book or article and just moving a section of text into R.)
Based upon a suggestion by Ken Knoblauch to the R-help listserve.
If the input file that was copied into the clipboard was an Excel file with blanks for missing data, then read.clipboard.tab() will correctly replace the blanks with NAs. Similarly for a csv file with blank entries, read.clipboard.csv will replace empty fields with NA.
read.clipboard.lower and read.clipboard.upper are adapted from John Fox's read.moments function in the sem package. They will read a lower (or upper) triangular matrix from the clipboard and return a full, symmetric matrix for use by factanal, factor. pa, ICLUST, etc. If the diagonal is false, it will be replaced by 1.0 s. These two function were added to allow easy reading of examples from various texts and manuscripts with just triangular output.
read.clipboard.fwf will read fixed format files from the clipboard. It includes a patch to read.fwf which will not read from the clipboard or from remote file. See read.fwf for documentation of how to specify the widths.

## Value

the contents of the clipboard.

## Author(s)

William Revelle

## Examples

```
#my.data <- read.clipboad()
#my.data <- read.clipboard.csv()
#my.data <- read.clipboad(header=FALSE)
```

```
rescale
```

Function to convert scores to "conventional " metrics

## Description

Psychologists frequently report data in terms of transformed scales such as "IQ" (mean=100, sd=15, "SAT/GRE" (mean=500, sd=100), "ACT" (mean=18, sd=6), "T-scores" (mean=50, sd=10), or "Stanines" (mean $=5, \mathrm{sd}=2$ ). The rescale function converts the data to standard scores and then rescales to the specified mean(s) and standard deviation(s).

## Usage

rescale(x, mean $=100, \mathrm{sd}=15, \mathrm{df}=\mathrm{TRUE})$
reverse.code

## Arguments

| $x$ | A matrix or data frame |
| :--- | :--- |
| mean | Desired mean of the rescaled scores- may be a vector |
| $s d$ | Desired standard deviation of the rescaled scores |
| $d f$ | if TRUE, returns a data frame, otherwise a matrix |

## Value

A data.frame (default) or matrix of rescaled scores.

## Author(s)

William Revelle

## See Also

See Also scale

## Examples

```
T <- rescale(attitude,50,10) #all put on same scale
describe(T)
T1 <- rescale(attitude,seq(0,300,50),seq(10,70,10)) #different means and sigmas
describe(T1)
```

reverse.code Reverse the coding of selected items prior to scale analysis

## Description

Some IRT functions require all items to be coded in the same direction. Some data sets have items that need to be reverse coded (e.g., $6->1,1->6$ ). reverse.code will flip items based upon a keys vector of 1 s and -1 s . Reversed items are subtracted from the item max + item min. These may be specified or may be calculated.

## Usage

reverse.code(keys, items, mini $=$ NULL, maxi $=$ NULL)

## Arguments

keys A vector of 1s and -1 s. -1 implies reverse the item
items A data set of items
mini if NULL, the empirical minimum for each item. Otherwise, a vector of minima
$\operatorname{maxi} \quad \mathrm{f}$ NULL, the empirical maximum for each item. Otherwise, a vector of maxima

## Details

Not a very complicated function, but useful in the case that items need to be reversed prior to using IRT functions from the ltm or eRM packages. Most psych functions do not require reversing prior to analysis, but will do so within the function.

## Value

The corrected items.

## Examples

```
original <- matrix(sample(6,50,replace=TRUE),10,5)
keys <- c(1,1,-1,-1,1) #reverse the 3rd and 4th items
new <- reverse.code(keys,original,mini=rep (1,5),maxi=rep (6,5))
original[1:3,]
new[1:3,]
```

sat.act

3 Measures of ability: SATV, SATQ, ACT

## Description

Self reported scores on the SAT Verbal, SAT Quantitative and ACT were collected as part of the Synthetic Aperture Personality Assessment (SAPA) web based personality assessment project. Age, gender, and education are also reported. The data from 700 subjects are included here as a demonstration set for correlation and analysis.

## Usage

```
data(sat.act)
```


## Format

A data frame with 700 observations on the following 6 variables.
gender males $=1$, females $=2$
education self reported education $1=$ high school $. . .5=$ graduate work
age age
ACT ACT composite scores may range from 1-36. National norms have a mean of 20.
SATV SAT Verbal scores may range from 200-800.
SATQ SAT Quantitative scores may range from 200-800

## Details

hese items were collected as part of the SAPA project to develop online measures of ability (Revelle, Wilt and Rosenthal, 2009). The score means are higher than national norms suggesting both self selection for people taking on line personality and ability tests and a self reporting bias in scores.
See also the iq.items data set.

## Source

```
http://personality-project.org
```


## References

Revelle, William, Wilt, Joshua, and Rosenthal, Allen (2009) Personality and Cognition: The PersonalityCognition Link. In Gruszka, Alexandra and Matthews, Gerald and Szymura, Blazej (Eds.) Handbook of Individual Differences in Cognition: Attention, Memory and Executive Control, Springer.

## Examples

```
data(sat.act)
describe(sat.act)
pairs.panels(sat.act)
```

scaling.fits Test the adequacy of simple choice, logistic, or Thurstonian scaling.

## Description

Given a matrix of choices and a vector of scale values, how well do the scale values capture the choices? That is, what is size of the squared residuals given the model versus the size of the squared choice values?

## Usage

```
scaling.fits(model, data, test = "logit", digits = 2, rowwise = TRUE)
```


## Arguments

```
model A vector of scale values
data A matrix or dataframe of choice frequencies
test "choice", "logistic", "normal"
digits Precision of answer
rowwise Are the choices ordered by column over row (TRUE) or row over column False)
```


## Details

How well does a model fit the data is the classic problem of all of statistics. One fit statistic for scaling is the just the size of the residual matrix compared to the original estimates.

## Value

| GF | Goodness of fit of the model |
| :--- | :--- |
| original | Sum of squares for original data |
| resid | Sum of squares for residuals given the data and the model |
| residual | Residual matrix |

## Note

Mainly for demonstration purposes for a course on psychometrics

## Author(s)

William Revelle

## References

Revelle, W. (in preparation) Introduction to psychometric theory with applications in R, Springer. http://personality-project.org/r/book

## See Also

thurstone, vegetables

Schmid 12 variables created by Schmid and Leiman to show the SchmidLeiman Transformation

## Description

John Schmid and John M. Leiman (1957) discuss how to transform a hierarchical factor structure to a bifactor structure. Schmid contains the example $12 \times 12$ correlation matrix. schmid.leiman is a $12 \times 12$ correlation matrix with communalities on the diagonal. This can be used to show the effect of correcting for attenuation. Two additional data sets are taken from Chen et al. (2006).

## Usage

data(Schmid)

## Details

Two artificial correlation matrices from Schmid and Leiman (1957). One real and one artificial covariance matrices from Chen et al. (2006).

- Schmid: a $12 \times 12$ artificial correlation matrix created to show the Schmid-Leiman transformation.
- schmid.leiman: A $12 \times 12$ matrix with communalities on the diagonal. Treating this as a covariance matrix shows the $6 \times 6$ factor solution
- Chen: An $18 \times 18$ covariance matrix of health related quality of life items from Chen et al. (2006). Number of observations $=403$. The first item is a measure of the quality of life. The remaining 17 items form four subfactors: The items are (a) Cognition subscale: "Have difficulty reasoning and solving problems?" "React slowly to things that were said or done?"; "Become confused and start several actions at a time?" "Forget where you put things or appointments?"; "Have difficulty concentrating?" (b) Vitality subscale: "Feel tired?" "Have enough energy to do the things you want?" (R) "Feel worn out?" ; "Feel full of pep?" (R). (c)

Mental health subscale: "Feel calm and peaceful?"(R) "Feel downhearted and blue?"; "Feel very happy"(R) ; "Feel very nervous?" ; "Feel so down in the dumps nothing could cheer you up? (d) Disease worry subscale: "Were you afraid because of your health?"; "Were you frustrated about your health?"; "Was your health a worry in your life?" .

- West: A $16 \times 16$ artificial covariance matrix from Chen et al. (2006).


## Source

John Schmid Jr. and John. M. Leiman (1957), The development of hierarchical factor solutions.Psychometrika, 22, 83-90.
F.F. Chen, S.G. West, and K.H. Sousa.(2006) A comparison of bifactor and second-order models of quality of life. Multivariate Behavioral Research, 41(2):189-225, 2006.

## References

Y.-F. Yung, D.Thissen, and L.D. McLeod. (1999) On the relationship between the higher-order factor model and the hierarchical factor model. Psychometrika, 64(2):113-128, 1999.

## Examples

```
data(Schmid)
cor.plot(Schmid,TRUE)
print(fa(Schmid,6,rotate="oblimin"), cut=0) #shows an oblique solution
round(cov2cor(schmid.leiman),2)
cor.plot(cov2cor(West),TRUE)
```

```
schmid Apply the Schmid Leiman transformation to a correlation matrix
```


## Description

One way to find omega is to do a factor analysis of the original data set, rotate the factors obliquely, do a Schmid Leiman transformation, and then find omega. Here is the code for Schmid Leiman. The S-L transform takes a factor or PC solution, transforms it to an oblique solution, factors the oblique solution to find a higher order ( g ) factor, and then residualizes $g$ out of the the group factors.

## Usage

schmid(model, $\mathrm{nfactors}=3, \mathrm{fm}=$ "minres", digits=2, rotate="oblimin", n. obs=NA, optior

## Arguments

model A correlation matrix
nfactors Number of factors to extract
fm the default is to do minres. fm="pa" for principal axes, fm="pc" for principal components, $\mathrm{fm}=$ "minres" for minimum residual (OLS), $\mathrm{pc}=" \mathrm{ml}$ " for maximum likelihood

| digits | if digits not equal NULL, rounds to digits <br> rotate |
| :--- | :--- |
| n.obs | The default, oblimin, produces somewhat more correlated factors than the alter- <br> native, simplimax. The third option is the promax criterion |
| option | Number of observations, used to find fit statistics if specified. Will be calculated <br> if input is raw data <br> When asking for just two group factors, option can be for "equal", "first" or <br> "second" |
| Phi | If Phi is specified, then the analysis is done on a pattern matrix with the asso- <br> ciated factor intercorrelation (Phi) matrix. This allows for reanalysess of pub- <br> lished results |
| ... | Allows additional parameters to be passed to the factoring routines |

## Details

Schmid Leiman orthogonalizations are typical in the ability domain, but are not seen as often in the non-cognitive personality domain. S-L is one way of finding the loadings of items on the general factor for estimating omega.
A typical example would be in the study of anxiety and depression. A general neuroticism factor $(\mathrm{g})$ accounts for much of the variance, but smaller group factors of tense anxiety, panic disorder, depression, etc. also need to be considerd.
An alternative model is to consider hierarchical cluster analysis techniques such as ICLUST.
Requires the GPArotation package.
Although 3 factors are the minimum number necessary to define the solution uniquely, it is occasionally useful to allow for a two factor solution. There are three possible options for this condition: setting the general factor loadings between the two lower order factors to be "equal" which will be the sqrt(oblique correlations between the factors) or to "first" or "second" in which case the general factor is equated with either the first or second group factor. A message is issued suggesting that the model is not really well defined.

A diagnostic tool for testing the appropriateness of a hierarchical model is p 2 which is the percent of the common variance for each variable that is general factor variance. In general, p2 should not have much variance.

## Value

| sl | loadings on $\mathrm{g}+$ nfactors group factors, communalities, uniqueness, percent of <br> g 2 of h 2 |
| :--- | :--- |
| orthog | original orthogonal factor loadings |
| oblique | oblique factor loadings |
| phi | correlations among the transformed factors |
| gload | loadings of the lower order factors on $g$ |
| $\ldots$ |  |

## Author(s)

William Revelle

## References

http://personality-project.org/r/r.omega.html gives an example taken from Jensen and Weng, 1994 of a S-L transformation.

## See Also

omega, omega.graph, fa.graph, ICLUST,VSS

## Examples

```
jen <- sim.hierarchical() #create a hierarchical demo
if(!require(GPArotation)) {message("I am sorry, you must have GPArotation installed to use
p.jen <- schmid(jen,rotate="promax") #use the promax rotation
}
```

score.alpha Score scales and find Cronbach's alpha as well as associated statistics

## Description

Given a matrix or data.frame of $k$ keys for $m$ items $(-1,0,1)$, and a matrix or data.frame of items scores for $m$ items and $n$ people, find the sum scores or average scores for each person and each scale. In addition, report Cronbach's alpha, the average r , the scale intercorrelations, and the item by scale correlations. (Superseded by score.items).

## Usage

score.alpha(keys, items, labels = NULL, totals=TRUE, digits = 2)

## Arguments

keys A matrix or dataframe of $-1,0$, or 1 weights for each item on each scale
items Data frame or matrix of raw item scores
labels column names for the resulting scales
totals Find sum scores (default) or average score
digits $\quad$ Number of digits for answer (default =2)

## Details

The process of finding sum or average scores for a set of scales given a larger set of items is a typical problem in psychometric research. Although the structure of scales can be determined from the item intercorrelations, to find scale means, variances, and do further analyses, it is typical to find the sum or the average scale score.

Various estimates of scale reliability include "Cronbach's alpha", and the average interitem correlation. For $\mathrm{k}=$ number of items in a scale, and av.r = average correlation between items in the scale,
alpha $=\mathrm{k} *$ av.r/(1+ (k-1)*av.r). Thus, alpha is an increasing function of test length as well as the test homeogeneity.

Alpha is a poor estimate of the general factor saturation of a test (see Zinbarg et al., 2005) for it can seriously overestimate the size of a general factor, and a better but not perfect estimate of total test reliability because it underestimates total reliability. None the less, it is a useful statistic to report.

## Value

scores Sum or average scores for each subject on the k scales
alpha Cronbach's coefficient alpha. A simple (but non-optimal) measure of the internal consistency of a test. See also beta and omega.
av.r The average correlation within a scale, also known as alpha 1 is a useful index of the internal consistency of a domain.
n.items $\quad$ Number of items on each scale
cor
The intercorrelation of all the scales
item.cor
The correlation of each item with each scale. Because this is not corrected for item overlap, it will overestimate the amount that an item correlates with the other items in a scale.

## Author(s)

William Revelle

## References

An introduction to psychometric theory with applications in R (in preparation). http://personality-project. org/r/book

## See Also

score.items, alpha.scale, correct.cor, alpha.scale, cluster.loadings, omega

## Examples

```
y <- attitude #from the datasets package
keys <- matrix(c(rep (1, 7),rep (1, 4),rep (0,7),rep (-1, 3)), ncol=3)
labels <- c("first","second","third")
x <- score.alpha(keys,y,labels)
```

```
score.items Score item composite scales and find Cronbach's alpha, Guttman lambda 6 and item whole correlations
```


## Description

Given a matrix or data.frame of k keys for m items ( $-1,0,1$ ), and a matrix or data.frame of items scores for $m$ items and $n$ people, find the sum scores or average scores for each person and each scale. In addition, report Cronbach's alpha, Guttman's Lambda 6, the average r, the scale intercorrelations, and the item by scale correlations (raw and corrected for item overlap). Replace missing values with the item median or mean if desired. Will adjust scores for reverse scored items. See make. keys for a convenient way to make the keys file. If the input is a square matrix, then it is assumed that the input is a covariance or correlation matix and scores are not found, but the item statistics are reported. (Similar functionality to cluster.cor). response.frequencies reports the frequency of item endorsements fore each response category for polytomous or multiple choice items.

## Usage

```
score.items(keys, items, totals = FALSE, ilabels = NULL, missing = TRUE,impute="mec
response.frequencies(items,max=10)
```


## Arguments

| keys | A matrix or dataframe of $-1,0$, or 1 weights for each item on each scale. May <br> be created by hand, or by using make. keys |
| :--- | :--- |
| items | Matrix or dataframe of raw item scores |
| totals | if TRUE find total scores, if FALSE (default), find average scores |
| ilabels | a vector of item labels. |
| missing | TRUE: Replace missing values with the corresponding item median or mean. <br> FALSE: do not score that subject <br> impute="median" replaces missing values with the item median, impute = "mean" <br> replaces values with the mean response. |
| min | May be specified as minimum item score allowed, else will be calculated from <br> data |
| max | May be specified as maximum item score allowed, else will be calculated from <br> data. Alternatively, in response frequencies, it is maximum number of alterna- <br> tive responses to count. |
| digits | Number of digits to report <br> short |
|  | if short is TRUE, then just give the item and scale statistics and do not report the <br> scores |

## Details

The process of finding sum or average scores for a set of scales given a larger set of items is a typical problem in psychometric research. Although the structure of scales can be determined from the item intercorrelations, to find scale means, variances, and do further analyses, it is typical to find scores based upon the sum or the average item score. For some strange reason, personality scale scores are typically given as totals, but attitude scores as averages. The default for score.items is the average.
Various estimates of scale reliability include "Cronbach's alpha", Guttman's Lambda 6, and the average interitem correlation. For $k=$ number of items in a scale, and av.r $=$ average correlation between items in the scale, alpha $=\mathrm{k} *$ av.r/( $1+(\mathrm{k}-1) * \mathrm{av} . r)$. Thus, alpha is an increasing function of test length as well as the test homeogeneity.
Surprisingly, 106 years after Spearman (1904) introduced the concept of reliability to psychologists, there are still multiple approaches for measuring it. Although very popular, Cronbach's $\alpha$ (1951) underestimates the reliability of a test and over estimates the first factor saturation.
$\alpha$ (Cronbach, 1951) is the same as Guttman's $\lambda 3$ (Guttman, 1945) and may be found by

$$
\lambda_{3}=\frac{n}{n-1}\left(1-\frac{\operatorname{tr}(\vec{V})_{x}}{V_{x}}\right)=\frac{n}{n-1} \frac{V_{x}-\operatorname{tr}\left(\vec{V}_{x}\right)}{V_{x}}=\alpha
$$

Perhaps because it is so easy to calculate and is available in most commercial programs, alpha is without doubt the most frequently reported measure of internal consistency reliability. Alpha is the mean of all possible spit half reliabilities (corrected for test length). For a unifactorial test, it is a reasonable estimate of the first factor saturation, although if the test has any microstructure (i.e., if it is "lumpy") coefficients $\beta$ (Revelle, 1979; see ICLUST) and $\omega_{h}$ (see omega) are more appropriate estimates of the general factor saturation. $\omega_{t}$ (see omega) is a better estimate of the reliability of the total test.

Guttman's Lambda 6 (G6) considers the amount of variance in each item that can be accounted for the linear regression of all of the other items (the squared multiple correlation or smc), or more precisely, the variance of the errors, $e_{j}^{2}$, and is

$$
\lambda_{6}=1-\frac{\sum e_{j}^{2}}{V_{x}}=1-\frac{\sum\left(1-r_{s m c}^{2}\right)}{V_{x}}
$$

The squared multiple correlation is a lower bound for the item communality and as the number of items increases, becomes a better estimate.

G6 is also sensitive to lumpyness in the test and should not be taken as a measure of unifactorial structure. For lumpy tests, it will be greater than alpha. For tests with equal item loadings, alpha > G6, but if the loadings are unequal or if there is a general factor, G6 $>$ alpha.

Alpha and G6 are both positive functions of the number of items in a test as well as the average intercorrelation of the items in the test. When calculated from the item variances and total test variance, as is done here, raw alpha is sensitive to differences in the item variances. Standardized alpha is based upon the correlations rather than the covariances.
More complete reliability analyses of a single scale can be done using the omega function which finds $\omega_{h}$ and $\omega_{t}$ based upon a hierarchical factor analysis.

Alpha is a poor estimate of the general factor saturation of a test (see Revelle and Zinbarg, 2009; Zinbarg et al., 2005) for it can seriously overestimate the size of a general factor, and a better but
not perfect estimate of total test reliability because it underestimates total reliability. None the less, it is a useful statistic to report.
Correlations between scales are attenuated by a lack of reliability. Correcting correlations for reliability (by dividing by the square roots of the reliabilities of each scale) sometimes help show structure.

By default, missing values are replaced with the corresponding median value for that item. Means can be used instead (impute="mean"), or subjects with missing data can just be dropped (missing = FALSE).

Value
scores Sum or average scores for each subject on the k scales
alpha Cronbach's coefficient alpha. A simple (but non-optimal) measure of the internal consistency of a test. See also beta and omega. Set to 1 for scales of length 1.
av.r The average correlation within a scale, also known as alpha 1 is a useful index of the internal consistency of a domain. Set to 1 for scales with 1 item.
G6 Guttman's Lambda 6 measure of reliability
n. items Number of items on each scale
item.cor The correlation of each item with each scale. Because this is not corrected for item overlap, it will overestimate the amount that an item correlates with the other items in a scale.
cor The intercorrelation of all the scales
corrected The correlations of all scales (below the diagonal), alpha on the diagonal, and the unattenuated correlations (above the diagonal)
item.corrected
The item by scale correlations for each item, corrected for item overlap by replacing the item variance with the smc for that item
response.freq
The response frequency (based upon number of non-missing responses) for each alternative.

## Author(s)

William Revelle

## References

An introduction to psychometric theory with applications in R (in preparation). http://personality-project. org/r/book
Revelle W. and R.E. Zinbarg. (2009) Coefficients alpha, beta, omega and the glb: comments on Sijtsma. Psychometrika, 74(1):145-154.
Zinbarg, R. E., Revelle, W., Yovel, I. and Li, W. (2005) Cronbach's alpha, Revelle's beta, and McDonald's omega h, Their relations with each other and two alternative conceptualizations of reliability, Psychometrika, 70, 123-133.

## See Also

make. keys for a convenient way to create the keys file, score.multiple.choice for multiple choice items,
alpha.scale, correct.cor, cluster.cor, cluster.loadings, omega for item/scale analysis

## Examples

```
#see the example including the bfi data set
data(bfi)
    keys.list <- list(agree=c(-1,2:5), conscientious=c(6:8,-9,-10), extraversion=c(-11,-12,13:15)
    keys <- make.keys(28,keys.list,item.labels=colnames(bfi))
    scores <- score.items(keys,bfi)
    summary(scores)
    #to get the response frequencies, we need to not use the age variable
    scores <- score.items(keys[1:27,],bfi[1:27])
    scores
```

score.multiple.choice

Score multiple choice items and provide basic test statistics

## Description

Ability tests are typically multiple choice with one right answer. score.multiple.choice takes a scoring key and a data matrix (or data.frame) and finds total or average number right for each participant. Basic test statistics (alpha, average r , item means, item-whole correlations) are also reported.

## Usage

score.multiple.choice(key, data, score = TRUE, totals = FALSE, ilabels = NULL, miss

## Arguments

| key | A vector of the correct item alternatives |
| :--- | :--- |
| data | a matrix or data frame of items to be scored. |
| score | score=FALSE, just convert to right (1) or wrong (0). <br> score=TRUE, find the totals or average scores and do item analysis <br> total=FALSE: find the average number correct <br> total=TRUE: find the total number correct |
| ilabels | item labels <br> missing |
| missing=TRUE: missing values are replaced with means or medians <br> mising=FALSE missing values are not scored |  |

impute impute="median", replace missing items with the median score impute="mean": replace missing values with the item mean
digits
short short=TRUE, just report the item statistics, short=FALSE, report item statistics and subject scores as well

## Details

Basically combines score. items with a conversion from multiple choice to right/wrong.
The item-whole correlation is inflated because of item overlap.

## Value

| scores | Subject scores on one scale |
| :--- | :--- |
| missing | Number of missing items for each subject |
| item.stats | scoring key, response frequencies, item whole correlations, n subjects scored, <br> mean, sd, skew, kurtosis and se for each item |
| alpha | Cronbach's coefficient alpha |
| av.r | Average interitem correlation |

## Author(s)

William Revelle

## See Also

```
score.items,omega
```


## Examples

```
data(iqitems)
iq.keys <- c(4,4,3,1,4,3,2,3,1,4,1,3,4,3)
score.multiple.choice(iq.keys,iqitems)
#just convert the items to true or false
iq.tf <- score.multiple.choice(iq.keys,iqitems,score=FALSE)
describe(iq.tf) #compare to previous results
```

```
scrub A utility for basic data cleaning. Changes values outside of minimum and maximum limits to \(N A\)
```


## Description

A tedious part of data analysis is addressing the problem of miscoded data that needs to be converted to NA. For a given data.frame or matrix, scrub will set all values of columns from=from to to=to that are less than a set (vector) of min values or more than a vector of max values to NA.

## Usage

scrub(x, where, min, max,isvalue)

## Arguments

x
where
min
max
isvalue
a data frame or matrix
The variables to examine. (Can be by name or by column number)
a vector of minimum values that are acceptable
a vector of maximum values that are acceptable
a vector of values to be converted to NA (one per variable)

## Details

Solves a tedious problem that can be done directly but that is sometimes awkward.

## Value

The corrected data frame.

## Note

Probably could be optimized to avoid one loop

## Author(s)

William Revelle

## See Also

reverse. code, rescale for other simple utilities.

## Examples

```
x <- matrix(1:150,ncol=10,byrow=TRUE)
#get rid of a complicated set of cases
y <- scrub(x,where=2:4,min=c (20,30,40),max= c(120,110,100),isvalue=c (32,43,54))
Y
```

Find the Standard deviation for a vector, matrix, or data.frame - do not return error if there are no cases

## Description

Find the standard deviation of a vector, matrix, or data.frame. In the latter two cases, return the sd of each column. Unlike the sd function, return NA if there are no observations rather than throw an error.

## Usage

SD (x, na.rm = TRUE) \#deprecated

## Arguments

$\begin{array}{ll}\mathrm{x} & \text { a vector, data.frame, or matrix } \\ \text { na.rm } & \text { na.rm is assumed to be TRUE }\end{array}$

## Details

Finds the standard deviation of a vector, matrix, or data.frame. Returns NA if no cases.
Just an adaptation of the stats:sd function to return the functionality found in $\mathrm{R}<2.7 .0$ or $\mathrm{R}>=$ 2.8.0 Because this problem seems to have been fixed, SD will be removed eventually.

## Value

The standard deviation

## Note

Until R 2.7.0, sd would return a NA rather than an error if no cases were observed. SD brings back that functionality. Although unusual, this condition will arise when analyzing data with high rates of missing values. This function will probably be removed as 2.7 .0 becomes outdated.

## Author(s)

William Revelle

## See Also

These functions use SD rather than sd: describe.by, skew, kurtosi

## Examples

```
data(attitude)
sd(attitude) #all complete
attitude[,1] <- NA
SD(attitude) #missing a column
describe(attitude)
```

sim Functions to simulate psychological/psychometric data.

## Description

A number of functions in the psych package will generate simulated data. These functions include sim for a factor simplex, and sim.simplex for a data simplex, sim. circ for a circumplex structure, sim. congeneric for a one factor factor congeneric model, sim. dichot to simulate dichotomous items, sim.hierarchical to create a hierarchical factor model, sim.item a more general item simulation, sim.minor to simulate major and minor factors, sim.omega to test various examples of omega, sim. parallel to compare the efficiency of various ways of deterimining the number of factors, sim.rasch to create simulated rasch data, sim.irt to create general 1 to 4 parameter IRT data by calling sim.npl 1 to 4 parameter logistic IRT or sim.npn 1 to 4 paramater normal IRT, sim. structural a general simulation of structural models, and sim. anova for ANOVA and lm simulations, and sim.VSS. Some of these functions are separately documented and are listed here for ease of the help function. See each function for more detailed help.

## Usage

```
sim(fx=NULL,Phi=NULL, fy=NULL, n=0,mu=NULL,raw=TRUE)
sim.simplex(nvar =12, r=.8,mu=NULL, n=0)
sim.minor(nvar=12,nfact=3,n=0,fbig=NULL,fsmall = c(-.2,. 2),bipolar=TRUE)
sim.omega(nvar=12,nfact=3,n=0,fbig=NULL,fsmall = c(-. 2,.2),bipolar=TRUE,om.fact=3,f
sim.parallel(ntrials=10, nvar = c(12,24,36,48),nfact = c(1,2,3,4,6),
n = c(200,400))
sim.rasch(nvar = 5,n = 500, low=-3,high=3,d=NULL, a=1,mu=0,sd=1)
sim.irt(nvar = 5, n = 500, low=-3, high=3,a=NULL, c=0, z=1,d=NULL,mu=0,sd=1,mod="logi
sim.npl(nvar = 5, n = 500, low=-3,high=3,a=NULL, c=0, z=1,d=NULL,mu=0,sd=1)
sim.npn(nvar = 5, n = 500, low=-3,high=3,a=NULL, c=0,z=1,d=NULL,mu=0,sd=1)
```


## Arguments

fx
Phi
fy
mu
n
raw

The measurement model for x . If NULL, a 4 factor model is generated
The structure matrix of the latent variables
The measurement model for $y$
The means structure for the fx factors
Number of cases to simulate. If $\mathrm{n}=0$ or NULL, the population matrix is returned.
if raw=TRUE, raw data are returned as well.

| nvar | Number of variables for a simplex structure |
| :---: | :---: |
| nfact | Number of large factors to simulate in sim.minor |
| r | the base correlation for a simplex |
| fbig | Factor loadings for the main factors. Default is a simple structure with loadings sampled from ( $.8, .6$ ) for nvar/nfact variables and 0 for the remaining. If fbig is specified, then each factor has loadings sampled from it. |
| bipolar | if TRUE, then positive and negative loadings are generated from fbig |
| om.fact | Number of factors to extract in omega |
| flip | In omega, should item signs be flipped if negative |
| option | In omega, for the case of two factors, how to weight them? |
| fsmall | nvar/2 small factors are generated with loadings sampled from (-.2,0,.2) |
| ntrials | Number of replications per level |
| low | lower difficulty for sim.rasch or sim.irt |
| high | higher difficulty for sim.rasch or sim.irt |
| a | if not specified as a vector, the descrimination parameter $\mathrm{a}=\alpha$ will be set to 1.0 for all items |
| d | if not specified as a vector, item difficulties ( $\mathrm{d}=\delta$ ) will range from low to high |
| c | the gamma parameter: if not specified as a vector, the guessing asymptote is set to 0 |
| z | the zeta parameter: if not specified as a vector, set to 1 |
| sd | the standard deviation for the underlying latent variable in the irt simulations |
| mod | which IRT model to use, mod="logistic" simulates a logistic function, otherwise, a normal function |

## Details

Simulation of data structures is a very useful tool in psychometric research and teaching. By knowing "truth" it is possible to see how well various algorithms can capture it. For a much longer discussion of the use of simulation in psychometrics, see the accompany vignettes.
The default values for sim.structure is to generate a 4 factor, 12 variable data set with a simplex structure between the factors.
Two data structures that are particular challenges to exploratory factor analysis are the simplex structure and the presence of minor factors. Simplex structures sim. simplex will typically occur in developmental or learning contexts and have a correlation structure of $r$ between adjacent variables and $\mathrm{r}^{\wedge} \mathrm{n}$ for variables n apart. Although just one latent variable (r) needs to be estimated, the structure will have nvar- 1 factors.
Many simulations of factor structures assume that except for the major factors, all residuals are normally distributed around 0 . An alternative, and perhaps more realistic situation, is that the there are a few major (big) factors and many minor (small) factors. The challenge is thus to identify the major factors. sim.minor generates such structures. The structures generated can be thought of as havinga a major factor structure with some small correlated residuals.
Although coefficient
is a very useful indicator of the general factor saturation of a unifactorial test (one with perhaps several sub factors), it has problems with the case of multiple, independent factors. In this situation, one of the factors is labelled as "general" and the omega estimate is too large. This situation may be explored using the sim. omega function.
The four irt simulations, sim.rasch, sim.irt, sim.npl and sim.npn, simulate dichotomous items following the Item Response model. sim.irt just calls either sim.npl (for logistic models) or sim.npn (for normal models) depending upon the specification of the model.
The logistic model is

$$
P(i, j)=\gamma+\frac{\zeta-\gamma}{1+e^{\alpha(\delta-\theta)}}
$$

where $\gamma$ is the lower asymptote or guesssing parameter, $\zeta$ is the upper asymptote (normally 1 ), $\alpha$ is item discrimination and $\delta$ is item difficulty. For the 1 Paramater Logistic (Rasch) model, gamma=0, zeta $=1$, alpha $=1$ and item difficulty is the only free parameter to specify.
For the 2 PL and 2 PN models, $\mathrm{a}=\alpha$ and $\mathrm{d}=\delta$ are specified.
For the 3PL or 3PN models, items also differ in their guessing parameter $\mathrm{c}=\gamma$.
For the 4PL and 4PN models, the upper asymptote, $\mathrm{z}=\zeta$ is also specified.
(Graphics of these may be seen in the demonstrations for the logistic function.)
The normal model (irt.npn calculates the probability using pnorm instead of the logistic function used in irt.npl, but the meaning of the parameters are otherwise the same. With the $\mathrm{a}=\alpha$ parameter $=1.702$ in the logitistic model the two models are practically identical.

Other simulation functions in psych are:
sim. structure A function to combine a measurement and structural model into one data matrix. Useful for understanding structural equation models. Combined with structure. diagram to see the proposed structure.
sim. congeneric A function to create congeneric items/tests for demonstrating classical test theory. This is just a special case of sim.structure.
sim.hierarchical A function to create data with a hierarchical (bifactor) structure.
sim. item A function to create items that either have a simple structure or a circumplex structure.
sim.circ Create data with a circumplex structure.
sim. dichot Create dichotomous item data with a simple or circumplex structure.
sim.minor Create a factor structure for nvar variables defined by nfact major factors and nvar/2 "minor" factors for n observations.
Although the standard factor model assumes that K major factors ( K « nvar) will account for the correlations among the variables

$$
R=F F^{\prime}+U^{2}
$$

where $R$ is of rank $P$ and $F$ is a $P \times K$ matrix of factor coefficients and $U$ is a diagonal matrix of uniquenesses. However, in many cases, particularly when working with items, there are many small factors (sometimes referred to as correlated residuals) that need to be considered as well. This leads to a data structure such that

$$
R=F F^{\prime}+M M^{\prime}+U^{2}
$$

where R is a $\mathrm{P} \times \mathrm{P}$ matrix of correlations, F is a $\mathrm{P} \times \mathrm{K}$ factor loading matrix, M is a $\mathrm{P} \times \mathrm{P} / 2$ matrix of minor factor loadings, and U is a diagonal matrix ( $\mathrm{P} \times \mathrm{P}$ ) of uniquenesses.

Such a correlation matrix will have a poor $\chi^{2}$ value in terms of goodness of fit if just the K factors are extracted, even though for all intents and purposes, it is well fit.
sim.minor will generate such data sets with big factors with loadings of .6 to .8 and small factors with loadings of -.2 to .2 . These may both be adjusted.
sim. parallel Create a number of simulated data sets using sim.minor to show how parallel analysis works. The general observation is that with the presence of minor factors, parallel analysis is probably best done with component eigen values rather than factor eigen values, even when using the factor model.
sim. anova Simulate a 3 way balanced ANOVA or linear model, with or without repeated measures. Useful for teaching research methods and generating teaching examples.

## Author(s)

William Revelle

## References

Revelle, W. (in preparation) An Introduction to Psychometric Theory with applications in R. Springer. at http://personality-project.org/r/book/

## See Also

See above

## Examples

```
simplex <- sim.simplex() #create the default structure
round(simplex,2) #the correlation matrix
```

```
congeneric <- sim.congeneric()
```

congeneric <- sim.congeneric()
round(congeneric,2)
round(congeneric,2)
R <- sim.hierarchical()
R <- sim.hierarchical()
R
R
fx <- matrix(c(.9,.8,.7,rep (0,6),c(.8,.7,.6)),ncol=2)
fx <- matrix(c(.9,.8,.7,rep (0,6),c(.8,.7,.6)),ncol=2)
fy <- c(.6,.5,.4)
fy <- c(.6,.5,.4)
Phi <- matrix(c(1,0,.5,0,1,.4,0,0,0), ncol=3)
Phi <- matrix(c(1,0,.5,0,1,.4,0,0,0), ncol=3)
R <- sim.structure(fx,Phi,fy)
R <- sim.structure(fx,Phi,fy)
cor.plot(R$model) #show it graphically
cor.plot(R$model) \#show it graphically
simp <- sim.simplex()
simp <- sim.simplex()
\#show the simplex structure using cor.plot
\#show the simplex structure using cor.plot
cor.plot(simp,colors=TRUE)

```
cor.plot(simp,colors=TRUE)
```

sim.anova
Simulate a 3 way balanced ANOVA or linear model, with or without repeated measures.

## Description

For teaching basic statistics, it is useful to be able to generate examples suitable for analysis of variance or simple linear models. sim.anova will generate the design matrix of three independent variables (IV1, IV2, IV3) with an arbitrary number of levels and effect sizes for each main effect and interaction. IVs can be either continuous or categorical and can have linear or quadratic effects. Either a single dependent variable or multiple (within subject) dependent variables are generated according to the specified model. The repeated measures are assumed to be tau equivalent with a specified reliability.

## Usage

sim.anova(es1 = 0, es $2=0$, es $3=0$, es $12=0$, es $13=0$, es23 = 0, es $123=0$, es $11=0$, es $22=0$, es $33=0, \mathrm{n}=2, \mathrm{n} 1=2, \mathrm{n} 2=2$, $\mathrm{n} 3=2$, withir

## Arguments

| es 1 | Effect size of IV1 |
| :---: | :---: |
| es2 | Effect size of IV2 |
| es 3 | Effect size of IV3 |
| es12 | Effect size of the IV1 x IV2 interaction |
| es13 | Effect size of the IV1 x IV3 interaction |
| es23 | Effect size of the IV2 x IV3 interaction |
| es123 | Effect size of the IV1 x IV2 * IV3 interaction |
| es11 | Effect size of the quadratric term of IV1 |
| es22 | Effect size of the quadratric term of IV2 |
| es33 | Effect size of the quadratric term of IV3 |
| n | Sample size per cell (if all variables are categorical) or (if at least one variable is continuous), the total sample size |
| n1 | Number of levels of IV1 (0) if continuous |
| n2 | Number of levels of IV2 |
| n3 | Number of levels of IV3 |
| within | if not NULL, then within should be a vector of the means of any repeated measures. |
| r | the correlation between the repeated measures (if they exist). This can be thought of as the reliablility of the measures. |
| factors | report the IVs as factors rather than numeric |
| center | center=TRUE provides orthogonal contrasts, center=FALSE adds the minimum value +1 to all contrasts |
| std | Standardize the effect sizes by standardizing the IVs |

## Details

A simple simulation for teaching about ANOVA, regression and reliability. A variety of demonstrations of the relation between anova and 1 lm can be shown.
The default is to produce categorical IVs (factors). For more than two levels of an IV, this will show the difference between the linear model and anova in terms of the comparisons made.
The within vector can be used to add congenerically equivalent dependent variables. These will have intercorrelations (reliabilities) of $r$ and means as specified as values of within.
To demonstrate the effect of centered versus non-centering, make factors = center=FALSE. The default is to center the IVs. By not centering them, the lower order effects will be incorrect given the higher order interaction terms.

## Value

y.df is a data.frame of the 3 IV values as well as the DV values.

```
IV1 ... IV3 Independent variables 1 ... 3
DV If there is a single dependent variable
DV.1 ... DV.n
    If within is specified, then the n within subject dependent variables
```


## Author(s)

William Revelle

## See Also

The general set of simulation functions in the psych package sim

## Examples

```
set.seed(42)
data.df <- sim.anova(es1=1,es2=.5,es13=1) # one main effect and one interaction
describe(data.df)
pairs.panels(data.df) #show how the design variables are orthogonal
#
summary(lm(DV~IV1*IV2*IV3,data=data.df))
summary(aov(DV~IV1*IV2*IV3,data=data.df))
set.seed(42)
data.df <- sim.anova(es1=1,es2=.5,es13=1,center=FALSE) # demonstrate the effect of not cent
describe(data.df)
#
summary(lm(DV~IV1*IV2*IV3,data=data.df)) #this one is incorrect, because the IVs are not cen
summary(aov(DV~IV1*IV2*IV3,data=data.df)) #compare with the lm model
#now examine multiple levels and quadratic terms
set.seed(42)
data.df <- sim.anova(es1=1,es13=1,n2=3,n3=4,es22=1)
summary(lm(DV~IV1*IV2*IV3,data=data.df))
summary(aov(DV~IV1*IV2*IV3,data=data.df))
pairs.panels(data.df)
```

```
#
data.df <- sim.anova(es1=1,es2=-.5,within=c(-1,0,1),n=10)
pairs.panels(data.df)
```

sim.congeneric Simulate a congeneric data set

## Description

Classical Test Theory (CTT) considers four or more tests to be congenerically equivalent if all tests may be expressed in terms of one factor and a residual error. Parallel tests are the special case where (usually two) tests have equal factor loadings. Tau equivalent tests have equal factor loadings but may have unequal errors. Congeneric tests may differ in both factor loading and error variances.

## Usage

sim.congeneric(loads $=c(0.8,0.7,0.6,0.5), N=N U L L, ~ e r r=N U L L, ~ s h o r t=T R U E, ~ c a t$

## Arguments

N
loads
err A vector of error variances - if NULL then error $=1-$ loading 2
short short=TRUE: Just give the test correlations, short=FALSE, report observed test scores as well as the implied pattern matrix
categorical continuous or categorical (discrete) variables.
low values less than low are forced to low
high values greater than high are forced to high

## Details

When constructing examples for reliability analysis, it is convenient to simulate congeneric data structures. These are the most simple of item structures, having just one factor. Mainly used for a discussion of reliability theory as well as factor score estimates.
The implied covariance matrix is just pattern $\% * \% \mathrm{t}$ (pattern).

## Value

| model | The implied population correlation matrix if N=NULL or short=FALSE, other- <br> wise the sample correlation matrix |
| :--- | :--- |
| pattern | The pattern matrix implied by the loadings and error variances |
| $r$ | The sample correlation matrix for long output |
| observed | a matrix of test scores for $n$ tests |
| latent | The latent trait and error scores |

## Author(s)

William Revelle

## References

Revelle, W. (in prep) An introduction to psychometric theory with applications in R. To be published by Springer. (working draft available at http://personality-project.org/r/book/

## See Also

item.sim for other simulations, fa for an example of factor scores, irt.fa and polychoric for the treatment of item data with discrete values.

## Examples

```
test <- sim.congeneric(c(.9,.8,.7,.6)) #just the population matrix
test <- sim.congeneric(c(.9,.8,.7,.6),N=100) # a sample correlation matrix
test <- sim.congeneric(short=FALSE, N=100)
round(cor(test$observed),2) # show a congeneric correlation matrix
f1=fa(test$observed, scores=TRUE)
round(cor(f1$scores,test$latent),2) #factor score estimates are correlated with but not equ
set.seed(42)
items <- sim.congeneric(N=500,short=FALSE,low=-2,high=2,categorical=TRUE) #500 responses to
d4 <- irt.fa(items$observed) #item response analysis of congeneric measures
```

sim.hierarchical Create a population or sample correlation matrix, perhaps with hierarchical structure.

## Description

Create a population orthogonal or hierarchical correlation matrix from a set of factor loadings and factor intercorrelations. Samples of size $n$ may be then be drawn from this population. Return either the sample data, sample correlations, or population correlations. This is used to create sample data sets for instruction and demonstration.

## Usage

```
sim.hierarchical(gload=NULL, fload=NULL, n = 0, raw = FALSE,mu = NULL)
make.hierarchical(gload=NULL, fload=NULL, n = 0, raw = FALSE) #deprecated
```


## Arguments

gload Loadings of group factors on a general factor
fload Loadings of items on the group factors
$\mathrm{n} \quad$ Number of subjects to generate: $\mathrm{N}=0=>$ population values
raw raw=TRUE, report the raw data, raw=FALSE, report the sample correlation matrix.
$\mathrm{mu} \quad$ means for the individual variables

## Details

Many personality and cognitive tests have a hierarchical factor structure. For demonstration purposes, it is useful to be able to create such matrices, either with population values, or sample values.

Given a matrix of item factor loadings (fload) and of loadings of these factors on a general factor (gload), we create a population correlation matrix by using the general factor law ( $\mathrm{R}=\mathrm{F}$ ' theta F where theta $=g^{\prime} g$ ).

To create sample values, we use the mvrnorm function from MASS.
The default is to return population correlation matrices. Sample correlation matrices are generated if $\mathrm{n}>0$. Raw data are returned if raw $=$ TRUE.

The default values for gload and fload create a data matrix discussed by Jensen and Weng, 1994.
Although written to create hierarchical structures, if the gload matrix is all 0 , then a non-hierarchical structure will be generated.

## Value

a matrix of correlations or a data matrix

## Author(s)

William Revelle

## References

http://personality-project.org/r/r.omega.html
Jensen, A.R., Weng, L.J. (1994) What is a Good g? Intelligence, 18, 231-258.

## See Also

omega, schmid, ICLUST, VSS for ways of analyzing these data. Also see sim. structure to simulate a variety of structural models (e.g., multiple correlated factor models). The simulation uses the mvrnorm function from the MASS package.

## Examples

```
gload <- gload<-matrix(c(.9,.8,.7),nrow=3) # a higher order factor matrix
fload <-matrix(c( #a lower order (oblique) factor matrix
    .8,0,0,
    .7,0,.0,
    .6,0,.0,
        0,.7,.0,
        0,.6,.0,
        0,.5,0,
        0,0,.6,
        0,0,.5,
        0,0,.4), ncol=3,byrow=TRUE)
jensen <- sim.hierarchical(gload,fload) #the test set used by omega
round(jensen, 2)
fload <- matrix(c(c(c(.9,.8,.7,.6),rep (0, 20)),c(c(.9,.8,.7,.6),rep (0, 20)),c(c(.9,.8,.7,.6),r
gload <- matrix(rep (0,5))
five.factor <- sim.hierarchical(gload,fload,500,TRUE) #create sample data set
```


## Description

Rotations of factor analysis and principal components analysis solutions typically try to represent correlation matrices as simple structured. An alternative structure, appealing to some, is a circumplex structure where the variables are uniformly spaced on the perimeter of a circle in a two dimensional space. Generating simple structure and circumplex data is straightforward, and is useful for exploring alternative solutions to affect and personality structure.

## Usage

```
sim.item(nvar = 72, nsub = 500, circum = FALSE, xloading = 0.6, yloading = 0.6, glc
sim.circ(nvar = 72, nsub = 500, circum = TRUE, xloading = 0.6, yloading = 0.6, gloc
sim.dichot(nvar = 72, nsub = 500, circum = FALSE, xloading = 0.6, yloading = 0.6, %
item.dichot(nvar = 72, nsub = 500, circum = FALSE, xloading = 0.6, yloading = 0.6,
```


## Arguments

nvar $\quad$ Number of variables to simulate
nsub Number of subjects to simulate
circum circum=TRUE is circumplex structure, FALSE is simple structure
xloading the average loading on the first dimension
yloading Average loading on the second dimension

| gloading | Average loading on a general factor (default=0) |
| :--- | :--- |
| xbias | To introduce skew, how far off center is the first dimension |
| ybias | To introduce skew on the second dimension |
| categorical | continuous or categorical variables. |
| low | values less than low are forced to low (or 0 in item.dichot) |
| high | values greater than high are forced to high (or 1 in item.dichot) |
| truncate | Change all values less than cutpoint to cutpoint. |
| cutpoint | What is the cutpoint |

## Details

This simulation was originally developed to compare the effect of skew on the measurement of affect (see Rafaeli and Revelle, 2005). It has been extended to allow for a general simulation of affect or personality items with either a simple structure or a circumplex structure. Items can be continuous normally distributed, or broken down into $n$ categories (e.g, $-2,-1,0,1,2$ ). Items can be distorted by limiting them to these ranges, even though the items have a mean of (e.g., 1).
The addition of item.dichot allows for testing structures with dichotomous items of different difficulty (endorsement) levels. Two factor data with either simple structure or circumplex structure are generated for two sets of items, one giving a score of 1 for all items greater than the low (easy) value, one giving a 1 for all items greater than the high (hard) value. The default values for low and high are 0 . That is, all items are assumed to have a 50 percent endorsement rate. To examine the effect of item difficulty, low could be -1 , high 1 . This will lead to item endorsements of .84 for the easy and .16 for the hard. Within each set of difficulties, the first $1 / 4$ are assigned to the first factor factor, the second to the second factor, the third to the first factor (but with negative loadings) and the fourth to the second factor (but with negative loadings).

## Value

A data matrix of (nsub) subjects by (nvar) variables.

## Author(s)

William Revelle

## References

Variations of a routine used in Rafaeli and Revelle, 2006; Rafaeli, E. \& Revelle, W. (2006). A premature consensus: Are happiness and sadness truly opposite affects? Motivation and Emotion.
Acton, G. S. and Revelle, W. (2004) Evaluation of Ten Psychometric Criteria for Circumplex Structure. Methods of Psychological Research Online, Vol. 9, No. 1 http://www.dgps.de/ fachgruppen/methoden/mpr-online/issue22/mpr110_10.pdf

## See Also

See Also the implementation in this to generate numerous simulations. simulation.circ, circ.tests as well as other simulations (sim.structural sim.hierarchical)

## Examples

```
round(cor(circ.sim(nvar=8,nsub=200)),2)
plot(factor.pa(circ.sim(16,500),2)$loadings,main="Circumplex Structure") #circumplex structu
#
#
plot(factor.pa(item.sim(16,500),2)$loadings,main="Simple Structure") #simple structure
#
cluster.plot(factor.pa(item.dichot(16,low=0,high=1), 2))
```

```
sim.structure Create correlation matrices or data matrices with a particular mea-
    surement and structural model
```


## Description

Structural Equation Models decompose correlation or correlation matrices into a measurement (factor) model and a structural (regression) model. sim.structural creates data sets with known measurement and structural properties. Population or sample correlation matrices with known properties are generated. Optionally raw data are produced.
It is also possible to specify a measurement model for a set of $x$ variables separately from a set of $y$ variables. They are then combined into one model with the correlation structure between the two sets.

## Usage

```
sim.structural(fx=NULL,Phi=NULL, fy=NULL,f=NULL, n=0,raw=FALSE)
make.structural(fx=NULL,Phi=NULL,fy=NULL,f=NULL,n=0,raw=FALSE) #deprecated
```


## Arguments

| fx | The measurement model for x |
| :--- | :--- |
| Phi | The structure matrix of the latent variables |
| fy | The measurement model for y |
| f | The measurement model |
| n | Number of cases to simulate. If $\mathrm{n}=0$, the population matrix is returned. |
| raw | if raw=TRUE, raw data are returned as well. |

## Details

Given the measurement model, fx and the structure model Phi, the model is f
Given the model, raw data are generated using the mvnorm function.
A special case of a structural model are one factor models such as parallel tests, tau equivalent tests, and congneneric tests. These may be created by letting the structure matrix $=1$ and then defining a vector of factor loadings. Alternatively, make.congeneric will do the same.

## Value

| model | The implied population correlation matrix |
| :--- | :--- |
| reliability | The population reliability values |
| $r$ | The sample correlation matrix |
| observed | If raw=TRUE, a sample data matrix |

## Author(s)

William Revelle

## References

Revelle, W. (in preparation) An Introduction to Psychometric Theory with applications in R. Springer. at http://personality-project.org/r/book/

## See Also

make. hierarchical for another structural model and make. congeneric for the one factor case. structure.list and structure.list for making symbolic structures.

## Examples

```
fx <-matrix(c( .9,.8,.6,rep (0,4),.6,.8,-.7),ncol=2)
fy <- matrix(c(.6,.5,.4),ncol=1)
rownames(fx) <- c("V","Q","A","nach","Anx")
rownames(fy)<- c("gpa","Pre","MA")
Phi <-matrix( c(1,0,.7,.0,1,.7,.7,.7,1),ncol=3)
gre.gpa <- sim.structural(fx,Phi,fy)
print(gre.gpa,2)
round(correct.cor(gre.gpa$model,gre.gpa$reliability),2) #correct for attenuation to see str
congeneric <- sim.structural(f=c(.9,.8,.7,.6)) # a congeneric model
congeneric
```

```
sim.VSS create VSS like data
```


## Description

Simulation is one of most useful techniques in statistics and psychometrics. Here we simulate a correlation matrix with a simple structure composed of a specified number of factors. Each item is assumed to have complexity one. See circ.sim and item.sim for alternative simulations.

## Usage

```
sim.VSS(ncases=1000, nvariables=16, nfactors=4, meanloading=.5,dichot=FALSE,cut=0)
```


## Arguments

ncases number of simulated subjects
nvariables Number of variables
nfactors Number of factors to generate
meanloading with a mean loading
dichot dichot=FALSE give continuous variables, dichot=TRUE gives dichotomous variables
cut if dichotomous $=$ TRUE, then items with values $>$ cut are assigned 1 , otherwise 0 .

## Value

a ncases x nvariables matrix

## Author(s)

William Revelle

## See Also

```
VSS, ICLUST
```


## Examples

```
## Not run:
simulated <- sim.VSS (1000,20,4,.6)
vSS <- VSS (simulated,rotate="varimax")
VSS.plot(vss)
## End(Not run)
```

```
simulation.circ Simulations of circumplex and simple structure
```


## Description

Rotations of factor analysis and principal components analysis solutions typically try to represent correlation matrices as simple structured. An alternative structure, appealing to some, is a circumplex structure where the variables are uniformly spaced on the perimeter of a circle in a two dimensional space. Generating these data is straightforward, and is useful for exploring alternative solutions to affect and personality structure.

## Usage

```
simulation.circ(samplesize=c(100,200,400,800), numberofvariables=c(16,32,48,72))
```


## Arguments

samplesize a vector of sample sizes to simulate
numberofvariables
vector of the number of variables to simulate

## Details

"A common model for representing psychological data is simple structure (Thurstone, 1947). According to one common interpretation, data are simple structured when items or scales have nonzero factor loadings on one and only one factor (Revelle \& Rocklin, 1979). Despite the commonplace application of simple structure, some psychological models are defined by a lack of simple structure. Circumplexes (Guttman, 1954) are one kind of model in which simple structure is lacking.
"A number of elementary requirements can be teased out of the idea of circumplex structure. First, circumplex structure implies minimally that variables are interrelated; random noise does not a circumplex make. Second, circumplex structure implies that the domain in question is optimally represented by two and only two dimensions. Third, circumplex structure implies that variables do not group or clump along the two axes, as in simple structure, but rather that there are always interstitial variables between any orthogonal pair of axes (Saucier, 1992). In the ideal case, this quality will be reflected in equal spacing of variables along the circumference of the circle (Gurtman, 1994; Wiggins, Steiger, \& Gaelick, 1981). Fourth, circumplex structure implies that variables have a constant radius from the center of the circle, which implies that all variables have equal communality on the two circumplex dimensions (Fisher, 1997; Gurtman, 1994). Fifth, circumplex structure implies that all rotations are equally good representations of the domain (Conte \& Plutchik, 1981; Larsen \& Diener, 1992)." (Acton and Revelle, 2004)

Acton and Revelle reviewed the effectiveness of 10 tests of circumplex structure and found that four did a particularly good job of discriminating circumplex structure from simple structure, or circumplexes from ellipsoidal structures. Unfortunately, their work was done in Pascal and is not easily available. Here we release R code to do the four most useful tests:

The Gap test of equal spacing
Fisher's test of equality of axes
A test of indifference to Rotation
A test of equal Variance of squared factor loadings across arbitrary rotations.
Included in this set of functions are simple procedure to generate circumplex structured or simple structured data, the four test statistics, and a simple simulation showing the effectiveness of the four procedures.
circ.sim.plot compares the four tests for circumplex, ellipsoid and simple structure data as function of the number of variables and the sample size. What one can see from this plot is that although no one test is sufficient to discriminate these alternative structures, the set of four tests does a very good job of doing so. When testing a particular data set for structure, comparing the results of all four tests to the simulated data will give a good indication of the structural properties of the data.

## Value

A data.frame with simulation results for circumplex, ellipsoid, and simple structure data sets for each of the four tests.

## Note

The simulations default values are for sample sizes of $100,200,400$, and 800 cases, with $16,32,48$ and 72 items.

## Author(s)

William Revelle

## References

Acton, G. S. and Revelle, W. (2004) Evaluation of Ten Psychometric Criteria for Circumplex Structure. Methods of Psychological Research Online, Vol. 9, No. 1 http://www.dgps.de/ fachgruppen/methoden/mpr-online/issue22/mpr110_10.pdf

## See Also

See also circ.tests, sim.circ, sim.structural, sim.hierarchical

## Examples

```
demo <- simulation.circ()
    boxplot(demo[3:14])
    title("4 tests of Circumplex Structure",sub="Circumplex, Ellipsoid, Simple Structure")
circ.sim.plot(demo[3:14]) #compare these results to real data
```


## Description

Find the skew and kurtosis for each variable in a data.frame or matrix. Unlike skew and kurtosis in e1071, this calculates a different skew for each variable or column of a data.frame/matrix.

## Usage

```
skew(x, na.rm = TRUE)
kurtosi(x, na.rm = TRUE)
```


## Arguments

x
A data.frame or matrix
na.rm
how to treat missing data

## Details

given a matrix or data.frame x , find the skew or kurtosis for each column.

## Value

if input is a matrix or data.frame, skew (kurtosi) is a vector of skews (kurtosi)

## Note

The mean function supplies means for the columns of a data.frame, but the overall mean for a matrix. Mean will throw a warning for non-numeric data, but colMeans stops with non-numeric data. Thus, the function uses either mean (for data frames) or colMeans (for matrices). This is true for skew and kurtosi as well.

## Author(s)

William Revelle

## See Also

```
describe, describe.by,
```


## Examples

```
round(skew(attitude), 2)
round(kurtosi(attitude),2)
```

```
smc
```

Find the Squared Multiple Correlation (SMC) of each variable with the remaining variables in a matrix

## Description

The squared multiple correlation of a variable with the remaining variables in a matrix is sometimes used as initial estimates of the communality of a variable.
SMCs are also used when estimating reliability using Guttman's lambda 6 guttman coefficient.
The SMC is just $1-1 / \operatorname{diag}$ (R.inv) where R.inv is the inverse of R.

## Usage

$\operatorname{smc}(R$, covar=FALSE)

## Arguments

R
covar

A correlation matrix or a dataframe. In the latter case, correlations are found.
if covar $=$ TRUE and $R$ is either a covariance matrix or data frame, then return the smc * variance for each item

## Value

a vector of squared multiple correlations. Or, if covar=TRUE, a vector of squared multiple correlations * the item variances
If the matrix is not invertible, then a vector of 1 s is returned

## Author(s)

William Revelle

## See Also

```
mat.regress, factor.pa
```


## Examples

```
R <- make.hierarchical()
round(smc(R),2)
```

```
structure.diagram Draw a structural equation model specified by two measurement mod-
els and a structural model
```


## Description

Graphic presentations of structural equation models are a very useful way to conceptualize sem and confirmatory factor models. Given a measurement model on $x$ (xmodel) and on y (ymodel) as well as a path model connecting $x$ and $y$ (phi), draw the graph. If ymodel is not specified, just draw the measurement model (xmodel + phi). If the Rx or Ry matrices are specified, show the correlations between the x variables, or y variables.
Perhaps even more usefully, the function returns a model appropriate for running directly in the sem package written by John Fox. For this option to work directly, it is necessary to specfy that errrors=TRUE.
Input can be specified as matrices or the output from fa, factor.pa, factanal, or a rotation package such as GPArotation.

For symbolic graphs, the input matrices can be character strings or mixtures of character strings and numeric vectors.
As an option, for those without Rgraphviz installed, structure.sem will just create the sem model and skip the graph. (This functionality is now included in structure.diagram.)
structure.diagram will draw the diagram without using Rgraphviz and is probably the preferred option. structure.graph will be removed eventually.

## Usage

```
structure.diagram(fx, Phi=NULL,fy=NULL,labels=NULL, cut=.3,errors=FALSE,simple=TRUE,
structure.graph(fx, Phi = NULL,fy = NULL, out.file = NULL, labels = NULL, cut = 0.
structure.sem(fx, Phi = NULL, fy = NULL,out.file = NULL, labels = NULL, cut = 0.3,
```


## Arguments

| fx | a factor model on the x variables. |
| :---: | :---: |
| Phi | A matrix of directed relationships. Lower diagonal values are drawn. If the upper diagonal values match the lower diagonal, two headed arrows are drawn. For a single, directed path, just the value may be specified. |
| fy | a factor model on the y variables (can be empty) |
| Rx | The correlation matrix among the x variables |
| Ry | The correlation matrix among the y variables |
| out.file | name a file to send dot language instructions. |
| labels | variable labels if not specified as colnames for the matrices |
| cut | Draw paths for values > cut |
| errors | draw an error term for observerd variables |
| simple | Just draw one path per x or y variable |
| regression | Draw a regression diagram (observed variables cause Y) |
| $1 r$ | Direction of diagram is from left to right (lr=TRUE, default) or from bottom to top (lr=FALSE) |
| e.size | size of the ellipses in structure.diagram |
| main | main title of diagram |
| size | page size of graphic |
| node.font | font type for graph |
| edge.font | font type for graph |
| rank.direction |  |
|  | Which direction should the graph be oriented |
| digits | Number of digits to draw |
| title | Title of graphic |
|  | other options to pass to Rgraphviz |

## Details

The recommended function is structure.diagram which does not use Rgraphviz but which does not produce dot code either.
All three function return a matrix of commands suitable for using in the sem package. (Specify errors=TRUE to get code that will run directly in the sem package.)

The structure.graph output can be directed to an output file for post processing using the dot graphic language but requires that Rgraphviz is installed.
The figure is organized to show the appropriate paths between:
The correlations between the X variables (if Rx is specified)
The $X$ variables and their latent factors (if fx is specified)
The latent $X$ and the latent $Y$ (if Phi is specified)
The latent Y and the observed Y (if fy is specified)

The correlations between the Y variables (if Ry is specified)

A confirmatory factor model would specify just $f x$ and Phi, a structural model would include fx , Phi, and fy. The raw correlations could be shown by just including Rx and Ry.

Other diagram functions include fa.diagram, omega.diagram. All of these functions use the various dia functions such as dia.rect, dia.ellipse, dia. arrow, dia.curve, dia.curved.arrow, and dia.shape.

## Value

sem (invisible) a model matrix (partially) ready for input to John Fox's sem package. It is of class "mod" for prettier output.
dotfile If out.file is specified, a dot language file suitable for using in a dot graphics program such as graphviz or Omnigraffle.
A graphic structural diagram in the graphics window

## Author(s)

William Revelle

## See Also

fa.graph, omega.graph, sim.structural to create artificial data sets with particular structural properties.

## Examples

```
fx <- matrix(c(.9,.8,.6,rep (0,4),.6,.8,-.7),ncol=2)
fy <- matrix(c(.6,.5,.4),ncol=1)
Phi <- matrix(c(1,0,0,0,1,0,.7,.7,1),ncol=3,byrow=TRUE)
f1 <- structure.diagram(fx,Phi,fy,main="A structural path diagram")
#symbolic input
X2 <- matrix(c("a",0,0,"b","e1",0,0,"e2"),ncol=4)
colnames(X2) <- c("X1","X2","E1","E2")
phi2 <- diag(1,4,4)
phi2[2,1] <- phi2[1,2] <- "r"
f2 <- structure.diagram(X2,Phi=phi2,errors=FALSE,main="A symbolic model")
#symbolic input with error
X2 <- matrix(c("a",0,0,"b"),ncol=2)
colnames(X2) <- c("X1","X2")
phi2 <- diag(1,2,2)
phi2[2,1] <- phi2[1,2] <- "r"
f3 <- structure.diagram(X2,Phi=phi2,main="an alternative representation")
#and yet another one
X6 <- matrix(c("a","b","c",rep(0,6),"d","e","f"),nrow=6)
colnames(X6) <- c("L1","L2")
rownames(X6) <- c("x1","x2","x3","x4","x5","x6")
```

```
Y3 <- matrix(c("u","w","z"),ncol=1)
colnames(Y3) <- "Y"
rownames(Y3) <- c("y1","y2","y3")
phi21 <- matrix(c(1,0,"r1",0,1,"r2",0,0,1),ncol=3)
colnames(phi21) <- rownames(phi21) <- c("L1","L2","Y")
f4 <- structure.diagram(X6,phi21,Y3)
# and finally, a regression model
X7 <- matrix(c("a","b","c","d","e","f"), nrow=6)
f5 <- structure.diagram(X7,regression=TRUE)
#and a really messy regession model
x8 <- c("b1","b2","b3")
r8 <- matrix(c(1,"r12","r13","r12",1,"r23","r13","r23",1),ncol=3)
f6<- structure.diagram(x8,Phi=r8,regression=TRUE)
```

structure.list Create factor model matrices from an input list

## Description

When creating a structural diagram or a structural model, it is convenient to not have to specify all of the zero loadings in a structural matrix. structure.list converts list input into a design matrix. phi.list does the same for a correlation matrix. Factors with NULL values are filled with 0s.

## Usage

```
structure.list(nvars, f.list,f=NULL, f.labels = NULL, item.labels = NULL)
phi.list(nf,f.list, f.labels = NULL)
```


## Arguments

nvars $\quad$ Number of variables in the design matrix
f.list A list of items included in each factor (for structure.list, or the factors that correlate with the specified factor for phi.list
$\mathrm{f} \quad$ prefix for parameters - needed in case of creating an X set and a $Y$ set
f.labels Names for the factors
item.labels Item labels
$\mathrm{nf} \quad$ Number of factors in the phi matrix

## Details

This is almost self explanatory. See the examples.

## Value

factor.matrix
a matrix of factor loadings to model

## See Also

structure.graph for drawing it, or sim. structure for creating this data structure.

## Examples

```
fx <- structure.list(9,list (F1=C (1,2,3),F2=C (4,5,6),F3=C (7, 8,9)))
fy <- structure.list(3,list(Y=c (1, 2, 3)),"Y")
phi <- phi.list(4,list(F1=C(4),F2=C(1,4),F3=C(2),F4=C(1,2,3)))
fx
phi
fy
```


## Description

Given the matrices $n X m$, and jYk , form the super matrix of dimensions $(\mathrm{n}+\mathrm{j})$ and $(\mathrm{m}+\mathrm{k})$ with with elements x and y along the super diagonal. Useful when considering structural equations. The measurement models $x$ and $y$ can be combined into a larger measurement model of all of the variables.

## Usage

super.matrix(x, y)

## Arguments

| $x$ | Anx m matrix |
| :--- | :--- |
| $y$ | $A j x k$ matrix |

## Value

$A(n+j) x(m+k)$ matrix with appropriate row and column names

## Author(s)

William Revelle

## See Also

```
sim.structural,structure.graph
```


## Examples

```
mx <- matrix(c(.9,.8,.7,rep(0,4),.8,.7,.6),ncol=2)
my <- matrix(c(.6,.5,.4))
colnames(mx) <- paste("X",1:dim(mx)[2],sep="")
rownames(mx) <- paste("Xv",1:dim(mx)[1],sep="")
colnames(my) <- "Y"
rownames(my) <- paste("Yv",1:3,sep="")
super.matrix(mx,my)
```

table2matrix

Convert a table with counts to a matrix or data.frame representing those counts.

## Description

Some historical sets are reported as summary tables of counts in a limited number of bins. Transforming these tables to data.frames representing the original values is useful for pedagogical purposes. (E.g., transforming the original Galton table of height $x$ cubits in order to demonstrate regression.) The column and row names must be able to be converted to numeric values.

## Usage

table2matrix(x, labs = NULL)
table2df(x, count=NULL, labs = NULL)

## Arguments

x
count if present, then duplicate each row count times
labs Labels for the rows and columns. These will be used for the names of the two columns of the resulting matrix

## Details

The original Galton (1888) of heights by cubits (arm length) is in tabular form. To show this as a correlation or as a scatter plot, it is useful to convert the table to a matrix or data frame of two columns.
This function may also be used to convert an item response pattern table into a data table. e.g., the Bock data set bock.

## Value

A matrix (or data.frame) of sum(x) rows and two columns.

## Author(s)

William Revelle

## See Also

cubits and bock data sets

## Examples

```
data(cubits)
cubit <- table2matrix(cubits,labs=c("height","cubit"))
describe(cubit)
ellipses(cubit,n=1)
data(bock)
responses <- table2df(bock.table[,2:6], count=bock.table[,7],labs= paste("lsat6.",1:5,sep="")
describe(responses)
```

test.psych Testing of functions in the psych package

## Description

Test to make sure the psych functions run on basic test data sets

## Usage

test.psych(first=1,last=5, short=TRUE)

## Arguments

first first=1: start with dataset first
last last=5: test for datasets until last
short short=TRUE - don't return any analyses

## Details

When modifying the psych package, it is useful to make sure that adding some code does not break something else. The test.psych function tests the major functions on various standard data sets. It also shows off a number of the capabilities of the psych package.
Uses 5 standard data sets:
USArrests Violent Crime Rates by US State (4 variables)
attitude The Chatterjee-Price Attitude Data
Harman23.corl\$cov Harman Example 2.38 physical measurements
Harman74.corl\$cov Harman Example 7.424 mental measurements
ability.cov $\backslash \$ \operatorname{cov} 8$ Ability and Intelligence Tests

## Value

out if short=FALSE, then list of the output from all functions tested

## Warning

Warning messages will be thrown by fa.parallel and sometimes by factor.pa for random datasets.

## Note

Although test.psych may be used as a quick demo of the various functions in the psych packge, in general, it is better to try the specific functions themselves. The main purpose of test.psych is to make sure functions throw error messages or correct for weird conditions.
The datasets tested are part of the standard R data sets and represent some of the basic problems encountered.

## Author(s)

William Revelle

## Examples

```
test <- test.psych()
```

```
tetrachoric Tetrachoric, polychoric, biserial and polyserial correlations from var-
``` ious types of input

\section*{Description}

The tetrachoric correlation is the inferred Pearson Correlation from a two x two table with the assumption of bivariate normality. The polychoric correlation generalizes this to the \(\mathrm{n} \times \mathrm{m}\) table. Particularly important when doing Item Response Theory or converting comorbidity statistics using normal theory to correlations. Input may be a \(2 \times 2\) table of cell frequencies, a vector of cell frequencies, or a data.frame or matrix of dichotomous data (for tetrachoric) or of numeric data (for polychoric). The biserial correlation is between a continuous y variable and a dichotmous x variable, which is assumed to have resulted from a dichotomized normal variable. Biserial is a special case of the polyserial correlation, which is the inferred latent correlation between a continuous variable (X) and a ordered categorical variable (e.g., an item response). Input for these later two are data frames or matrices.

\section*{Usage}
```

tetrachoric(x,correct=TRUE)
polychoric(x,polycor=FALSE, ML = FALSE, std.err=FALSE)
biserial(x,y)
polyserial(x,y)
poly.mat(x, short = TRUE, std.err = FALSE, ML = FALSE) \#deprecated use polychoric

```

\section*{Arguments}
std.err std.err=FALSE does not report the standard errors (faster)
ML \(\quad\) ML=FALSE do a quick two step procedure, ML=TRUE, do longer maximum

X

Y
short

The input may be in one of four forms:
a) a data frame or matrix of dichotmous data (e.g., the lsat6 from the bock data set) or discrete numerical (i.e., not too many levels, e.g., the big 5 data set, bfi) for polychoric, or continuous for the case of biserial and polyserial.
b) a \(2 \times 2\) table of cell counts or cell frequencies (for tetrachoric)
c) a vector with elements corresponding to the four cell frequencies (for tetrachoric)
d) a vector with elements of the two marginal frequencies (row and column) and the comorbidity (for tetrachoric)
A (matrix or dataframe) of discrete scores. In the case of tetrachoric, these should be dichotomous, for polychoric not too many levels, for biserial they should be discrete (e.g., item responses) with not too many ( \(<10\) ? ) categories.
correct Correct for continuity in the case of zero entry cell for tetrachoric
polycor If polycor=TRUE and the polycor package is installed, then use it when finding the polychoric correlations.
short=TRUE, just show the correlations, short=FALSE give the full hetcor output from John Fox's hetcor function if installed and if doing polychoric likelihood - very slow!

\section*{Details}

Tetrachoric correlations infer a latent Pearson correlation from a two \(x\) two table of frequencies with the assumption of bivariate normality. The estimation procedure is two stage ML. Cells with zero counts are replaced with .5 as a correction for continuity (correct=TRUE).
The data typically will be a raw data matrix of responses to a questionnaire scored either true/false (tetrachoric) or with a limited number of responses (polychoric). In both cases, the marginal frequencies are converted to normal theory thresholds and the resulting table for each item pair is converted to the (inferred) latent Pearson correlation that would produce the observed cell frequencies with the observed marginals. (See draw.tetra for an illustration.)
The tetrachoric correlation is used in a variety of contexts, one important one being in Item Response Theory (IRT) analyses of test scores, a second in the conversion of comorbity statistics to correlation coefficients. It is in this second context that examples of the sensitivity of the coefficient to the cell frequencies becomes apparent:

Consider the test data set from Kirk (1973) who reports the effectiveness of a ML algorithm for the tetrachoric correlation (see examples).
Examples include the 1sat6 and lsat7 data sets in the bock data.
The polychoric function forms matrices of polychoric correlations by either using John Fox's polychor function or by an local function (polyc) and will also report the tau values for each alternative.
polychoric replaces poly.mat and is recommended. poly.mat is an alternative wrapper to the polycor function.
biserial and polyserial correlations are the inferred latent correlations equivalent to the observed point-biserial and point-polyserial correlations (which are themselves just Pearson correlations).

The polyserial function is meant to work with matrix or dataframe input and treats missing data by finding the pairwise Pearson r corrected by the overall (all observed cases) probability of response frequency. This is particularly useful for SAPA procedures with large amounts of missing data and no complete cases.

Ability tests and personality test matrices will typically have a cleaner structure when using tetrachoric or polychoric correlations than when using the normal Pearson correlation.

A biserial correlation (not to be confused with the point-biserial correlation which is just a Pearson correlation) is the latent correlation between x and y where y is continuous and x is dichotomous but assumed to represent an (unobserved) continuous normal variable. Let \(\mathrm{p}=\) probability of x level 1 , and \(\mathrm{q}=1-\mathrm{p}\). Let \(\mathrm{zp}=\) the normal ordinate of the z score associated with p . Then, \(r b i=r s * \sqrt{(p q)} / z p\).

The 'ad hoc' polyserial correlation, rps is just \(r=r * \operatorname{sqrt}(n-1) / n) \sigma y / \sum(z p i)\) where zpi are the ordinates of the normal curve at the normal equivalent of the cut point boundaries between the item responses. (Olsson, 1982)

All of these were inspired by (and adapted from) John Fox's polychor package which should be used for precise ML estimates of the correlations. See, in particular, the hetcor function in the polychor package.

\section*{Value}
\[
\begin{array}{ll}
\text { rho } & \text { The (matrix) of tetrachoric/polychoric/biserial correlations } \\
\text { tau } & \text { The normal equivalent of the cutpoints }
\end{array}
\]

\section*{Note}

For tetrachoric, in the degenerate case of a cell entry with zero observations, a correction for continuity is applied and .5 is added to the cell entry. A warning is issued. If correct=FALSE the correction is not applied.

\section*{Author(s)}

William Revelle

\section*{References}
A. Gunther and M. Hofler. Different results on tetrachorical correlations in mplus and stata-stata announces modified procedure. Int J Methods Psychiatr Res, 15(3):157-66, 2006.

David Kirk (1973) On the numerical approximation of the bivariate normal (tetrachoric) correlation coefficient. Psychometrika, 38, 259-268.
U.Olsson, F.Drasgow, and N.Dorans (1982). The polyserial correlation coefficient. Psychometrika, 47:337-347.

\section*{See Also}

See also the polychor function in the polycor package. irt. fa uses the tetrachoric function to do item analysis with the fa factor analysis function. draw. tetra shows the logic behind a tetrachoric correlation (for teaching purpuses.)

\section*{Examples}
```

if(require(mvtnorm)) {
data(bock)
tetrachoric(lsat6)
polychoric(lsat6) \#values should be the same
tetrachoric(matrix(c(44268,193,14,0),2,2)) \#MPLUS reports.24
tetrachoric(matrix(c(44268,193,14,0),2,2),FALSE) \#Do not apply continuity correction -- com
tetrachoric(matrix(c(61661,1610,85,20),2,2)) \#Mplus reports . 35
tetrachoric(matrix(c(62503,105,768,0),2,2)) \#Mplus reports -.10
tetrachoric(matrix(c(24875,265,47,0),2,2)) \#Mplus reports 0
tetrachoric(matrix(c(24875,265,47,0),2,2),FALSE) \#Do not apply continuity correction- compar
tetrachoric(c(0.02275000, 0.0227501320, 0.500000000))
tetrachoric(c(0.0227501320, 0.0227501320, 0.500000000)) } else {message("Sorry, you must hav

# 4 plots comparing biserial to

set.seed(42)
x.4 <- sim.congeneric(loads =c(.9,.6,.3,0),N=1000,short=FALSE)
y <- x.4$latent[,1]
for(i in 1:4) {
x <- x.4$observed[,i]
r <- round(cor(x,y),1)
ylow <- y[x<= 0]
yhigh <- y[x > 0]
yc <- c(ylow,yhigh)
rpb <- round(cor((x>=0),y),2)
rbis <- round(biserial(y, (x>=0)),2)
ellipses(x,y,ylim=c (-3,3),xlim=c(-4,3),pch=21 - (x>0),main =paste("r = ",r,"rpb = ",rpb,"rbi
dlow <- density(ylow)
dhigh <- density(yhigh)
points(dlow$y*5-4,dlow$x,typ="l",lty="dashed")
lines(dhigh$y*5-4,dhigh$x,typ="l")
}

```

\section*{Description}

Thurstone Case V scaling allows for a scaling of objects compared to other objects. As one of the cases considered by Thurstone, Case V makes the assumption of equal variances and uncorrelated distributions.

\section*{Usage}
thurstone (x, ranks = FALSE, digits = 2)

\section*{Arguments}
\(x \quad\) A square matrix or data frame of preferences, or a rectangular data frame or matrix rank order choices.
ranks TRUE if rank orders are presented
digits number of digits in the goodness of fit

\section*{Details}

Louis L. Thurstone was a pioneer in psychometric theory and measurement of attitudes, interests, and abilities. Among his many contributions was a systematic analysis of the process of comparative judgment (thurstone, 1927). He considered the case of asking subjects to successively compare pairs of objects. If the same subject does this repeatedly, or if subjects act as random replicates of each other, their judgments can be thought of as sampled from a normal distribution of underlying (latent) scale scores for each object, Thurstone proposed that the comparison between the value of two objects could be represented as representing the differences of the average value for each object compared to the standard deviation of the differences between objects. The basic model is that each item has a normal distribution of response strength and that choice represents the stronger of the two response strengths. A justification for the normality assumption is that each decision represents the sum of many independent inputs and thus, through the central limit theorem, is normally distributed.
Thurstone considered five different sets of assumptions about the equality and independence of the variances for each item (Thurston, 1927). Torgerson expanded this analysis slightly by considering three classes of data collection (with individuals, between individuals and mixes of within and between) crossed with three sets of assumptions (equal covariance of decision process, equal correlations and small differences in variance, equal variances).
The data may be either a square matrix of dataframe of preferences (as proportions with the probability of the column variable being chosen over the row variable) or a matrix or dataframe of rank orders ( 1 being prefered to 2 , etc.)

\section*{Value}

GF Goodness of fit \(1=1-\) sum(squared residuals/squared original) for lower off diagonal.

Goodness of fit \(2=1-\) sum(squared residuals/squared original) for full matrix.
residual square matrix of residuals (of class dist)
data The original choice data

\section*{Author(s)}

William Revelle

\section*{References}

Thurstone, L. L. (1927) A law of comparative judgments. Psychological Review, 34, 273-286.
Revelle, W. An introduction to psychometric theory with applications in R. (in preparation), Springer. http://personality-project.org/r/book

\section*{Examples}
```

data(vegetables)
thurstone(veg)

```
tr

Find the trace of a square matrix

\section*{Description}

Hardly worth coding, if it didn't appear in so many formulae in psychometrics, the trace of a (square) matrix is just the sum of the diagonal elements.

\section*{Usage}
tr (m)

\section*{Arguments}
\(m \quad\) A square matrix

\section*{Details}

The \(\operatorname{tr}\) function is used in various matrix operations and is the sum of the diagonal elements of a matrix.

\section*{Value}

The sum of the diagonal elements of a square matrix.
i.e. \(\operatorname{tr}(m)<-\operatorname{sum}(\operatorname{diag}(m))\).

\section*{Examples}
```

m <- matrix(1:16,ncol=4)
m
tr (m)

```

\section*{Description}

Tucker and Lewis (1973) introduced a reliability coefficient for ML factor analysis. Their example data set was previously reported by Tucker (1958) and taken from Thurstone and Thurstone (1941). The correlation matrix is a \(9 \times 9\) for 710 subjects and has two correlated factors of ability: Word Fluency and Verbal.

\section*{Usage}
```

    data(Tucker)
    ```

\section*{Format}

A data frame with 9 observations on the following 9 variables.
t 42 Prefixes
t54 Suffixes
t 45 Chicago Reading Test: Vocabulary
t46 Chicago Reading Test: Sentences
t23 First and last letters
t24 First letters
t27 Four letter words
t10 Completion
t51 Same or Opposite

\section*{Details}

The correlation matrix from Tucker (1958) was used in Tucker and Lewis (1973) for the TuckerLewis Index of factoring reliability.

\section*{Source}

Tucker, Ledyard (1958) An inter-battery method of factor analysis, Psychometrika, 23, 111-136.

\section*{References}
L. \(\sim\) Tucker and C. \(\sim\) Lewis. (1973) A reliability coefficient for maximum likelihood factor analysis. Psychometrika, 38(1):1-10.
F.~J. Floyd and K.~F. Widaman. (1995) Factor analysis in the development and refinement of clinical assessment instruments., Psychological Assessment, 7(3):286-299.

\section*{Examples}
```

data(Tucker)
fa(Tucker,2,n.obs=710)
omega(Tucker,2)

```
vegetables

\section*{Description}

A classic data set for demonstrating Thurstonian scaling is the preference matrix of 9 vegetables from Guilford (1954). Used by Guiford, Nunnally, and Nunally and Bernstein, this data set allows for examples of basic scaling techniques.

\section*{Usage}
data(vegetables)

\section*{Format}

A data frame with 9 choices on the following 9 vegetables. The values reflect the perecentage of times where the column entry was preferred over the row entry.

Turn Turnips
Cab Cabbage
Beet Beets
Asp Asparagus
Car Carrots
Spin Spinach
S.Beans String Beans

Peas Peas
Corn Corn

\section*{Details}

Louis L. Thurstone was a pioneer in psychometric theory and measurement of attitudes, interests, and abilities. Among his many contributions was a systematic analysis of the process of comparative judgment (thurstone, 1927). He considered the case of asking subjects to successively compare pairs of objects. If the same subject does this repeatedly, or if subjects act as random replicates of each other, their judgments can be thought of as sampled from a normal distribution of underlying (latent) scale scores for each object, Thurstone proposed that the comparison between the value of two objects could be represented as representing the differences of the average value for each object compared to the standard deviation of the differences between objects. The basic model is that each item has a normal distribution of response strength and that choice represents the stronger of the two
response strengths. A justification for the normality assumption is that each decision represents the sum of many independent inputs and thus, through the central limit theorem, is normally distributed.
Thurstone considered five different sets of assumptions about the equality and independence of the variances for each item (Thurston, 1927). Torgerson expanded this analysis slightly by considering three classes of data collection (with individuals, between individuals and mixes of within and between) crossed with three sets of assumptions (equal covariance of decision process, equal correlations and small differences in variance, equal variances).
This vegetable data set is used by Guilford and by Nunnally to demonstrate Thurstonian scaling.

\section*{Source}

Guilford, J.P. (1954) Psychometric Methods. McGraw-Hill, New York.

\section*{References}

Nunnally, J. C. (1967). Psychometric theory., McGraw-Hill, New York.

Revelle, W. An introduction to psychometric theory with applications in R. (in preparation), Springer. http://personality-project.org/r/book

\section*{See Also}
thurstone

\section*{Examples}
```

data(vegetables)
thurstone(veg)

```

Apply the Very Simple Structure and MAP criteria to determine the appropriate number of factors.

\section*{Description}

There are multiple ways to determine the appropriate number of factors in exploratory factor analysis. Routines for the Very Simple Structure (VSS) criterion allow one to compare solutions of varying complexity and for different number of factors. Graphic output indicates the "optimal" number of factors for different levels of complexity. The Velicer MAP criterion is another good choice.

\section*{Usage}
```

vSS(x, n = 8, rotate = "varimax", diagonal = FALSE, fm = "minres", n.obs=NULL,plot=
VSS(x, n = 8, rotate = "varimax", diagonal = FALSE, fm = "minres", n.obs=NULL,plot=

```

\section*{Arguments}
\begin{tabular}{ll}
x & a correlation matrix or a data matrix \\
n & Number of factors to extract - should be more than hypothesized! \\
rotate & \begin{tabular}{l} 
what rotation to use c("none", "varimax", "oblimin","promax") \\
diagonal \\
fm
\end{tabular} \\
n. obs & \begin{tabular}{l} 
Should we fit the diagonal as well \\
factoring method - fm="pa" Principal Axis Factor Analysis, fm = "minres" min- \\
imum residual (OLS) factoring fm="mle" Maximum Likelihood FA, fm="pc" \\
Principal Components"
\end{tabular} \\
plot & \begin{tabular}{l} 
Number of observations if doing a factor analysis of correlation matrix. This \\
value is ignored by VSS but is necessary for the ML factor analysis package. \\
plot=TRUE Automatically call VSS.plot with the VSS output, otherwise don't \\
plot
\end{tabular} \\
title & \begin{tabular}{l} 
a title to be passed on to VSS.plot \\
parameters to pass to the factor analysis program The most important of these is
\end{tabular} \\
if using a correlation matrix is covmat= xx
\end{tabular}

\section*{Details}

Determining the most interpretable number of factors from a factor analysis is perhaps one of the greatest challenges in factor analysis. There are many solutions to this problem, none of which is uniformly the best. "Solving the number of factors problem is easy, I do it everyday before breakfast. But knowing the right solution is harder" (Kaiser, 195x).
Techniques most commonly used include
1) Extracting factors until the chi square of the residual matrix is not significant.
2) Extracting factors until the change in chi square from factor \(n\) to factor \(n+1\) is not significant.
3) Extracting factors until the eigen values of the real data are less than the corresponding eigen values of a random data set of the same size (parallel analysis) fa. parallel.
4) Plotting the magnitude of the successive eigen values and applying the scree test (a sudden drop in eigen values analogous to the change in slope seen when scrambling up the talus slope of a mountain and approaching the rock face.
5) Extracting principal components until the eigen value \(<1\).
6) Extracting factors as long as they are interpetable.
7) Using the Very Structure Criterion (VSS).
8) Using Wayne Velicer's Minimum Average Partial (MAP) criterion.

Each of the procedures has its advantages and disadvantages. Using either the chi square test or the change in square test is, of course, sensitive to the number of subjects and leads to the nonsensical condition that if one wants to find many factors, one simply runs more subjects. Parallel analysis is partially sensitive to sample size in that for large samples the eigen values of random factors will be very small. The scree test is quite appealling but can lead to differences of interpretation as to when the scree "breaks". The eigen value of 1 rule, although the default for many programs, seems to be a rough way of dividing the number of variables by 3 . Extracting interpretable factors means that the number of factors reflects the investigators creativity more than the data. VSS, while very
simple to understand, will not work very well if the data are very factorially complex. (Simulations suggests it will work fine if the complexities of some of the items are no more than 2).
Most users of factor analysis tend to interpret factor output by focusing their attention on the largest loadings for every variable and ignoring the smaller ones. Very Simple Structure operationalizes this tendency by comparing the original correlation matrix to that reproduced by a simplified version (S) of the original factor matrix (F). \(\mathrm{R}=\mathrm{SS}{ }^{\prime}+\mathrm{U} 2 . \mathrm{S}\) is composed of just the c greatest (in absolute value) loadings for each variable. C (or complexity) is a parameter of the model and may vary from 1 to the number of factors.
The VSS criterion compares the fit of the simplified model to the original correlations: VSS \(=1\) -sumsquares \(\left(\mathrm{r}^{*}\right) /\) sumsquares(r) where \(\mathrm{R}^{*}\) is the residual matrix \(\mathrm{R}^{*}=\mathrm{R}-\mathrm{SS}^{\prime}\) and \(\mathrm{r}^{*}\) and r are the elements of \(\mathrm{R}^{*}\) and R respectively.
VSS for a given complexity will tend to peak at the optimal (most interpretable) number of factors (Revelle and Rocklin, 1979).
Although originally written in Fortran for main frame computers, VSS has been adapted to micro computers (e.g., Macintosh OS 6-9) using Pascal. We now release R code for calculating VSS.
Note that if using a correlation matrix (e.g., my.matrix) and doing a factor analysis, the parameters n.obs should be specified for the factor analysis: e.g., the call is VSS(my.matrix,n.obs=500). Otherwise it defaults to 1000 .
Wayne Velicer's MAP criterion has been added as an additional test for the optimal number of components to extract. Note that VSS and MAP will not always agree as to the optimal number.

A variety of rotation options are available. These include varimax, promax, and oblimin. Others can be added. Suggestions are welcome.

\section*{Value}

A data.frame with entries: map: Velicer's MAP values (lower values are better) dof: degrees of freedom (if using FA)
chisq: chi square (from the factor analysis output (if using FA)
prob: probability of residual matrix \(>0\) (if using FA)
sqresid: squared residual correlations
fit: factor fit of the complete model
cfit.1: VSS fit of complexity 1
cfit.2: VSS fit of complexity 2
...
cfit.8: VSS fit of complexity 8
cresidiual.1: sum squared residual correlations for complexity 1
...: sum squared residual correlations for complexity 2 .. 8

\section*{Author(s)}

William Revelle

\section*{References}
http://personality-project.org/r/vss.html, Revelle, W. An introduction to psychometric theory with applications in R (in prep) Springer. Draft chapters available at http:

\section*{//personality-project.org/r/book/}

Revelle, W. and Rocklin, T. 1979, Very Simple Structure: an Alternative Procedure for Estimating the Optimal Number of Interpretable Factors, Multivariate Behavioral Research, 14, 403-414. http://personality-project.org/revelle/publications/vss.pdf
Velicer, W. (1976) Determining the number of components from the matrix of partial correlations. Psychometrika, 41, 321-327.

\section*{See Also}

VSS.plot, ICLUST, omega, fa.parallel

\section*{Examples}
```

test.data <- Harman74.cor\$cov
my.vss <- VSS(test.data,title="VSS of 24 mental tests")
\#print(my.vss[,1:12],digits =2)
\#VSS.plot(my.vss, title="VSS of 24 mental tests")
\#now, some simulated data with two factors
VSS(sim.circ(nvar=24),fm="mle" ,title="VSS of 24 circumplex variables")
VSS(sim.item(nvar=24),fm="mle" ,title="VSS of 24 simple structure variables")

```
```

VSS.parallel Compare real and random VSS solutions

```

\section*{Description}

Another useful test for the number of factors is when the eigen values of a random matrix are greater than the eigen values of a a real matrix. Here we show VSS solutions to random data. A better test is probably fa.parallel.

\section*{Usage}
```

VSS.parallel(ncases, nvariables,scree=FALSE,rotate="none")

```

\section*{Arguments}
\begin{tabular}{ll} 
ncases & Number of simulated cases \\
nvariables & number of simulated variables \\
scree & Show a scree plot for random data - see omega \\
rotate & rotate="none" or rotate="varimax"
\end{tabular}

\section*{Value}

VSS like output to be plotted by VSS.plot

\section*{Author(s)}

William Revelle

\section*{References}

Very Simple Structure (VSS)

\section*{See Also}
fa.parallel, VSS.plot, ICLUST, omega

\section*{Examples}
```

\#VSS.plot(VSS.parallel(200,24))

```
```

VSS.plot Plot VSS fits

```

\section*{Description}

The Very Simple Structure criterion (VSS) for estimating the optimal number of factors is plotted as a function of the increasing complexity and increasing number of factors.

\section*{Usage}

VSS.plot(x, title = "Very Simple Structure", line = FALSE)

\section*{Arguments}
\(x \quad\) output from VSS
title any title
line connect different complexities

\section*{Details}

Item-factor models differ in their "complexity". Complexity 1 means that all except the greatest (absolute) loading for an item are ignored. Basically a cluster model (e.g., ICLUST). Complexity 2 implies all except the greatest two, etc.
Different complexities can suggest different number of optimal number of factors to extract. For personality items, complexity 1 and 2 are probably the most meaningful.
The Very Simple Structure criterion will tend to peak at the number of factors that are most interpretable for a given level of complexity. Note that some problems, the most interpretable number of factors will differ as a function of complexity. For instance, when doing the Harman 24 psychological variable problems, an unrotated solution of complexity one suggests one factor (g), while a complexity two solution suggests that a four factor solution is most appropriate. This latter probably reflects a bi-factor structure.
For examples of VSS.plot output, see http://personality-project.org/r/r.vss. html

\section*{Value}

A plot window showing the VSS criterion varying as the number of factors and the complexity of the items.

\section*{Author(s)}

Maintainer: William Revelle <revelle@northwestern.edu>

\section*{References}
```

http://personality-project.org/r/r.vss.html

```

\section*{See Also}
```

VSS, ICLUST, omega

```

\section*{Examples}
```

test.data <- Harman74.cor\$cov
my.vss <- VSS(test.data) \#suggests that 4 factor complexity two solution is optimal
VSS.plot(my.vss,title="VSS of Holzinger-Harmon problem") \#see the graphics

```
    VSS.scree Plot the successive eigen values for a scree test

\section*{Description}

Cattell's scree test is one of most simple ways of testing the number of components or factors in a correlation matrix. Here we plot the eigen values of a correlation matrix as well as the eigen values of a factor analysis.

\section*{Usage}
scree(rx,factors=TRUE, main="Scree plot", add=FALSE)
VSS.scree(rx, main = "scree plot")

\section*{Arguments}
\begin{tabular}{ll} 
rx & \begin{tabular}{l} 
a correlation matrix or a data matrix. If data, then correlations are found using \\
pairwise deletions.
\end{tabular} \\
factors & If true, draw the scree for factors and components \\
main & Title \\
add & Should multiple plots be drawn?
\end{tabular}

\section*{Details}

Among the many ways to choose the optimal number of factors is the scree test. A better function to show the scree as well as compare it to randomly parallel solutions is found found in fa.parallel

\section*{Author(s)}

William Revelle

\section*{References}
http://personality-project.org/r/vss.html

\section*{See Also}
fa.parallel VSS.plot, ICLUST, omega

\section*{Examples}
```

scree(attitude)
\#VSS.scree(cor(attitude)

```
winsor \(\quad\)\begin{tabular}{l} 
Find the Winsorized scores, means, sds or variances for a vector, ma- \\
trix, or data.frame
\end{tabular}

\section*{Description}

Among the robust estimates of central tendency are trimmed means and Winsorized means. This function finds the Winsorized scores. The top and bottom trim values are given values of the trimmed and 1- trimmed quantiles. Then means, sds, and variances are found.

\section*{Usage}
```

winsor(x, trim = 0.2, na.rm = TRUE)
winsor.mean(x, trim = 0.2, na.rm = TRUE)
winsor.means(x, trim = 0.2, na.rm = TRUE)
winsor.sd(x, trim = 0.2, na.rm = TRUE)
winsor.var(x, trim = 0.2, na.rm = TRUE)

```

\section*{Arguments}
\begin{tabular}{ll}
x & A data vector, matrix or data frame \\
trim & Percentage of data to move from the top and bottom of the distributions \\
na.rm & Missing data are removed
\end{tabular}

\section*{Details}

Among the many robust estimates of central tendency, some recommend the Winsorized mean. Rather than just dropping the top and bottom trim percent, these extreme values are replaced with values at the trim and 1- trim quantiles.

\section*{Value}

A scalar or vector of winsorized scores or winsorized means, sds, or variances (depending upon the call).

\section*{Author(s)}

William Revelle with modifications suggested by Joe Paxton and a further correction added (January, 2009) to preserve the original order for the winsor case.

\section*{References}

Wilcox, Rand R. (2005) Introduction to robust estimation and hypothesis testing. Elsevier/Academic Press. Amsterdam ; Boston.

\section*{See Also}
```

interp.median

```

\section*{Examples}
```

data(sat.act)
winsor.means(sat.act) \#compare with the means of the winsorized scores
y <- winsor(sat.act)
describe(y)
xy <- data.frame(sat.act,y)
pairs.panels(xy) \#to see the effect of winsorizing
x <- matrix(1:100, ncol=5)
winsor(x)
winsor.means(x)
y <- 1:11
winsor(y,trim=.5)

```

From a two by two table, find the Yule coefficients of association, convert to phi, or polychoric, recreate table the table to create the Yule coefficient.

\section*{Description}

One of the many measures of association is the Yule coefficient. Given a two \(x\) two table of counts
a b
c d

Yule Q is \((\mathrm{ad}-\mathrm{bc}) /(\mathrm{ad}+\mathrm{bc})\).
Conceptually, this is the number of pairs in agreement (ad) - the number in disagreement (bc) over the total number of paired observations. Warren (2008) has shown that Yule's Q is one of the "coefficients that have zero value under statistical independence,maximum value unity, and minimum value minus unity independent of the marginal distributions" (p 787).
\(\mathrm{ad} / \mathrm{bc}\) is the odds ratio and \(\mathrm{Q}=(\mathrm{OR}-1) /(\mathrm{OR}+1)\)
Yule's coefficient of colligation is \(Y=(\operatorname{sqrt}(\mathrm{OR})-1) /(\mathrm{sqrt}(\mathrm{OR})+1)\) Yule.inv finds the cell entries for a particular Q and the marginals \((a+b, c+d, a+c, b+d)\). This is useful for converting old tables of correlations into more conventional phi or polychoric correlations.
Yule2phi and Yule2poly convert the Yule Q with set marginals to the correponding phi or polychoric correlation.

\section*{Usage}
```

Yule(x,Y=FALSE) \#find Yule given a two by two table of frequencies
Yule.inv(Q,m) \#find the frequencies that produce a Yule Q given the Q and margir
Yule2phi(Q,m) \#find the phi coefficient that matches the Yule Q given the margir
Yule2poly(Q,m) \#Find the tetrachoric correlation given the Yule Q and the margina

```

\section*{Arguments}
\(\mathrm{x} \quad\) A vector of four elements or a two by two matrix
Y Y=TRUE return Yule's Y coefficient of colligation
Q The Yule coefficient
\(m \quad\) A two \(x\) two matrix of marginals or a four element vector of marginals

\section*{Details}

Yule developed two measures of association for two by two tables. Both are functions of the odds ratio

\section*{Value}

Q The Yule Q coefficient
\(R \quad\) A two by two matrix of counts

\section*{Note}

Yule.inv is currently done by using the optimize function, but presumably could be redone by solving a quadratic equation.

\section*{Author(s)}

William Revelle

\section*{References}

Yule, G. Uday (1912) On the methods of measuring association between two attributes. Journal of the Royal Statistical Society, LXXV, 579-652
Warrens, Matthijs (2008), On Association Coefficients for \(2 \times 2\) Tables and Properties That Do Not Depend on the Marginal Distributions. Psychometrika, 73, 777-789.

\section*{See Also}

See Also as phi, Yule2poly.matrix, Yule2phi.matrix

\section*{Examples}
```

Nach <- matrix(c(40,10,20,50), ncol=2,byrow=TRUE)
Yule(Nach)
Yule.inv(.81818,c(50,70,60,60))
Yule2phi(.81818,c(50,70,60,60))
if(require(polycor)) Yule2poly(.81818,c(50,70,60,60))
phi(Nach) \#much less

```

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