

# Psychology 454: Psychological Measurement

## An introduction to latent variable modeling

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## 1 Background

Much of the current literature in psychology and related disciplines includes variables which are not directly observable but rather are inferred from the observed variables. That is, our observable measures are thought to reflect some underlying *latent* process or processes. Our theories tend to be formulated in terms of these

latent processes. The problem then is how to measure or model these unobservable processes. This is the challenge of this course.

## 2 Objectives

To understand the fundamental concepts in latent variable modeling in order to make you a better consumer and producer of latent variable models in your research. This includes a number of latent variable techniques, including, but not limited to Exploratory Factor Analysis (EFA), Confirmatory Factor Analysis (CFA), Structural Equation Modeling (SEM) and Item Response Theory (IRT). By necessity, it also includes fundamental issues in reliability.

Thus, the goal of this course is to help you understand how to evaluate the quality of models when applied to data by understanding various sources of variability as measured by goodness of fit tests. The emphasis will be upon model comparison rather than model confirmation.

A further goal is to learn how to apply these concepts to real data sets using a variety of standard statistical packages (in particular in R ([R Core Team, 2022](#)) using the *lavaan* package ([Rosseel, 2012](#))). Some examples will be given from other software (e.g., EQS, Lisrel, Prelis, Amos, and MPlus).

## 3 Text, readings, and requirements

### 3.1 Text

Loehlin, J. C. and Alexander Beaujean, Latent Variable Models (5th ed). Routledge, N.J. 2017

This book is well worth purchasing and a quick web search will show a variety of options ranging from [unauthorized](#) pdf copies to used copies to the latest version.

Revelle, W. (in preparation) An introduction to psychometric theory with applications in R. Springer. Draft chapters available at <http://personality-project.org/r/book>.

Revelle, W. (in preparation) Adventures in Latent Variable Modeling using R. Draft chapters available at <http://personality-project.org/r/book>.

### 3.2 Readings

Multiple web based readings including, but not limited to the ones listed in the references. This list will be added to throughout the quarter.

Syllabus and handouts available at <http://personality-project.org/courses/454.syllabus.pdf> as well as the NU CANVAS website. In the spirit of open science and its basic principle that materials should be shared, all readings and assignments will be available through the <http://personality-project.org/courses/454/> website and are linked from here.

### 3.3 Requirements

The most important requirement is the willingness to ask questions and to try the examples.

Additional requirements include:

1. Some basic knowledge of psychometric theory (to be reviewed in week 1). This course is a natural sequel to [Psychology 405: Psychometric Theory](#). Some of the web readings will be taken from the 405 syllabus.
2. Familiarity with linear (matrix) algebra (to be reviewed in week 1). Or at least a willingness learn a little bit about linear algebra.

3. Willingness to use computer packages that allow for structural equation modeling. These can either be downloaded to your computer (e.g, the open source packages R , and *lavaan*), or ones that you have a license for (e.g., MPLUS, EQS?)
4. Willingness to ask questions and add to the class discussion.

As might be expected, most of my examples and lectures will make use of the powerful statistical system, R ([R Core Team, 2022](#)). This is because the open source nature of R allows us to see (if we want) how the calculations are actually done, and to add new features to existing packages. Most importantly, from the point of open science, by documenting your manuscripts with the code you used, other people can replicate your work. R makes the providing of scripts in a manuscript very easy.

We will use two packages a great deal. The R packages *psych* ([Revelle, 2022](#)) and *lavaan* ([Rosseel, 2012](#)). Most of our data sets are included in the *psychTools* package. These packages are all available for down load from [CRAN](#).

### 3.4 Accessibility

Any student requesting accommodations related to a disability or other condition is required to register with AccessibleNU ([accessiblenu@northwestern.edu](mailto:accessiblenu@northwestern.edu); 847-467-5530) and provide me with an accommodation notification from AccessibleNU, preferably within the first two weeks of class. All information will remain confidential.

### 3.5 Office Hours

Tuesdays (and most Thursdays) from 2-5. If we ever are meeting physically, my office is Swift Hall 315. Otherwise I am available for “office hours” by Zoom. I strongly encourage you to take advantage of this time to ask additional questions.

### 3.6 Evaluation

Homework assignments will be given weekly. These are your benefit and will be graded on a completed, not completed basis.

Students will be expected to write a short paper demonstrating the use of structural equation techniques applied to their particular research interests. They will also be asked to present their use of latent variable models in short (15-30 minute) presentations in the last few weeks of the course.

This is a hands on course. You will be expected to try the various programs on simulated and real data sets.

## 4 Privacy considerations

This class will be recorded by me for educational purposes. These recordings will be shared only with students enrolled in the course and will be deleted at the end of the end of the Fall Quarter, 2020 course. The recordings will be available in CANVAS so that you can review the materials after class as well. In addition, I hope to make publicly available a set of short tutorials on various aspects of R and the use of the *psych* package.

### 4.1 Recording of Class Sessions by Students

Unauthorized student recording is prohibited. Faculty should not grant individual requests for students to record class sessions. Students requesting the use of assistive technology as an accommodation should direct such requests to AccessibleNU.

As part of our communication to students, all instructors have been asked to include the following statement on all Syllabi for Fall Quarter, 2020 classes:

Unauthorized student recording of classroom or other academic activities (including advising sessions or office hours) is prohibited. Unauthorized recording is unethical and may also be a violation of University policy and state law. Students requesting the use of assistive technology as an accommodation should contact AccessibleNU. Unauthorized use of classroom recordings – including distributing or posting them – is also prohibited. Under the University’s Copyright Policy, faculty own the copyright to instructional materials – including those resources created specifically for the purposes of instruction, such as syllabi, lectures and lecture notes, and presentations. Students cannot copy, reproduce, display, or distribute these materials. Students who engage in unauthorized recording, unauthorized use of a recording, or unauthorized distribution of instructional materials will be referred to the appropriate University office for follow-up.

Having said that, materials on CANVAS follow that policy. Materials I release on the <https://personality-project.org/courses/454> will be in the public domain.

## 5 Outline (to be added to frequently – keep checking)

Updates:

Current version as of September 19, 2022.

Week	Topic	Reading	Lecture Notes	Homework
1	Background  Correlation/Regression Reliability	<a href="#">Computers and Psychology</a>  <a href="#">Correlation/Regression Reliability</a> <a href="#">Revelle and Condon, 2018 2019</a> <a href="#">Linear Algebra</a>	<a href="#">Overview</a>  <a href="#">Correlation and Regression</a> <a href="#">Introduction to R</a>  <a href="#">Linear Algebra</a> <a href="#">inverse of a matrix.</a>	<a href="#">Loehlin Chapter 1</a>  Problem set 1 Exercises from Loehlin 1-12 (page 32-34)
2	Basic Model fitting	<a href="#">Loehlin Chapter 2</a> <a href="#">regression vignettes</a>	<a href="#">Path models (deprecated)</a> <a href="#">Week 2 slides</a> Model fitting – regression vs. latent variables <a href="#">regression/mediation/latent</a> Using R	Problem set 2
3	Simple models	<a href="#">Loehlin Chapter 3</a>	<a href="#">OLS/WLS/MLE</a> <a href="#">regression and sem</a> Even more on regression <a href="#">Simulating structural data.</a> <a href="#">A general factor of personality?</a>	<a href="#">6.3.1</a> Problem set 3
4	Exploratory FA	<a href="#">Loehlin Chapter 5</a> <a href="#">Hierarchical factor models</a>	<a href="#">EFA/CFA</a> <a href="#">EFA/CFA – psych, sem and lavaan</a>	Problem set 4
5	EFA (continued)	<a href="#">Loehlin Chapter 6</a>  <a href="#">Goodness of fit</a>	<a href="#">items vs continuous measures</a> <a href="#">Item Response Models</a> <a href="#">Guttman models</a> more on skew change and more on change	Problem set 5
6	CFA  Multiple Groups	<a href="#">Loehlin Chapter 4</a>  <a href="#">one good and one bad example of modeling complex structures</a> <a href="#">R and LISREL models</a>	<a href="#">SEM = regression + reliability</a> How to define a model  <a href="#">analysis and critique</a>  <a href="#">R and lavaan models</a> More on change types of variables, types of models	Problem set 6
7	Goodness of Fit  Evaluating Alternatives	<a href="#">Loehlin Chapter 7</a>  <a href="#">More on change</a> <a href="#">latent change</a>	<a href="#">Advanced modeling with Lavaan</a> <a href="#">measurement invariance</a>	Problem set 7
8		<a href="#">Loehlin Chapter</a>	<a href="#">Causality?</a> <a href="#">More on IRT</a>	Problem set 8
10	Latent Classes		<a href="#">Latent class and latent growth</a> <a href="#">Review of SEM</a>	

## 6 Detailed Notes

### 6.1 Week 1 Introduction to latent variables (405 in a week).

The past century has seen a revolution in the way scientists can analyze data using [computers](#). Over the past 50 years psychologists have gone from using main frame computers that took hours to days to analyze simple problems to the use of lap tops to analyze much more complex problems. With the introduction of the open source statistical system R ([R Core Team, 2022](#)) we can now do in classes as exercises what used to be months of research. We will do so.

This will require installing R on your computer (go the [CRAN project](#)), download the appropriate version of R) and becoming familiar with the basic syntax of R commands. You will probably also want to install [R studio](#) at the same time.

Although there are many introductions to the R language, I have written one for [personality researchers](#). It needs to be updated, but the current version is useful.

The concept of latent variables has been with us since at least [Plato \(1892\)](#) and the cave. The need to see beyond appearances and to think in higher dimensions was brilliantly (and humorously) discussed by [Abbott \(1884\)](#) (see also [a pdf version](#)). The difficulty in thinking in higher dimensions is that once one gets beyond three dimensions our intuition breaks down ([Del Giudice, 2021](#)).

Review of [Correlation and Regression](#) and classical reliability theory ([Revelle and Condon, 2018, 2019](#)). See also Chapter 4 on [Correlation and regression](#) as well as Chapter 5 on [multiple correlation and regression](#).

Review of linear/matrix algebra ([Appendix E](#)) See the [Linear Algebra](#) slides.

What is a latent variable? Some opposing viewpoints ([Bartholomew et al., 2009](#); [Loevinger, 1957](#); [Markon and Jonas, 2016](#); [Jonas and Markon, 2016](#)).

### 6.2 Week 2

Application of linear algebra to pattern and structure. [Chapter 6: Exploratory factor analysis](#) as a basic latent variable model. Finding the [inverse of a matrix](#). For a review of factor analysis, see <http://personality-project.org/courses/405/405.efa.pdf>.

### 6.3 Week 3

Structural models and goodness of fit tests. [Barrett \(2007\)](#), Examples with simulated data.

[How to simulate structural data](#). This has been revised with a correction for two factor simulations and with a more extensive analysis of the effects of sample size on estimating parameters in the two factor model.

Using basic sem programs to find structure and apply goodness of fit tests. Using the *sem* ([Fox et al., 2013](#)) and *lavaan* ([Rosseel, 2012](#)) packages.

#### 6.3.1 Homework: Problem set 3

Try to simulate a one factor model and then factor it using EFA and the CFA (using *lavaan*). First simulate with no error and then add error by specifying a sample size.

### 6.4 Week 4

Perhaps the fundamental issue of latent variable analysis is why use latent variables. The classic development of latent variable analysis was ([Spearman, 1904](#)) with the development of what has come to be called “Exploratory Factor Analysis”. While a useful descriptive technique to describe the “common” part of variables, EFA can be made a testable technique using “Confirmatory Factor Analysis” which is the root of

most SEM packages. See [EFA/CFA](#) for an overview of PCA, EFA, and then the basics of CFA. [EFA/CFA – psych, sem and lavaan](#).

The problem of hierarchical representations of data. It has been known since [Spearman \(1904\)](#) that ability measures may be represented hierarchically. This is true at the data level as well as the genetic level ([de la Fuente et al., 2020](#)). However, this does not imply that there is one causal ‘g’ ([Bartholomew et al., 2009](#); [Thomson, 1916](#); [Revelle et al., 2020](#)).

Many people claim a “general factor” of personality in analogy to the ‘g’ factor of ability. This has been disputed ([Revelle and Wilt, 2013](#)). See also [A general factor of personality?](#) talk given at an “Experts Meeting” on personality structure. Also see [Analysis of hierarchical factor models](#) using hierarchical and bifactor solutions. The lecture notes for week 4 are [here](#) and prior notes are [prior year notes](#)

## 6.5 Week 5

Exploratory and confirmatory factor analysis, continued. The lecture notes for week 5 are [here](#).

Considering issues of using items rather than continuous measures. [items vs continuous measures](#). Unfortunately, items have serious problems with [skew](#).

One of the most powerful applications of sem is the analysis of [change](#).

## 6.6 Week 6

Comparing three examples from the literature: a [short example](#) ([Erdle et al., 2009](#)) of how not to report factor analysis, [a sem paper which](#) which actually fails to identify the model correctly ([Erdle et al., 2010](#)) and [another](#) ([Marsh et al., 2010](#)) which systematically compares models. This last one includes a good discussion of how to do measurement invariance.

## 6.7 Week 7

*lavaan* uses many examples from the MPlus manual (<http://www.statmodel.com/ugexcerpts.shtml>). See in particular the example data sets at <http://www.statmodel.com/usersguide/chapter5.shtml>. The notes describing *lavaan* output for these examples are [available here](#).

## 6.8 Week 8

Comparing sem in [R](#) and LISREL ([Jöreskog and Sörbom, 1999](#)). Consideration of goodness of fit tests ([Barratt et al., 2007](#)) (Click on Issue 5 in the left hand column). [R and LISREL lecture notes](#)

For a very good discussion of latent change estimation in R see [Ghisletta and McArdle, \(2014\)](#) ([Ghisletta and McArdle, 2012](#)). Also see the lecture notes from Yves Rosseel [Modeling change with lavaan](#)

For an example of modeling change in cognitive ability and depression to examine the temporal sequencing of the effects, look at [Aichele et al. \(2018\)](#).

An excellent set of lecture notes on testing for invariance comes from [Tutorial on measurement invariance](#) Kate Xu.

## 6.9 Week 10

[Course review](#)

## 6.10 Software

Commercial software for structural equation modeling: [EQS Bentler \(1995\)](#), LISREL ([Jöreskog and Sörbom, 1999](#)) [MPlus \(Muthén and Muthén, 2007\)](#).

## 6.11 R advice

The [R tutorial](#) gives a short introduction to the use of R.

- (Macs and PCs) For this, or any other package to work, you must activate it by either using the Package Manager or the “library” command:
  - `type library(psych)`
  - If loading the psych package works, function such as “describe” and “pairs.panels” should work (or at least give an error message that is NOT “could not find function”).
  - entering `?psych` will give a list of the functions available in the psych package.

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