Psychometric Theory

Homework on reliability

Correlations, means, sds (alphas on diagonal)

	GREV	GREQ	GREA	N-Ach	Anx	GPA	MA	Pre
GREV	0.81							
GRE Q	0.72	0.64						
GRE A	0.54	0.48	0.72					
N-Ach	0.00	0.00	0.42	0.49				
ANX	0.00	0.00	-0.48	-0.56	0.64			
GPA	0.38	0.34	0.55	0.34	-0.39	0.49		
MA	0.32	0.29	0.47	0.29	-0.34	0.42	0.36	
Pre	0.27	0.24	0.39	0.25	-0.28	0.35	0.30	0.25
Means	600	650	700	5	12	3.2	6	5
Sigma	80	100	50	2	4	0.5	2	3

Basic descriptives from a matrix

1) What is the measure with the greatest reliability?
2) What is the measure with the lowest reliability?
3) What is the correlation between GRE Advanced and the rated quality of Prelims?
4) If a person has a GRE verbal score of 680, then what would you expect his/her GRE quantitative score to be?
5) For a person with an anxiety score of 16, what is the expected GPA?
6) Assuming the classical model of partial correlations, what is the correlation between GRE Quantitative and GPA with GRE Verbal held constant?
7) What is the multiple correlation of GRE V and GRE Q with rated quality of the MA?
8) What is the unit weighted correlation of GREV and GRE Q with MA.
 9) What is the unattenuated correlation between Anxiety and GPA?

Predicting scores

- If a person has a GRE verbal score of 680, then what would you expect his/her GRE quantitative score to be?

- Mean GRE V = 600   SD = 80   r = .72

- Mean GRE Q = 650   SD = 100

- z_{GREV} = (680- 600)/80 = 1.0

- predicted $z_{GRE\ Q}$ = $r_{xy}\ z_x$ = .72 * (1) =.72

- predicted $_{GRE\ Q}$ = .72 * 100 + 650 = 722

Predicting scores

- 5)   For a person with an anxiety score of 16, what is the expected GPA?

- Anxiety   Mean = 12   sd = 4   r = -.39

- GPA   Mean = 3.2 sd = .5

- z_{anx} = (16- 12)/4 = 1.0

- predicted z_{gpa} = r_{xy} z_x = -.3.9 * (1) =-.39

- predicted $_{gpa}$ = -.39 * .5 + 3.2 = 3.005

Part and Partial correlation

- 6) Assuming the classical model of partial correlations, what is the correlation between GRE Quantitative and GPA with GRE Verbal held constant?

- $r_{\text{GRE Q,GPA}} = .34$

- $r_{\text{GRE Q,V}} = .72$ $\qquad r_{\text{GREV,GPA}} = .38$

- partial $r_{xy.z} =$

  - $(r_{xy} - r_{xz} * r_{yx})/\text{sqrt}((1 - r_{xz}^2) * (1 - r_{yz}^2))$

- $r_{\text{qgpa.v}} = (.34 - .72 * .38)/\text{sqrt}(.482 * .856) = .103$ partial

- part $r = (r_{xy} - r_{xz} * r_{yx})/\text{sqrt}((1 - r_{xz}^2) = .096$ part

Multiple correlation

- 7) What is the multiple correlation of GRE V and GRE Q with rated quality of the MA?
- $r_{GRE\ V,\ MA} = .32$ $r_{GRE\ Q,\ MA} = .29$ $r_{GRE\ V,Q} = .72$
- beta y.x = $(r_{xy} - r_{xz}*r_{yz})/ (1-r_{xz}^2)$
- beta $_{GRE\ V,\ MA} = (.32- .72*.29)/(1-.72^2) = .231$
- beta $_{GRE\ Q,\ MA} = (.29- .72*.32)/(1-.72^2) = .124$
- R^2 = beta$_{y.x}$ * r_{xy} + beta $_{y.z}$ *r_{yz} ...
- R^2 = beta $_{GRE\ Q,\ MA}$ * $r_{GRE\ Q,\ MA}$ + beta $_{GRE\ V,\ MA}$ * $r_{GRE\ V,\ MA}$ =
- R^2 = .124 * .29 + .231 * .32 = .108
- R = .329

Unit weighted multiple R

What is the unit weighted correlation of GREV and GRE Q with MA

	GREV	GREQ	MA
GRE V	1.0	.72	.32
GRE Q	.72	1.0	.29
MA	.32	.29	1.0

$$r_{xy} = Cov_{xy}/Sqrt(V_x*V_y)$$

$Cov_{xy} = (.32 + .29)$ $V_{(V+Q)} = 1.0 + 1.0 + .72 + .72 = 3.44$

$$R = .61/sqrt*(3.44*1) = .329$$

Reliability and attenuation

9) What is the unattenuated correlation between Anxiety and GPA?

	Anx	GPA
Anx	.64	-.39
GPA	-.39	.49

$$r_{xtyt} = r_{xy}/\text{sqrt}(r_{xx}*r_{yy})$$

$$r_{\text{true anx ,true gpa}} = -.39/\text{sqrt}(.64*.49)=$$
$$-.39/.56 = -.70$$

Effect of reliability

10) If we could increase the reliability of N-ach from .49 to .81, what would happen to the correlation of N-ach with Prelims?

11) If we selected the 10 best students on the basis of their GRE Q scores and found that their mean was 750, what would weexpect them to get if we gave them the GRE Q again?

12) What would be the coefficient alpha of the composite performance measure made up of standard scores of the three measures of performance?   (Note that there are two ways of finding this. Assume that you do not know the reliabilities of the single measures.)

13) What is the correlation of GPA with this composite?

14) Assuming that all the component scores were standardized, what is the variance of the composite of the first three measures

15) What is the coefficient alpha of this composite (again, assuming no knowledge of their separate reliabilities.)

16) What is the correlation of this composite with the performance composite found in question 12?

17) If a ten item test has an average inter-item r of .15, and the items have been standardized, a) what is the variance of the test? b) what is coefficient alpha for this test?

18) If another test with 10 items has an average inter-item r of . 2 and an average item correlation with the first test (see above) of .1, a) what is the reliability of the second test? b) what is the covariance of the first with the second test? c) what is the correlation between these two tests? d) what is the correlation corrected for attenuation?

Correcting for attenuation

10) If we could increase the reliability of N-ach from .49 to .81, what would happen to the correlation of N-ach with Prelims?

	Need Ach	Prelims
Need Ach	.49	.25
Prelims	.25	.25

$$(\text{True } r) = r_{xtyt} = r_{xy}/\text{sqrt}(r_{xx}*r_{yy})$$

$$\text{True } r = .25/\text{sqrt}(.49*.25) = .71 = r_{xy}/\text{sqrt}(.81*.25)$$

$$r_{xy} = .71*.9*.5 = .32$$

Reliability and regression

11) If we selected the 10 best students on the basis of their GRE Q scores and found that their mean was 750, what would weexpect them to get if we gave them the GRE Q again?

$$z_{estimated\ score} = r_{xx} * z_{observed}$$

z Qretest = .64 * (750-650)/100 = .64

estimated Q retest = z score * sd + mean
= .64 * 100 + 650 = 714

Reliability of composites

12) What would be the coefficient alpha of the composite performance measure made up of standard scores of the three measures of performance?   (Note that there are two ways of finding this. Assume that you do not know the reliabilities of the single measures.)

	GPA	MA	Pre
GPA	1.0	.42	.35
MA	.42	1.0	.30
Pre	.35	.30	1.0

$V_t = 5.14$

$\sum v_i = 3$

av r = 1.07/3
=.357

alpha = $\{(V_t - \sum v_i)/ V_t \}*\{n/(n-1)\}$ =
   $\{(5.14-3)/5.14\}*\{3/2)$ = .625

alpha = $n*r./(1+(n-1)*r.)$ = 3 * .357 /(1+2*.357) = .625

Reliability of composites

12) What would be the coefficient alpha of the composite performance measure made up of standard scores of the three measures of performance?   (If you know the reliabilities, then you can find a slightly better estimate.) This method is particularly appropriate if the reliabilities of the subtests are much higher than the intercorrelations.

	GPA	MA	Pre
GPA	.49	.42	.35
MA	.42	.36	.30
Pre	.35	.30	.25

V reliable =3.24

$V_t = 5.14$

reliability of a "hodgepodge" =
reliable variance/total variance = 3.24/5.14 = .63

Part whole correlations

13) What is the correlation of GPA with this composite?

	GPA	MA	Pre
GPA	1.0	.42	.35
MA	.42	1.0	.30
Pre	.35	.30	1.0

$$r_{xy} = C_{xy}/sqrt(V_x * V_y)$$

$$C_{GPA,Total} = 1.00 + .42 + .35 = 1.77 \qquad V_t = 5.14$$

$$r_{GPA,Total} = C_{GPA,Total}/sqrt(V_t * V_{gpa})$$

$$r_{GPA,Total} = 1.77/sqrt(5.14*1) = .781$$

Variance and alpha

14) Assuming that all the component scores were standardized, what is the variance of the composite of the first three measures
15) What is the coefficient alpha of this composite (again, assuming no knowledge of their separate reliabilities.)

	GREV	GREQ	GREA
GRE V	1.00	.72	.54
GRE Q	.72	1.00	.48
GRE A	.54	.48	1.00

$V_t = 6.48$

$$\text{alpha} = \{(V_t - \sum v_i)/V_t\} * \{n/(n-1)\} =$$
$$\{(6.48-3)/6.48\} * \{3/2\} = .806$$

Correlations of composites

	GRE	GRE	GRE	GPA	PRE	MA
GRE	1.00	.72	.54			
GRE	.72	1.00	.48			
GRE	.54	.48	1.00			
GPA	.38	.34	.55	1.00	.42	.35
PRE	.32	.29	.47	.42	1.00	.30
MA	.27	.24	.39	.35	.30	1.00

$V_x = 6.48$

$V_y = 5.14$

$C_{xy} = 3.25$

$r_{xy} = C_{xy}/\text{sqrt}(V_x * V_y)$

$r_{xy} = 3.25/\text{sqrt}(6.48 * 5.14) = .563$

Composite variances

17) If a ten item test has an average inter-item r of .15, and the items have been standardized, a) what is the variance of the test? b) what is coefficient alpha for this test?

$$V_t = k* (1.0) + k(k-1)*r.$$

$$\text{alpha} = n*r./(1+(n-1)*r.)$$

$$\text{alpha} = \{(V_t - \sum v_i)/V_t\}*\{n/(n-1)\}$$

	x_1	x_2	...	x_i	...	x_n
x_1	1.0	.15		.15		.15
x_2	.15	1.0				
...			...	.15		
x_i	.15		.15	1.0		.15
...					...	
x_n	.15			.15		1.0

$$V_t = 10 + 10*9*.15 = 23.5$$

$$\text{alpha} = 10*.15/(1+9*.15) = .638$$

$$\text{alpha} = \{(23.5-10)/23.5\} *(10/9) = .638$$

Correlations of composites

18) If another test with 10 items has an average inter-item r of .2 and an average item correlation with the first test (see above) of .1, a) what is the reliability of the second test? b) what is the covariance of the first with the second test? c) what is the correlation between these two tests? d) what is the correlation corrected for attenuation?

	Test 1 (10 items r. = .15)	Test 2 (10 items r. = .20)
Test 1	$V_1 = 23.5$	
Test 2	$C_{12} = 10$	$V_2 = 28$

$r_{xy} = C_{xy}/\sqrt{(V_x * V_y)}$

$r_{xtyt} = r_{xy}/\sqrt{(r_{xx} * r_{yy})}$

$r_{12} = 10/\sqrt{(23.5 * 28)} = .39$

$r_{1t2t} = .39/\sqrt{(.638 * .714)} = .577$

$r_{1t2t} = .1/\sqrt{(.15 * .2)}$

Interactions and scaling

19) When we break N-ach and GREV into high and low groups, we notice the following 2x2 table of GPAs:
The interaction of N-ach and GRE V is statistically significant. One inteprepretation of this interaction is that ability makes more difference for less motivated students than it does for more motivated students. Is this a reasonable interpretation of the data? Is there another plausible explanation?

	Low V	High V
Hi-Nach	3.7	3.9
Lo-Nach	3.0	3.5

Interactions and scaling

Suppose we take the previous data set and find that when we break it up by anxiety as well as GRE V and N-Ach that we have the following pattern:

Low Anxious

	low V	Hi V
Hi Nach	3.8	4.0
low Nach	3.0	3.5

High Anxious

	low V	Hi V
Hi Nach	3.0	3.2
low Nach	2.5	3.0

Does your interpretatation of the data set change with the addition of these anxiety results? How would you interpret the data now?

The original correlations (alpha on diagonal)

	GREV	GREQ	GREA	NAch	Anx	GPA	MA	Pre
1	0.81	0.72	0.54	0	0	0.38	0.32	0.27
2	0.72	0.64	0.48	0	0	0.34	0.29	0.24
3	0.54	0.48	0.72	0.42	-0.48	0.55	0.47	0.39
4	0	0	0.42	0.49	-0.56	0.34	0.29	0.25
5	0	0	-0.48	-0.56	0.64	-0.39	-0.34	-0.28
6	0.38	0.34	0.55	0.34	-0.39	0.49	0.42	0.35
7	0.32	0.29	0.47	0.29	-0.34	0.42	0.36	0.3
8	0.27	0.24	0.39	0.25	-0.28	0.35	0.3	0.25

Correct for attenuation to show structure

	GREV	GREQ	GREA	NAch	Anx	GPA	MA	Pre
1	0.81	1	0.71	0	0	0.6	0.59	0.6
2	0.72	0.64	0.71	0	0	0.61	0.6	0.6
3	0.54	0.48	0.72	0.71	-0.71	0.93	0.92	0.92
4	0	0	0.42	0.49	-1	0.69	0.69	0.71
5	0	0	-0.48	-0.56	0.64	-0.7	-0.71	-0.7
6	0.38	0.34	0.55	0.34	-0.39	0.49	1	1
7	0.32	0.29	0.47	0.29	-0.34	0.42	0.36	1
8	0.27	0.24	0.39	0.25	-0.28	0.35	0.3	0.25