

Psychology 405: Psychometric Theory

Examples of finding correlations from composites

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Outline

Composite variances/covariances

Regression

References

Consider the following correlation matrix

Variable	V1	V2	V3	V4
V1	1.0	0.4	0.3	0.2
V2	0.4	1.0	0.1	0.3
V3	0.3	0.1	1.0	0.5
V4	0.2	0.3	0.5	1.0

1. Find the Variance of the composite of V1 and V2
2. Find the Variance of the composite V3 and V4
3. Find the covariance of (V1, V2) with (V3, V4)
4. Find the correlation of (V1, V2) with (V3, V4)

Consider the following correlation matrix

Variable	V1	V2	V3	V4
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V2	0.4	1.0	0.1	0.3
V3	0.3	0.1	1.0	0.5
V4	0.2	0.3	0.5	1.0

1. The Variance of the composite of V1 and V2 = $1 + .4 + .4 + 1 = 2.8$
2. The Variance of the composite V3 and V4 = $1 + .5 + .5 + 1 = 3$
3. The covariance of (V1, V2) with (V3, V4) = $.3 + .1 + .2 + .3 = .9$
4. The correlation of (V1, V2) with (V3, V4) =
$$\frac{C_{xy}}{\sqrt{V_x V_y}} = \frac{.9}{\text{sqrt}2.8*3} = .31$$

Consider the following correlation matrix

Variable	V1	V2	V3	V4
V1	1.0	0.4	0.3	0.2
V2	0.4	1.0	0.1	0.3
V3	0.3	0.1	1.0	0.5
V4	0.2	0.3	0.5	1.0

1. What is the covariance of item 1 with the composite of V1 and V2
2. What is the covariance of item 1 with the composite of (V1 ... V4)
3. What is the variance of the total composite?
4. What is the correlation of item 1 with the composite of V1 and V2
5. What is the correlation of V1 with the entire set of items (V1 ... V4)

Consider the following correlation matrix

Variable	V1	V2	V3	V4
V1	1.0	0.4	0.3	0.2
V2	0.4	1.0	0.1	0.3
V3	0.3	0.1	1.0	0.5
V4	0.2	0.3	0.5	1.0

1. The covariance of item 1 with the composite of V1 and V2 = $1 + .4 = 1.4$
2. The covariance of item 1 with the composite of (V1 ... V4) = $1 + .4 + .3 + .2 = 1.9$
3. What is the variance of the total composite? = $(.4 + .3 + .2 + .1 + .3 + .5) * 2 + 4 = 7.6$
4. The correlation of item 1 with the composite of V1 and V2 = $\frac{C_{xy}}{\sqrt{V_x V_y}} = \frac{1.4}{\sqrt{2.8 * 1}} = .84$
5. What is the correlation of V1 with the entire set of items (V1 ... V4) = $\frac{C_{xy}}{\sqrt{V_x V_y}} = \frac{1.9}{\sqrt{7.6 * 1}} = .69$

Another example

Variable	V1	V2	V3	V4
V1	1	0	0	0
V2	0	1	0	0
V3	0	0	1	0
V4	0	0	0	1

1. Find the Variance of the composite of V1 and V2
2. Find the Variance of the composite V3 and V4
3. Find the covariance of (V1, V2) with (V3, V4)
4. Find the correlation of (V1, V2) with (V3, V4)

Another example

Variable	V1	V2	V3	V4
V1	1	0	0	0
V2	0	1	0	0
V3	0	0	1	0
V4	0	0	0	1

1. Find the Variance of the composite of V1 and V2 = $1 + 1 + 0 + 0 = 2$
2. Find the Variance of the composite V3 and V4 = $1 + 1 + 0 + 0 = 2$
3. Find the covariance of (V1, V2) with (V3, V4) = $0 + 0 + 0 + 0 = 0$
4. Find the correlation of (V1, V2) with (V3, V4) =
$$\frac{C_{xy}}{\sqrt{V_x V_y}} = \frac{0}{\sqrt{2*2}} = 0$$

Another example

Variable	V1	V2	V3	V4
V1	1	0	0	0
V2	0	1	0	0
V3	0	0	1	0
V4	0	0	0	1

1. What is the covariance of item 1 with the composite of V1 and V2
2. What is the covariance of item 1 with the composite of (V1 ... V4)
3. What is the variance of the total composite?
4. What is the correlation of item 1 with the composite of V1 and V2
5. What is the correlation of V1 with the entire set of items (V1 ... V4)

Another example

Variable	V1	V2	V3	V4
V1	1	0	0	0
V2	0	1	0	0
V3	0	0	1	0
V4	0	0	0	1

1. What is the covariance of item 1 with the composite of V1 and V2 = $1 + 0 = 1$
2. What is the covariance of item 1 with the composite of (V1 ... V4) = $1 + 0 + 0 + 0 = 1$
3. What is the variance of the total composite? 4
4. What is the correlation of item 1 with the composite of V1 and V2 = $\frac{C_{xy}}{\sqrt{V_x V_y}} = \frac{1}{\sqrt{2*1}} = .71$
5. What is the correlation of V1 with the entire set of items (V1 ... V4) = $\frac{C_{xy}}{\sqrt{V_x V_y}} = \frac{1}{\sqrt{4}} = .5$

Multiple regression

Variable	V1	V2	V3	V4
V1	1.0	0.4	0.4	0.4
V2	0.4	1.0	0.2	0.0
V3	0.4	0.2	1.0	0.5
V4	0.4	0.0	0.5	1.0

1. What is the unit weighted correlation between the composite of V1 and V2 with V3?
2. What is the unit weighted correlation between the composite of V1 and V2 with V4?
3. What is the multiple correlation between the optimal composite of V1 and V2 predicting V3?
4. What is the multiple correlation between V1 and V2 predicting V4?

Multiple regression

Variable	V1	V2	V3	V4
V1	1.0	0.4	0.4	0.4
V2	0.4	1.0	0.2	0.0
V3	0.4	0.2	1.0	0.5
V4	0.4	0.0	0.5	1.0

1. What is the unit weighted correlation between the composite of V1 and V2 with V3?
2. This is just what does the composite $V1 + V2$ correlate V3
3. Find the variance of $V1 + V2 = 1 = 1 + .4 + .4 + 1 = 2.8$
4. Find the covariance of $(V1 + V2)$ with V3 $.4 + .2 = .6$
5. The unweighted correlation is therefore

$$= \frac{C_{xy}}{\sqrt{V_x V_y}} = \frac{.6}{\sqrt{2.8 * 1}} = .36$$
6. What is the unit weighted correlation between the composite of V1 and V2 with V4?
7. The unweighted correlation is $= \frac{C_{xy}}{\sqrt{V_x V_y}} = \frac{.4}{\sqrt{2.8 * 1}} = .24$
8. What is the multiple correlation between the optimal composite of V1 and V2 predicting V3?

Multiple regression

Variable	V1	V2	V3	V4
V1	1.0	0.4	0.4	0.4
V2	0.4	1.0	0.2	0.0
V3	0.4	0.2	1.0	0.5
V4	0.4	0.0	0.5	1.0

- $\beta_{13.2} = \frac{r_{13} - r_{23}r_{12}}{1 - r_{12}^2} = \frac{.4 - .2 * .4}{1 - .4^2} = \frac{.32}{.84} = .38$
- $\beta_{23.1} = \frac{r_{23} - r_{13}r_{12}}{1 - r_{12}^2} = \frac{.2 - .4 * .4}{1 - .4^2} = \frac{.04}{.84} = .05$
- $R^2 = \beta_{13.2}r_{13} + \beta_{23.1}r_{23} = .38 * .4 + .05 * .2 = .16 \Rightarrow R = .4$
- What is the unit weighted correlation between the composite of V1 and V2 with V4?
- The unweighted correlation is $= \frac{C_{xy}}{\sqrt{V_x V_y}} = \frac{.4}{\sqrt{2.8 * 1}} = .36$
- What is the multiple correlation between the optimal composite of V1 and V2 predicting V3?
- What is the multiple correlation between V1 and V2 predicting V4?

A table from the psych package in R

Variable	V1	V2	V3	V4
V1	1.0	0.4	0.4	0.4
V2	0.4	1.0	0.2	0.0
V3	0.4	0.2	1.0	0.5
V4	0.4	0.0	0.5	1.0

R code

```
setCor(V3 ~ V1 + V2, data = R)
```

Multiple Regression from matrix input

```
DV = V3
      slope  VIF
V1  0.38 1.19
V2  0.05 1.19
```

```
Multiple Regression
      R    R2  Ruw R2uw
V3 0.4 0.16 0.36 0.13
```