Psychology 454: Latent Variable Modeling
Adventures in good and bad modeling

William Revelle

Department of Psychology
Northwestern University
Evanston, Illinois USA

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Outline

Reading sem articles critically
- GFP articles as examples of what not to do

Structural Equation Modeling of these data

Dimensionality of Self Esteem

Examples taken from the general factor of personality controversy

1. The great debate: How many factors of personality?
   - Two-Three factor models Eysenck (1952, 1967, 1981)
   - 12-16+ Cattell (1956, 1957)
   - Comrey (1995)
   - Five factors Tupes & Christal (1961); Norman (1963); Digman (1990); Goldberg (1990); Costa & McCrae (1992)
2. Plasticity and Stability
   - Digman (1997)
   - DeYoung, Peterson & Higgins (2002); DeYoung (2010)
3. The Great One: a general factor of personality
   - Original meta-analysis by Musek (2007) of Big 5 data claimed a General Factor of Personality (GFP). This was followed by a torrent of research by Rushton and his associates (Rushton & Irwing, 2008; Rushton, Bons & Hur, 2008; Rushton & Irwing, 2009).
   - Review articles in the Handbook of Individual Differences (Ferguson, Chamorro-Premuzic, Pickering & Weiss, 2011; Rushton & Irwing, 2011) and elsewhere (Just, 2011).

Erdle example

1. Erdle, Irwing, Rushton & Park (2010) The general factor of personality and its relation to self-esteem in 628,640 internet respondents.
   - 628,640 subjects taken from a web survey.
   - Big Five Inventory John, Donahue & Kentle (1991); Benet-Martínez & John (1998); John, Naumann & Soto (2008)
   - note that these are not the references given in the article, which cites another paper (John & Srivastava, 1999)).
   - Self esteem was measured by one item: "I see myself as someone who has high self esteem" with a five point scale (strongly disagree–strongly agree).
2. Based upon a prior paper (Erdle, Gosling & Potter, 2009) reporting the same data set with two "factors", although they actually did a principal components analysis!
   - Lets first look at that paper.

The basic data as reported

```
  E    A       C  ES     SE
O .19  .09        .07  .08   .18
E      .15   .12  .26  .40
A            .26  .30  .13
C                 .27  .26
E                      .48

#select just the numbers
.19  .09          .07  .08   .18
      .15   .12  .26   .40
            .26  .30   .13
                 .27   .26
                       .48

 > es <- read.clipboard.upper(FALSE,FALSE)
Read 15 items
> es
     V1   V2   V3   V4   V5   V6
V1 1.00 0.19 0.09 0.07 0.08 0.18
V2 0.19 1.00 0.15 0.12 0.26 0.40
V3 0.09 0.15 1.00 0.26 0.30 0.13
V4 0.07 0.12 0.26 1.00 0.27 0.26
V5 0.08 0.26 0.30 0.27 1.00 0.48
V6 0.18 0.40 0.13 0.26 0.48 1.00
```

```
> colnames(es) <- rownames(es) <-
                c("O","E","A","C","S","ES")
> pr <- partial.r(es,1:5,6) #partial out self es
> es
>pr
      O    E    A    C    S   ES
O  1.00 0.19 0.09 0.07 0.08 0.18
E  0.19 1.00 0.15 0.12 0.26 0.40
A  0.09 0.15 1.00 0.26 0.30 0.13
C  0.07 0.12 0.26 1.00 0.27 0.26
S  0.08 0.26 0.30 0.27 1.00 0.48
ES 0.18 0.40 0.13 0.26 0.48 1.00
> pr
partial correlations
      O    E    A    C     S
O  1.00 0.13 0.07 0.02 -0.01
E  0.13 1.00 0.11 0.02  0.08
A  0.07 0.11 1.00 0.24  0.27
C  0.02 0.02 0.24 1.00  0.17
S -0.01 0.08 0.27 0.17  1.00
```

Erdle et al. (2009) claims to have done a "principal components factor analysis"

```
> p2 <- principal(es[-6,-6],2,n.obs=628240)  #this will do a varimax rotation
> p2

Principal Components Analysis
Call: principal(r = es[-6, -6], nfactors = 2, n.obs = 628240)
Standardized loadings (pattern matrix) based upon correlation matrix
    PC1   PC2   h2   u2
O -0.07  0.83 0.70 0.30
E  0.26  0.68 0.53 0.47
A  0.71  0.07 0.51 0.49
C  0.71 -0.02 0.50 0.50
S  0.70  0.21 0.54 0.46


                       PC1  PC2
SS loadings           1.57 1.21
Proportion Var        0.31 0.24
Cumulative Var        0.31 0.56
Proportion Explained  0.57 0.43
Cumulative Proportion 0.57 1.00

Test of the hypothesis that 2 components are sufficient.

The degrees of freedom for the null model are  10  and the objective function was  0.33
The degrees of freedom for the model are 1  and the objective function was  0.56
The number of observations was  628240  with Chi Square =  350853.6  with prob <  0

Fit based upon off diagonal values = 0.17
```

Two factors show a very clear solution – but what is it?

```
> f2 <- fa(es[-6,-6],2,n.obs=628240)  #this will do an oblique rotation
> f2
Factor Analysis using method =  minres
Call: fa(r = es[-6, -6], nfactors = 2,
           n.obs = 628240)
Standardized loadings (pattern matrix)
based upon correlation matrix
    MR1  MR2    h2    u2
O  0.16 0.10 0.045 0.955
E  1.00 0.00 0.995 0.005
A -0.03 0.55 0.292 0.708
C -0.05 0.50 0.237 0.763
S  0.08 0.53 0.320 0.680

                       MR1  MR2
SS loadings           1.04 0.85
Proportion Var        0.21 0.17
Cumulative Var        0.21 0.38
Proportion Explained  0.55 0.45
Cumulative Proportion 0.55 1.00

 With factor correlations of
     MR1  MR2
MR1 1.00 0.33
MR2 0.33 1.00
```

```
Test of the hypothesis that 2 factors are sufficient.

The degrees of freedom for the null model are  10  and th
                 objective function was  0.33 with
                                Chi Square of  206317.6
The degrees of freedom for the model are 1  and the
                objective function was  0

The root mean square of the residuals (RMSR) is  0
The df corrected root mean square of the residuals is  0.
The number of observations was  628240  with
                  Chi Square =  440.62  with prob <  7.9e

Tucker Lewis Index of factoring reliability =  0.979
RMSEA index =  0.026  and the 90
                % confidence intervals are  0.024 0.029
BIC =  427.27
Fit based upon off diagonal values = 1
Measures of factor score adequacy
                                                 MR1  MR2
Correlation of scores with factors              1.00 0.75
Multiple R square of scores with factors        1.00 0.57
Minimum correlation of possible factor scores   0.99 0.13
```

Compare the factor and components solutions

```
#the component loadings

    PC1   PC2   h2   u2
O -0.07  0.83 0.70 0.30
E  0.26  0.68 0.53 0.47
A  0.71  0.07 0.51 0.49
C  0.71 -0.02 0.50 0.50
S  0.70  0.21 0.54 0.46

                       PC1  PC2
SS loadings           1.57 1.21
Proportion Var        0.31 0.24
Cumulative Var        0.31 0.56
Proportion Explained  0.57 0.43
Cumulative Proportion 0.57 1.00

#think about the raw correlations
#and examine the commonalities

      O    E    A    C    S   ES
O  1.00 0.19 0.09 0.07 0.08 0.18
E  0.19 1.00 0.15 0.12 0.26 0.40
A  0.09 0.15 1.00 0.26 0.30 0.13
C  0.07 0.12 0.26 1.00 0.27 0.26
S  0.08 0.26 0.30 0.27 1.00 0.48
ES 0.18 0.40 0.13 0.26 0.48 1.00
```

```
Call: fa(r = es[-6, -6], nfactors = 2, n.obs = 6
Standardized loadings (pattern matrix) based upo
   MR1  MR2    h2    u2
O 0.18 0.11 0.045 0.955
E 0.99 0.09 0.995 0.005
A 0.10 0.53 0.292 0.708
C 0.08 0.48 0.237 0.763
S 0.22 0.52 0.320 0.680
                       MR1  MR2
SS loadings           1.08 0.80
Proportion Var        0.22 0.16
Cumulative Var        0.22 0.38
Proportion Explained  0.57 0.43
Cumulative Proportion 0.57 1.00

   MR1  MR2    h2    u2
O  0.16 0.10 0.045 0.955
E  1.00 0.00 0.995 0.005
A -0.03 0.55 0.292 0.708
C -0.05 0.50 0.237 0.763
S  0.08 0.53 0.320 0.680
                       MR1  MR2
SS loadings           1.04 0.85
Proportion Var        0.21 0.17
Cumulative Var        0.21 0.38
Proportion Explained  0.55 0.45
Cumulative Proportion 0.55 1.00
 With factor correlations of
     MR1  MR2
MR1 1.00 0.33
MR2 0.33 1.00
```

Compare the residuals from the factor and component models

```
> resid(p2)

  O     E     A     C     S
O  0.30
E -0.36  0.47
A  0.07 -0.09  0.49
C  0.13 -0.05 -0.24  0.50
S -0.05 -0.07 -0.21 -0.22  0.46
```

```
> resid(f2)

  O     E     A     C     S
O  0.96
E  0.00  0.00
A  0.01  0.00  0.71
C  0.00  0.00  0.00  0.76
S -0.02  0.00  0.00  0.00  0.68
```

Components fits the entire matrix, factors fit the off diagonal elements.

Components for the data with self esteem partialled out

These match what is reported

```
> class(pr) <- NULL
> pc2p <- principal(pr,2)
> pc2p
```

```
Principal Components Analysis
Call: principal(r = pr, nfactors = 2)
Standardized loadings (pattern matrix) based upon correlation matrix
    PC1   PC2   h2   u2
O -0.05  0.77 0.59 0.41
E  0.11  0.72 0.53 0.47
A  0.72  0.17 0.56 0.44
C  0.66 -0.07 0.44 0.56
S  0.70  0.01 0.49 0.51

                       PC1  PC2
SS loadings           1.46 1.15
Proportion Var        0.29 0.23
Cumulative Var        0.29 0.52
Proportion Explained  0.56 0.44
Cumulative Proportion 0.56 1.00

Test of the hypothesis that 2 components are sufficient.

The degrees of freedom for the null model are  10  and the objective function was  0.19
The degrees of freedom for the model are 1  and the objective function was  0.58

Fit based upon off diagonal values = -0.91
```

Factors on the partialed data

```
> f2p <- fa(pr,2,n.obs=628240)
> f2p

Factor Analysis using method =  minres
Call: fa(r = pr, nfactors = 2, n.obs = 628240)
Standardized loadings (pattern matrix)
        based upon correlation matrix
    MR1  MR2    h2    u2
O  1.00 0.00 0.995 0.005
E  0.12 0.15 0.039 0.961
A  0.02 0.62 0.390 0.610
C -0.01 0.37 0.139 0.861
S -0.04 0.45 0.202 0.798

                       MR1  MR2
SS loadings           1.01 0.75
Proportion Var        0.20 0.15
Cumulative Var        0.20 0.35
Proportion Explained  0.57 0.43
Cumulative Proportion 0.57 1.00

 With factor correlations of
     MR1  MR2
MR1 1.00 0.08
MR2 0.08 1.00
```

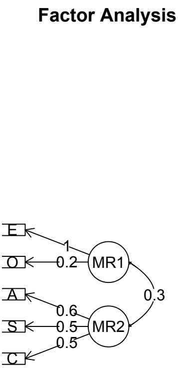
```
Test of the hypothesis that 2 factors are sufficien

The degrees of freedom for the null model are  10
  and the objective function was  0.19
       with Chi Square of  116266.5
The degrees of freedom for the model are 1
         and the objective function was  0

The root mean square of the residuals (RMSR) is  0.
The df corrected root mean square
                  of the residuals is  0.05
The number of observations was  628240
     with Chi Square =  1658.93  with prob <  0

Tucker Lewis Index of factoring reliability =  0.85
RMSEA index =  0.051  and the
       90 % confidence intervals are  0.049 0.053
BIC =  1645.58
Fit based upon off diagonal values = 0.99
Measures of factor score adequacy
                                                 MR1
Correlation of scores with factors              1.00
Multiple R square of scores with factors        1.00
Minimum correlation of possible factor scores   0.99
```

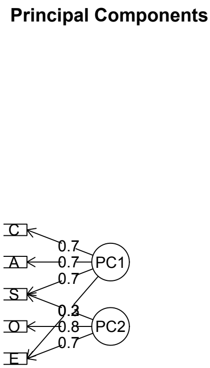
Compare component and factor solutions of the residualized Big 5 data

```
> pc2p

Principal Components Analysis
Call: principal(r = pr, nfactors = 2)
Standardized loadings (pattern matrix)
         based upon correlation matrix
    PC1   PC2   h2   u2
O -0.05  0.77 0.59 0.41
E  0.11  0.72 0.53 0.47
A  0.72  0.17 0.56 0.44
C  0.66 -0.07 0.44 0.56
S  0.70  0.01 0.49 0.51

                       PC1  PC2
SS loadings           1.46 1.15
Proportion Var        0.29 0.23
Cumulative Var        0.29 0.52
Proportion Explained  0.56 0.44
Cumulative Proportion 0.56 1.00

Test of the hypothesis that 2
         components are sufficient.

The degrees of freedom for the null model are  10
 and the objective function was  0.19
The degrees of freedom for the model are 1
 and the objective function was  0.58

Fit based upon off diagonal values = -0.91
```

```
> f2p

Factor Analysis using method =  minres
Call: fa(r = pr, nfactors = 2, n.obs = 628240)
Standardized loadings (pattern matrix)
         based upon correlation matrix
    MR1  MR2    h2    u2
O  1.00 0.00 0.995 0.005
E  0.12 0.15 0.039 0.961
A  0.02 0.62 0.390 0.610
C -0.01 0.37 0.139 0.861
S -0.04 0.45 0.202 0.798

                       MR1  MR2
SS loadings           1.01 0.75
Proportion Var        0.20 0.15
Cumulative Var        0.20 0.35
Proportion Explained  0.57 0.43
Cumulative Proportion 0.57 1.00
 With factor correlations of
     MR1  MR2
MR1 1.00 0.08
MR2 0.08 1.00
Test of the hypothesis that 2 factors are suffic

The degrees of freedom for the null model are  1
    and the objective function was  0.19
                with Chi Square of  116266.5
The degrees of freedom for the model are 1
         and the objective function was  0

The root mean square of the residuals (RMSR) is
```

Factor analysis vs. components analysis of Big5 data





How similar are these four solutions: Factor Congruence

$$r_c = \frac{\sum_1^n F_{xi} F_{y_i}}{\sqrt{\sum_1^n F_{xi}^2 \sum_1^n F_{yi}^2}}$$

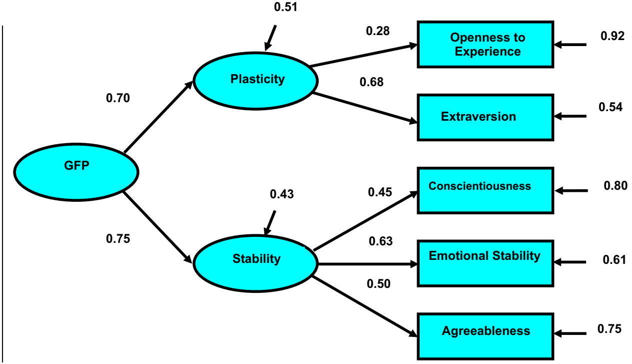
```
> factor.congruence(list(f2,p2,f2p,pc2p))
```

	MR1	MR2	PC1	PC2	MR1	MR2	PC1	PC2
MR1	1.00	0.02	0.20	0.74	0.27	0.17	0.08	0.77
MR2	0.02	1.00	0.96	0.22	0.09	0.96	0.98	0.15
PC1	0.20	0.96	1.00	0.22	-0.05	0.98	0.99	0.17
PC2	0.74	0.22	0.22	1.00	0.82	0.24	0.16	0.98
MR1	0.27	0.09	-0.05	0.82	1.00	0.01	-0.05	0.79
MR2	0.17	0.96	0.98	0.24	0.01	1.00	0.98	0.21
PC1	0.08	0.98	0.99	0.16	-0.05	0.98	1.00	0.10
PC2	0.77	0.15	0.17	0.98	0.79	0.21	0.10	1.00

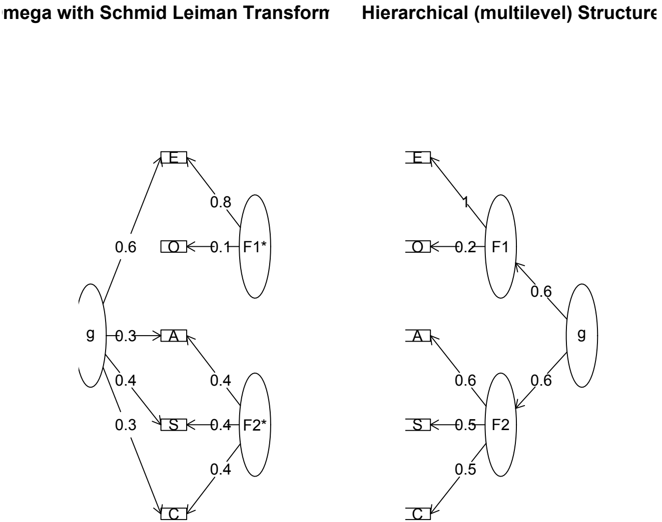
Subsequent paper (Erdle et al., 2010) looks for a general factor

- Same data set as before, but using sem
  - Two different models
  - One without Self Esteem
  - One with Self Esteem
- Lets redo their analyses
  - Examine model and alternative models
- Also, do the analysis as an exploratory higher level model

Erdle Model 1 – Is it actually defined?



Erdle data with an exploratory Omega solution



Fitting the model, part 1: Two correlated factors

```
> se.mod <- Plasticity =~ O + E
+           Stability =~ C + S + A
+          '
> fit.se <- cfa(se.mod,sample.cov=es,sample.nobs=628640)
> summary(fit.se,fit.measures=TRUE)

lavaan (0.4-14) converged normally after 38 iterations

  Number of observations                        628640

  Estimator                                         ML
  Minimum Function Chi-square                 6123.056
  Degrees of freedom                                 4
  P-value                                        0.000

Chi-square test baseline model:

  Minimum Function Chi-square               206450.068
  Degrees of freedom                                10
  P-value                                        0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    0.970
  Tucker-Lewis Index (TLI)                       0.926
```

```
Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)               -435
  Loglikelihood unrestricted model (H1)       -435

  Number of free parameters
  Akaike (AIC)                                8719
  Bayesian (BIC)                              8719
  Sample-size adjusted Bayesian (BIC)         8719

Root Mean Square Error of Approximation:

  RMSEA
  90 Percent Confidence Interval          0.048
  P-value RMSEA <= 0.05

Standardized Root Mean Square Residual:

  SRMR

Parameter estimates:

  Information                                 Ex
  Standard Errors                             St
```

With raw and standardized values that match Erdle et al. (2010)

```
                  Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  Plasticity =~
    O                1.000
    E                2.399    0.028   86.318    0.000
  Stability =~
    C                1.000
    S                1.404    0.007  195.667    0.000
    A                1.114    0.006  199.323    0.000

Covariances:
  Plasticity ~~
    Stability        0.066    0.001   82.587    0.000

Variances:
    O                0.921    0.002
    E                0.544    0.005
    C                0.800    0.002
    S                0.607    0.002
    A                0.752    0.002
    Plasticity       0.079    0.001
    Stability        0.200    0.002
```

```
> standardizedsolution(fit.se)

          lhs op        rhs est.std se  z pvalue
1  Plasticity =~          O   0.281 NA NA     NA
2  Plasticity =~          E   0.675 NA NA     NA
3   Stability =~          C   0.447 NA NA     NA
4   Stability =~          S   0.627 NA NA     NA
5   Stability =~          A   0.498 NA NA     NA
6           O ~~          O   0.921 NA NA     NA
7           E ~~          E   0.544 NA NA     NA
8           C ~~          C   0.800 NA NA     NA
9           S ~~          S   0.607 NA NA     NA
10          A ~~          A   0.752 NA NA     NA
11 Plasticity ~~ Plasticity   1.000 NA NA     NA
12  Stability ~~  Stability   1.000 NA NA     NA
13 Plasticity ~~  Stability   0.525 NA NA     NA
```

Fitting the model: part 2 – one higher order factor

```
> se.modg <- 'Plasticity =~ O + E
+            Stability =~ C + S + A
+            gfp =~  Plasticity  + Stability
+           '
> fit.se <- cfa(se.modg,sample.cov=es,sample.nobs=628640,std.lv=TRUE)
Error in solve.default(E) :
  system is computationally singular:
  reciprocal condition number = 4.16726e-18
Warning message:
In estimateVCOV(lavaanModel,
     samplestats = lavaanSampleStats, options = lavaanOptions,  :
  lavaan WARNING: could not compute standard errors!
  > summary(fit.se,fit.measures=TRUE)

lavaan (0.4-14) converged normally after 28 iterations

  Number of observations                          628640

  Estimator                                         ML
  Minimum Function Chi-square                   6123.056
  Degrees of freedom                                 3
  P-value                                        0.000

Chi-square test baseline model:

  Minimum Function Chi-square                 206450.068
  Degrees of freedom                                10
  P-value                                        0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    0.970
  Tucker-Lewis Index (TLI)                       0.901
```

```
Root Mean Square Error of Approximation:

  RMSEA
  90 Percent Confidence Interval          0.056
  P-value RMSEA <= 0.05

Standardized Root Mean Square Residual:

  SRMR

Parameter estimates:

  Information                                 Ex
  Standard Errors                             St

                   Estimate  Std.err  Z-value  P
Latent variables:
  Plasticity =~
    O                 0.194
    E                 0.464
  Stability =~
    C                 0.308
    S                 0.433
    A                 0.343
  gfp =~
    Plasticity        1.055
    Stability         1.048

Variances:
    O                 0.921
```

With standardized coefficients that partly match Erdle et al. (2010)

```
> standardizedsolution(fit.se)

          lhs op        rhs est.std se  z pvalue
1  Plasticity =~          O   0.281 NA NA     NA
2  Plasticity =~          E   0.675 NA NA     NA
3   Stability =~          C   0.447 NA NA     NA
4   Stability =~          S   0.627 NA NA     NA
5   Stability =~          A   0.498 NA NA     NA
6         gfp =~ Plasticity   0.726 NA NA     NA  <-
7         gfp =~  Stability   0.724 NA NA     NA  <-
8           O ~~          O   0.921 NA NA     NA
9           E ~~          E   0.544 NA NA     NA
10          C ~~          C   0.800 NA NA     NA
11          S ~~          S   0.607 NA NA     NA
12          A ~~          A   0.752 NA NA     NA
13 Plasticity ~~ Plasticity   0.473 NA NA     NA  <-
14  Stability ~~  Stability   0.476 NA NA     NA <-
15        gfp ~~        gfp   1.000 NA NA     NA
```

But the two loadings on the GFP are flexible

```
> se.modg <- 'Plasticity =~ O + E
+            Stability =~ C + S + A
+            gfp =~ 1* Plasticity  + Stability
+           '
> fit.se <- cfa(se.modg,sample.cov=es,sample.nobs=628640,std.lv=TRUE)
> summary(fit.se,fit.measures=TRUE)

> standardizedsolution(fit.se)
          lhs op        rhs est.std se  z pvalue
1  Plasticity =~          O   0.281 NA NA     NA
2  Plasticity =~          E   0.675 NA NA     NA
3   Stability =~          C   0.447 NA NA     NA
4   Stability =~          S   0.627 NA NA     NA
5   Stability =~          A   0.498 NA NA     NA
6         gfp =~ Plasticity   0.707 NA NA     NA
7         gfp =~  Stability   0.743 NA NA     NA
8           O ~~          O   0.921 NA NA     NA
9           E ~~          E   0.544 NA NA     NA
10          C ~~          C   0.800 NA NA     NA
11          S ~~          S   0.607 NA NA     NA
12          A ~~          A   0.752 NA NA     NA
13 Plasticity ~~ Plasticity   0.500 NA NA     NA
14  Stability ~~  Stability   0.448 NA NA     NA
15        gfp ~~        gfp   1.000 NA NA     NA
```

```
> se.modg <- 'Plasticity =~ O + E
+            Stability =~ C + S + A
+            gfp =~  Plasticity  + 1*Stability
+           '
> fit.se <- cfa(se.modg,sample.cov=es,sample.nob
> summary(fit.se,fit.measures=TRUE)

> standardizedsolution(fit.se)
          lhs op        rhs est.std se  z pvalue
1  Plasticity =~          O   0.281 NA NA     NA
2  Plasticity =~          E   0.675 NA NA     NA
3   Stability =~          C   0.447 NA NA     NA
4   Stability =~          S   0.627 NA NA     NA
5   Stability =~          A   0.498 NA NA     NA
6         gfp =~ Plasticity   0.743 NA NA     NA
7         gfp =~  Stability   0.707 NA NA     NA
8           O ~~          O   0.921 NA NA     NA
9           E ~~          E   0.544 NA NA     NA
10          C ~~          C   0.800 NA NA     NA
11          S ~~          S   0.607 NA NA     NA
12          A ~~          A   0.752 NA NA     NA
13 Plasticity ~~ Plasticity   0.448 NA NA     NA
14  Stability ~~  Stability   0.500 NA NA     NA
15        gfp ~~        gfp   1.000 NA NA     NA
```

Very flexible

```
> se.modg <- 'Plasticity =~ O + E
+            Stability =~ C + S + A
+            gfp =~  .6* Plasticity  + Stability
+           '
> fit.se <- cfa(se.modg,sample.cov=es,sample.nobs=628640,std.lv=TRUE)
Error in solve.default(E) :
  system is computationally singular: reciprocal condition number = 2.57205e-18
In addition: Warning message:
In lavaan(model = se.modg, std.lv = TRUE, sample.cov = es, sample.nobs = 628640,  :
  lavaan WARNING: model has NOT converged!
Warning message:
In estimateVCOV(lavaanModel, samplestats = lavaanSampleStats, options = lavaanOptions,  :
  lavaan WARNING: could not compute standard errors!

> standardizedsolution(fit.se)
          lhs op        rhs est.std se  z pvalue
1  Plasticity =~          O   0.279 NA NA     NA
2  Plasticity =~          E   0.685 NA NA     NA
3   Stability =~          C   0.447 NA NA     NA
4   Stability =~          S   0.627 NA NA     NA
5   Stability =~          A   0.497 NA NA     NA
6         gfp =~ Plasticity   0.514 NA NA     NA
7         gfp =~  Stability   1.000 NA NA     NA
8           O ~~          O   0.922 NA NA     NA
9           E ~~          E   0.531 NA NA     NA
10          C ~~          C   0.800 NA NA     NA
11          S ~~          S   0.606 NA NA     NA
12          A ~~          A   0.753 NA NA     NA
13 Plasticity ~~ Plasticity   0.735 NA NA     NA
14  Stability ~~  Stability   0.000 NA NA     NA
15        gfp ~~        gfp   1.000 NA NA     NA
```

```
> se.modg <- 'Plasticity =~ O + E
+            Stability =~ C + S + A
+            gfp =~   Plasticity  + .6*Stability
+           '
> fit.se <- cfa(se.modg,sample.cov=es,sample.nob
Error in solve.default(E) :
  system is computationally singular: reciprocal
In addition: Warning message:
In lavaan(model = se.modg, std.lv = TRUE, sample
  lavaan WARNING: model has NOT converged!
Warning message:
In estimateVCOV(lavaanModel, samplestats = lavaa
  lavaan WARNING: could not compute standard err

> standardizedsolution(fit.se)
          lhs op        rhs est.std se  z pvalue
1  Plasticity =~          O   0.279 NA NA     NA
2  Plasticity =~          E   0.685 NA NA     NA
3   Stability =~          C   0.447 NA NA     NA
4   Stability =~          S   0.627 NA NA     NA
5   Stability =~          A   0.497 NA NA     NA
6         gfp =~ Plasticity   1.000 NA NA     NA
7         gfp =~  Stability   0.514 NA NA     NA
8           O ~~          O   0.922 NA NA     NA
9           E ~~          E   0.531 NA NA     NA
10          C ~~          C   0.800 NA NA     NA
11          S ~~          S   0.606 NA NA     NA
12          A ~~          A   0.753 NA NA     NA
13 Plasticity ~~ Plasticity   0.000 NA NA     NA
14  Stability ~~  Stability   0.735 NA NA     NA
15        gfp ~~        gfp   1.000 NA NA     NA
>
```

Flexible fits that fit!

```
> se.modg <- 'Plasticity =~ O + E
+            Stability =~ C + S + A
+            gfp =~   Plasticity  + .62*Stability
+           '
> fit.se <- cfa(se.modg,sample.cov=es,
     sample.nobs=628640,std.lv=TRUE)
> summary(fit.se,fit.measures=TRUE)
> standardizedsolution(fit.se)

 Estimator                                         ML
  Minimum Function Chi-square                6123.056
  Degrees of freedom                                4
  P-value                                       0.000

          lhs op        rhs est.std se  z pvalue
1  Plasticity =~          O   0.281 NA NA     NA
2  Plasticity =~          E   0.675 NA NA     NA
3   Stability =~          C   0.447 NA NA     NA
4   Stability =~          S   0.627 NA NA     NA
5   Stability =~          A   0.498 NA NA     NA
6         gfp =~ Plasticity   0.997 NA NA     NA
7         gfp =~  Stability   0.527 NA NA     NA
8           O ~~          O   0.921 NA NA     NA
9           E ~~          E   0.544 NA NA     NA
10          C ~~          C   0.800 NA NA     NA
11          S ~~          S   0.607 NA NA     NA
12          A ~~          A   0.752 NA NA     NA
13 Plasticity ~~ Plasticity   0.006 NA NA     NA
14  Stability ~~  Stability   0.722 NA NA     NA
15        gfp ~~        gfp   1.000 NA NA     NA
```

```
> se.modg <- 'Plasticity =~ O + E
+            Stability =~ C + S + A
+            gfp =~   .62* Plasticity  + Stabilit
+           '
> fit.se <- cfa(se.modg,sample.cov=es,
  sample.nobs=628640,std.lv=TRUE)
> summary(fit.se,fit.measures=TRUE)
> standardizedsolution(fit.se)

 Estimator
  Minimum Function Chi-square                  61
  Degrees of freedom
  P-value
          lhs op        rhs est.std se  z pvalue
1  Plasticity =~          O   0.281 NA NA     NA
2  Plasticity =~          E   0.675 NA NA     NA
3   Stability =~          C   0.447 NA NA     NA
4   Stability =~          S   0.627 NA NA     NA
5   Stability =~          A   0.498 NA NA     NA
6         gfp =~ Plasticity   0.527 NA NA     NA
7         gfp =~  Stability   0.997 NA NA     NA
8           O ~~          O   0.921 NA NA     NA
9           E ~~          E   0.544 NA NA     NA
10          C ~~          C   0.800 NA NA     NA
11          S ~~          S   0.607 NA NA     NA
12          A ~~          A   0.752 NA NA     NA
13 Plasticity ~~ Plasticity   0.722 NA NA     NA
14  Stability ~~  Stability   0.006 NA NA     NA
15        gfp ~~        gfp   1.000 NA NA     NA
```

Finding ω from the models

```
> lm
  Plasticity Stability
O      0.281     0.000
E      0.675     0.000
A      0.000     0.447
C      0.000     0.627
S      0.000     0.498

> fm
           model 1 model 2 model 3 model 4
Plasticity   0.527   0.997   0.707   0.743
Stability    0.997   0.527   0.743   0.707

> round(lm %*% fm,2)
  model 1 model 2 model 3 model 4
O    0.15    0.28    0.20    0.21
E    0.36    0.67    0.48    0.50
A    0.45    0.24    0.33    0.32
C    0.63    0.33    0.47    0.44
S    0.50    0.26    0.37    0.35

> round( colSums(lm%*%fm)^2/sum(es[-6,-6]),2)
model 1 model 2 model 3 model 4
   0.50    0.37    0.40    0.39
```

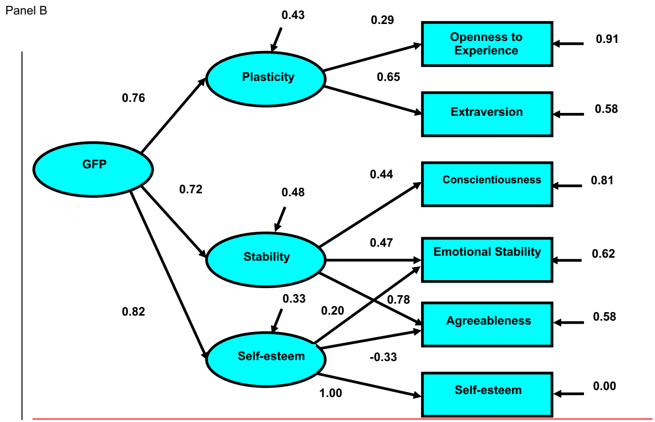
1. The factor loadings
2. The g loadings
3. % g for each item
4. $\omega = \sum(g)^2/V_t$

Compare with EFA omega

```
om <-omega(es[-6,-6],2)
> print(om,cut=.1)

Omega
Call: omega(m = es[-6, -6], nfactors = 2)
Alpha:                0.52
G.6:                  0.48
Omega Hierarchical:   0.31
Omega H asymptotic:   0.49
Omega Total           0.64

Schmid Leiman Factor loadings greater than  0.1
     g   F1*  F2*   h2   u2   p2
O 0.15  0.13      0.04 0.96 0.49
E 0.58  0.81      1.00 0.00 0.33
A 0.30       0.45 0.29 0.71 0.31
C 0.26       0.41 0.24 0.76 0.29
S 0.36       0.43 0.32 0.68 0.40

With eigenvalues of:
   g  F1*  F2*
0.64 0.69 0.56
```

```
general/max  0.93   max/min =   1.22
mean percent general =  0.36    with sd =  0.08 and cv of

The degrees of freedom are 1  and the fit is  0

The root mean square of the residuals is  0
The df corrected root mean square of the residuals is  0.

Compare this with the adequacy of just a general factor a
The degrees of freedom for just the general factor are 5

The root mean square of the residuals is  0.08
The df corrected root mean square of the residuals is  0.

Measures of factor score adequacy
                                                  g  F1*
Correlation of scores with factors             0.65 0.84
Multiple R square of scores with factors       0.42 0.70
Minimum correlation of factor score estimates -0.16 0.41
```

Erdle Model 2 – what does this mean wrt a general factor



Try with sem – one negative variance

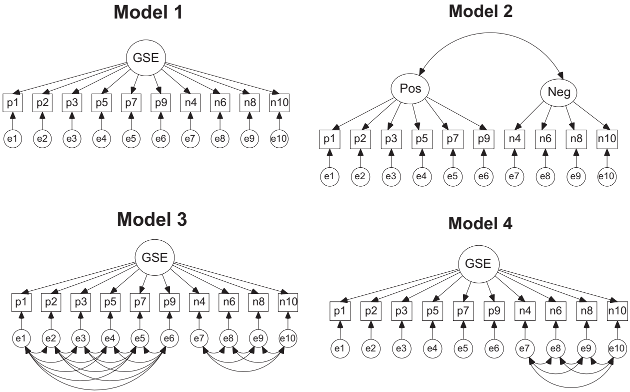
```
> se.modg <- 'Plasticity =~ O + E
+            Stability =~ C + S + A
+            Selfesteem =~ S + A + ES
+            gfp =~ Plasticity  + Stability + Selfesteem
+           '
> fit.se <- cfa(se.modg,sample.cov=es,sample.nobs=628640,std.lv=TRUE)
> summary(fit.se)

avaan (0.4-14) converged normally after 48 iterations

  Number of observations                          628640

  Estimator                                           ML
  Minimum Function Chi-square                   3370.726
  Degrees of freedom                                   4
  P-value                                          0.000

Parameter estimates:

  Information                                   Expected
  Standard Errors                               Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  Plasticity =~
    O                 0.193    0.002  128.052    0.000
    E                 0.423    0.004  115.929    0.000
  Stability =~
    C                 0.311    0.002  150.473    0.000
    S                 0.340    0.002  149.506    0.000
    A                 0.521    0.004  139.974    0.000
```

```
  Selfesteem =~
    S                 0.115    0.002   71.176
    A                -0.182    0.002  -75.568
    ES                0.639    0.009   68.334
  gfp =~
    Plasticity        1.155    0.011  104.436
    Stability         1.024    0.008  124.115
    Selfesteem        1.280    0.020   64.913

 Variances:
    O                 0.913    0.002
    E                 0.583    0.003
    C                 0.802    0.002
    S                 0.626    0.002
    A                 0.606    0.004
    ES               -0.079    0.012
    Plasticity        1.000
    Stability         1.000
    Selfesteem        1.000
    gfp               1.000
```

With standardardized loadings

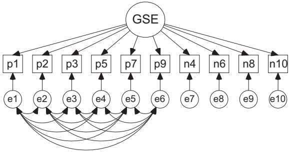
```
> standardizedsolution(fit.se)
          lhs op        rhs est.std se  z pvalue
1  Plasticity =~          O   0.294 NA NA     NA
2  Plasticity =~          E   0.645 NA NA     NA
3   Stability =~          C   0.445 NA NA     NA
4   Stability =~          S   0.486 NA NA     NA
5   Stability =~          A   0.745 NA NA     NA
6  Selfesteem =~          S   0.188 NA NA     NA
7  Selfesteem =~          A  -0.295 NA NA     NA
8  Selfesteem =~         ES   1.039 NA NA     NA
9         gfp =~ Plasticity   0.756 NA NA     NA
10        gfp =~  Stability   0.715 NA NA     NA
11        gfp =~ Selfesteem   0.788 NA NA     NA
12          O ~~          O   0.913 NA NA     NA
13          E ~~          E   0.583 NA NA     NA
14          C ~~          C   0.802 NA NA     NA
15          S ~~          S   0.626 NA NA     NA
16          A ~~          A   0.606 NA NA     NA
17         ES ~~         ES  -0.079 NA NA     NA
18 Plasticity ~~ Plasticity   0.429 NA NA     NA
19  Stability ~~  Stability   0.488 NA NA     NA
20 Selfesteem ~~ Selfesteem   0.379 NA NA     NA
21        gfp ~~        gfp   1.000 NA NA     NA
```

Four models of self esteem (from Marsh, Scalas & Nagengast (2010)

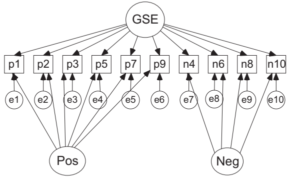


Four more models of self esteem (from Marsh et al. (2010))

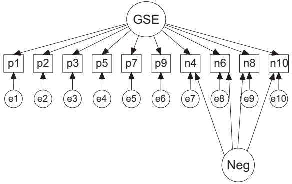
Model 5



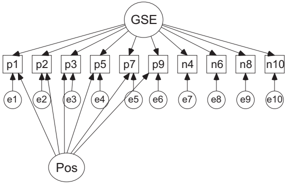
Model 6



Model 7



Model 8



Models 1 and 2 measurement invariance over 4 time points

Two correlated traits do better than one.

Confirmatory Factor Analyses and Invariance Tests

Model	χ^2	*df*	*cf*	*TLI*	*CFI*	*RMSEA*
Model 1 (one trait factor, no correlated uniqueness)						
Single-wave CFAs						
1.1 wave 1	651.57**	35		.700	.767	.089
1.2 wave 2	624.03**	35		.710	.775	.095
1.3 wave 3	539.94**	35		.752	.807	.090
1.4 wave 4	542.04**	35		.753	.808	.095
Longitudinal CFAs (Model 1.5)						
1.5a Unconstrained model (UM)	2,930.10**	674	1.203	.838	.860	.039
1.5b Factor loadings (FL)	2,971.55**	701	1.200	.843	.859	.038
1.5c FL & Variances (Var)	2,975.17**	704	1.199	.844	.859	.038
1.5d FL-Var-Uniquenesses (Uniq)	3,134.95**	724	1.203	.839	.850	.039
Model 2 (two trait factors: positive and negative correlated factors)						
Single-wave CFAs						
2.1 wave 1	111.70**	34		.961	.971	.032
2.2 wave 2	120.16**	34		.956	.967	.037
2.3 wave 3	161.46**	34		.936	.951	.046
2.4 wave 4	133.00**	34		.950	.962	.043
Longitudinal CFAs (Model 2.5)						
2.5a UM	1,116.35**	652	1.199	.965	.971	.018
2.5b FL	1,152.80**	676	1.194	.966	.970	.018
2.5c FL & Var	1,161.67**	682	1.194	.966	.970	.018
2.5d FL-Var-Uniq	1,313.80**	702	1.200	.958	.962	.020

Model 6 measurement invariance over 4 time points

But a general factor + two "method" factors is better. But is this really a method, or is there substance over and beyond general self esteem?

Model 6 (one trait factor plus positive and negative latent method factors)

Single-wave CFAs						
6.1 wave 1	69.62**	25		.970	.983	.028
6.2 wave 2	70.62**	25		.969	.983	.031
6.3 wave 3	88.48**	25		.956	.976	.038
6.4 wave 4	78.83**	25		.963	.980	.037
Longitudinal CFAs (Model 6.5)						
6.5–0 No correlations for the same method factor over time	1,483.37**	634	1.186	.935	.947	.025
6.5a UM	916.52**	622	1.182	.977	.982	.015
6.5b FL	962.06**	673	1.188	.979	.982	.014
6.5c FL & Var	1,000.90**	682	1.189	.977	.980	.015
6.5d FL-Var-Uniq	1,161.90**	702	1.196	.968	.971	.017

Is it a method or is it substance?

Is the stability over time of the positive and negative factors a sign of method effects being stable, or that there is content in the directionality of the answer?

	GSE1	GSE2	GSE3	GSE4	Pos1	Pos2	Pos3	Pos4	Neg1	Neg2	Neg3	Neg4
Model 6												
GSE1	—											
GSE2	.71	—										
GSE3	.64	.82	—									
GSE4	.55	.68	.80	—								
Pos1					—							
Pos2					.52	—						
Pos3					.48	.60	—					
Pos4					.43	.62	.60	—				
Neg1									—			
Neg2									.49	—		
Neg3									.39	.49	—	
Neg4									.39	.65	.60	—

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