Psychology 350: Special Topics
An introduction to R for psychological research
Latent variable models

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May, 2024

Outline

Models of data

1. Ockham's razor and data reduction
2. (MacCallum, 2004) "A factor analysis model is not an exact representation of real-world phenomena.
3. Always wrong to some degree, even in population.
4. At best, model is an approximation of real world."
5. Box (1979): "Models, of course, are never true, but fortunately it is only necessary that they be useful. For this it is usually needful only that they not be grossly wrong."
6. Tukey (1961): "In a single sentence, the moral is: Admit that complexity always increases, first from the model you fit to the data, thence to the model you use to think and plan about the experiment and its analysis, and thence to the true situation."

(From MacCallum, 2004); http://www.fa100.info/maccallum2.pdf

Observed Variables

X	Y
$\boxed{X_1}$	$\boxed{Y_1}$
$\boxed{X_2}$	$\boxed{Y_2}$
$\boxed{X_3}$	$\boxed{Y_3}$
$\boxed{X_4}$	$\boxed{Y_4}$
$\boxed{X_5}$	$\boxed{Y_5}$
$\boxed{X_6}$	$\boxed{Y_6}$

Latent Variables

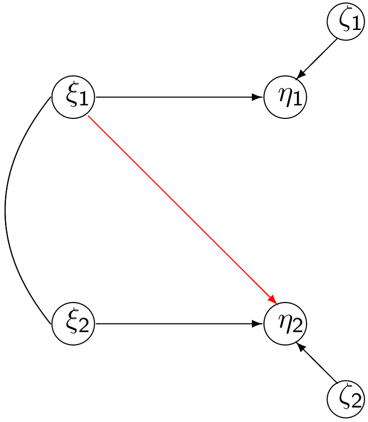
ξ η

ξ_1 η_1

ξ_2 η_2

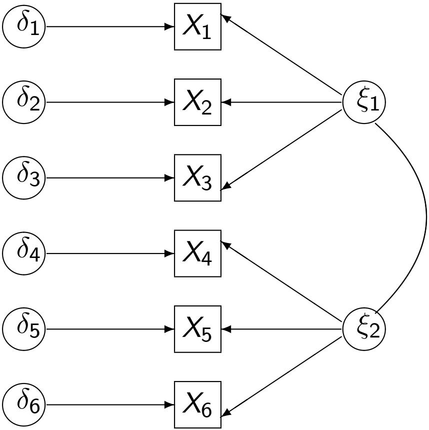
Theory: A regression model of latent variables

ξ η

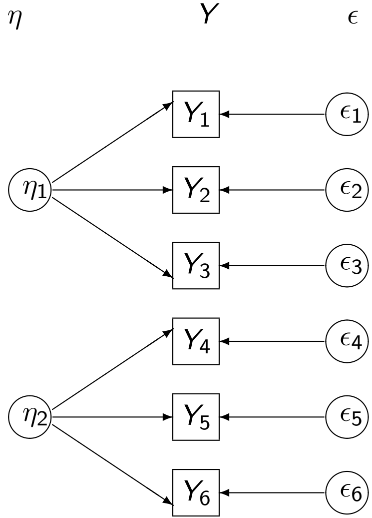


A measurement model for X

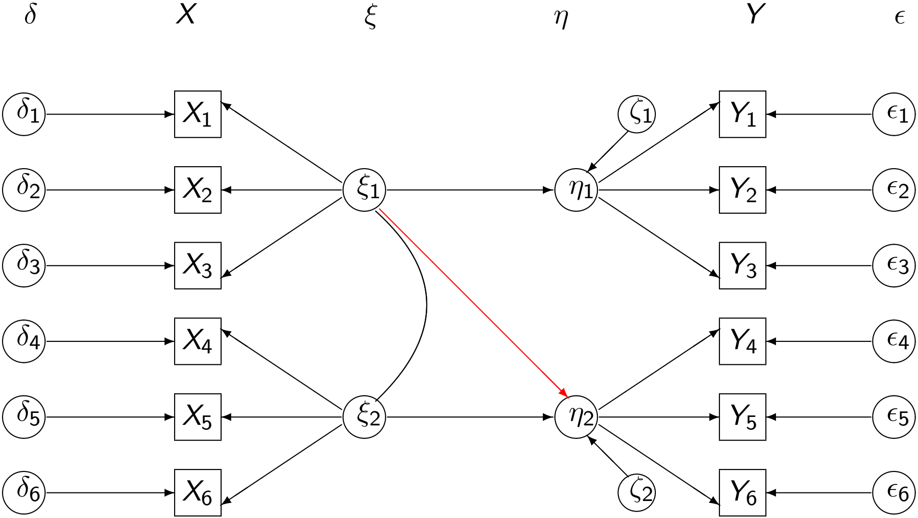
δ X ξ



A measurement model for Y



A complete structural model



Various measurement models

1. Observed variables models
   - Singular Value Decompostion
   - Eigen Value – Eigen Vector decomposition
   - Principal Components
   - First k principal components as an approximation
2. Latent variable models
   - Exploratory Factor analysis
   - Confirmatory Factor analysis
     - Growth Curve Models
     - Latent Class Models
   - Item Response Theory models
3. Interpretation of models
   - Choosing the appropriate number of components/factors
   - Transforming/rotating towards interpretable structures

See Loehlin & Beaujean (2017) for an excellent discussion of latent variable models. See also the syllabus for 454: latent variable models and for a general introduction to psychometric theory (Psychology 405),

Exploratory Factor Analysis (EFA) and Confirmatory approaches (CFA/SEM)

1. Traditional dimensional reduction was exploratory
2. Factors were estimated to reduce the complexity of the data
3. Goodness of fit, if reported, was merely an estimate of the size of residuals
4. Reliability theory allowed for correction for attenuation, but still exploratory
5. Regression models gave statistical measures of goodness of fit, but were hurt by unreliability of measurement.
6. Combining factor models of reliability with regression models of structure was developed and generically known as Structural Equation Models or Latent Variable Models

Latent Variable Modeling programs

- Commercial programs
  - LISREL/PRELIS (Jöreskog, 1978; Joreskog & Sorbom, 1993; Jöreskog & Sörbom, 1999) http://www.ssicentral.com/lisrel/techdocs/IPUG.pdf Users Manual
  - EQS (Bentler, 1995)
  - AMOS Arbuckle (1989, 1994) http://spss.wikia.com/wiki/SEM_(structural_equation_modeling)_-_Amos Amos wiki
  - MPLUS (Muthén & Muthén, 2007) http://www.statmodel.com/ugexcerpts.shtml User's guide with examples
- Open source
  - Mx (Neale, 1994)
  - OpenMx
  - sem (**?**)
  - lavaan (Rosseel, 2012)

lavaan 0.6.17 from CRAN. Description from the user's guide

http://lavaan.ugent.be

- The lavaan package is free open-source software. This means (among other things) that there is no warranty whatsoever.
- The numerical results of the lavaan package are typically very close, if not identical, to the results of the commercial package Mplus. If you wish to compare the results with other SEM packages, you can use the optional argument mimic="EQS" when calling the cfa, sem or growth functions
- The lavaan package is not finished yet. But it is already very useful for most users, or so we hope. There are a number of known minor issues and some features are simply not implemented yet.
- Some important features that have just been released in lavaan are:
    - support for hierarchical/multilevel datasets (multilevel cfa, multilevel sem)
    - support for discrete latent variables (mixture models, latent classes)

More about lavaan from the user's guide

1. We do not expect you to be an expert in R. In fact, the lavaan package is designed to be used by users that would normally never use R. Nevertheless, it may help to familiarize yourself a bit with R, just to be comfortable with it. Perhaps the most important skill that you may need to learn is how to import your own datasets (perhaps in an SPSS format) into R. There are many tutorials on the web to teach you just that. Once you have your data in R, you can start specifying your model. We have tried very hard to make it as easy as possible for users to fit their models. Of course, if you have suggestions on how we can improve things, please let us know.

Data sets available in lavaan

- HolzingerSwineford1939: A data frame with 301 observations of 15 variables.
  - The classic (Holzinger & Swineford, 1939) dataset consists of mental ability test scores of seventh- and eighth-grade children from two different schools (Pasteur and Grant-White). In the original dataset (available in the MBESS and *psychTools* packages, there are scores for 26 tests. However, a smaller subset with 9 variables is more widely used in the literature (for example in Joreskog's 1969 paper, which also uses the 145 subjects from the Grant-White school only.
- PoliticalDemocracy: A data frame of 75 observations of 11 variables.
  - The ''famous" Industrialization and Political Democracy dataset. This dataset is used throughout (Bollen, 1989) (see pages 12, 17, 36 in chapter 2, pages 228 and following in chapter 7, pages 321 and following in chapter 8). The dataset contains various measures of political democracy and industrialization in developing countries.

Lavaan syntax

surprisingly easy

```
Regression
y ~ f1 + f2 + x1 + x2
f1 ~ f2 + f3
f2 ~ f3 + x1 + x2

Latent variables
f1 =~ y1 + y2 + y3
f2 =~ y4 + y5 + y6
f3 =~ y7 + y8 + y9 + y10

Variances and covariances
y1 ~~ y1
y1 ~~ y2
f1 ~~ f2

Intercepts
y1 ~ 1
f1 ~ 1
```

Entering the model syntax as a string literal

```
myModel <- ' # regressions
y1 + y2 ~ f1 + f2 + x1 + x2
f1 ~ f2 + f3
f2 ~ f3 + x1 + x2
# latent variable definitions
f1 =~ y1 + y2 + y3
f2 =~ y4 + y5 + y6
f3 =~ y7 + y8 +
y9 + y10
# variances and covariances
y1 ~~ y1
y1 ~~ y2
f1 ~~ f2
# intercepts
y1 ~ 1
f1 ~ 1
'
#fit <- lavaan(myModel, mydata, ...}
```

The HolzingerSwineford data set

Using 9 tests from the 26 originally given to 301 7th and 8th graders. Rescaled to roughly equal ranges by transforming the original data. See holzinger.swinford help page in *psychTools* for a more full description of the data.

R code

```
HS.model <- '
visual =~ x1 + x2 + x3
 textual =~ x4 + x5 + x6
  speed =~ x7 + x8 + x9
 '
fit <- cfa(HS.model, data = HolzingerSwineford1939,std.lv=TRUE,
                std.ov=TRUE)

summary(fit, fit.measures = TRUE)
lavaan.diagram(fit)
```

Holzinger Swineford analysis

```
summary(fit, fit.measures = TRUE)
lavaan 0.6-3 ended normally after 21 iterations

  Optimization method                           NLMINB
  Number of free parameters                         21

  Number of observations                           301

  Estimator                                         ML
  Model Fit Test Statistic                      85.306
  Degrees of freedom                                24
  P-value (Chi-square)                           0.000

Model test baseline model:

  Minimum Function Test Statistic              918.852
  Degrees of freedom                                36
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.931
  Tucker-Lewis Index (TLI)                       0.896

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3422.624
  Loglikelihood unrestricted model (H1)      -3379.971

  Number of free parameters                         21
  Akaike (AIC)                                6887.248
  Bayesian (BIC)                              6965.097
  Sample-size adjusted Bayesian (BIC)         6898.497
```

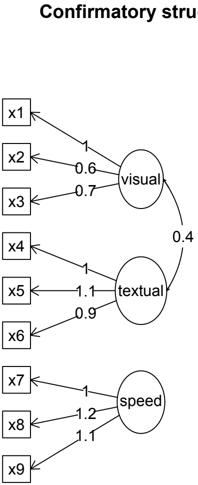
More goodness of fits

```
Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3422.624
  Loglikelihood unrestricted model (H1)      -3379.971

  Number of free parameters                         21
  Akaike (AIC)                                6887.248
  Bayesian (BIC)                              6965.097
  Sample-size adjusted Bayesian (BIC)         6898.497

Root Mean Square Error of Approximation:

  RMSEA                                          0.092
  90 Percent Confidence Interval          0.071  0.114
  P-value RMSEA <= 0.05                          0.001

Standardized Root Mean Square Residual:

  SRMR                                           0.065

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard
```

more parameters

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
    x1                0.771    0.069   11.127    0.000
    x2                0.423    0.066    6.429    0.000
    x3                0.580    0.066    8.817    0.000
  textual =~
    x4                0.850    0.049   17.474    0.000
    x5                0.854    0.049   17.576    0.000
    x6                0.837    0.049   17.082    0.000
  speed =~
    x7                0.569    0.064    8.903    0.000
    x8                0.722    0.065   11.090    0.000
    x9                0.664    0.064   10.305    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual ~~
    textual           0.459    0.064    7.189    0.000
    speed             0.471    0.073    6.461    0.000
  textual ~~
    speed             0.283    0.069    4.117    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                0.403    0.083    4.833    0.000
   .x2                0.818    0.073   11.146    0.000
   .x3                0.660    0.071    9.317    0.000
   .x4                0.274    0.035    7.779    0.000
   .x5                0.268    0.035    7.642    0.000
   .x6                0.297    0.036    8.277    0.000
   .x7                0.673    0.069    9.823    0.000
   .x8                0.476    0.072    6.573    0.000
```

Graphic output using lavaan.diagram



Redo with alternative parameterization

R code

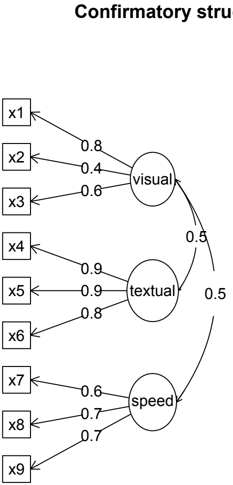
```
HS.model <- '
visual =~ x1 + x2 + x3
 textual =~ x4 + x5 + x6
 speed =~ x7 + x8 + x9
 '
fit <- cfa(HS.model, data = HolzingerSwineford1939,std.ov=TRUE,std.lv=TRUE)

summary(fit)   #shorter output
lavaan.diagram(fit)
```

```
lavaan 0.6-3 ended normally after 21 iterations

  Optimization method                           NLMINB
  Number of free parameters                         21

  Number of observations                           301

  Estimator                                         ML
  Model Fit Test Statistic                      85.306
  Degrees of freedom                                24
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard
```

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
    x1                0.771    0.069   11.127    0.000
    x2                0.423    0.066    6.429    0.000
    x3                0.580    0.066    8.817    0.000
  textual =~
    x4                0.850    0.049   17.474    0.000
    x5                0.854    0.049   17.576    0.000
    x6                0.837    0.049   17.082    0.000
  speed =~
    x7                0.569    0.064    8.903    0.000
    x8                0.722    0.065   11.090    0.000
    x9                0.664    0.064   10.305    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual ~~
    textual           0.459    0.064    7.189    0.000
    speed             0.471    0.073    6.461    0.000
  textual ~~
    speed             0.283    0.069    4.117    0.000
```

```
Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                0.403    0.083    4.833    0.000
   .x2                0.818    0.073   11.146    0.000
   .x3                0.660    0.071    9.317    0.000
   .x4                0.274    0.035    7.779    0.000
   .x5                0.268    0.035    7.642    0.000
   .x6                0.297    0.036    8.277    0.000
   .x7                0.673    0.069    9.823    0.000
   .x8                0.476    0.072    6.573    0.000
   .x9                0.556    0.069    8.003    0.000
    visual            1.000
    textual           1.000
    speed             1.000
```

The standardized solution to the Holzinger Swineford 1939 problem



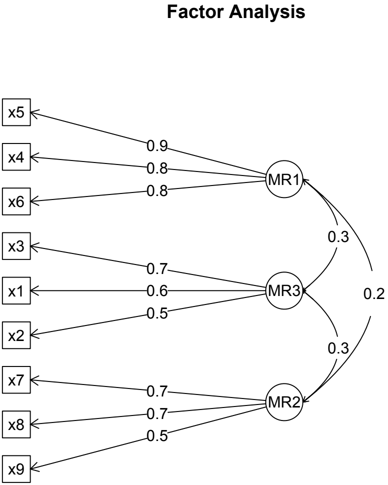
EFA of the Holzinger Swineford 1939 problem

```
f3 <- fa(HolzingerSwineford1939[7:15],3)
 f3
 diagram(f3,cut=.1)
```

```
Factor Analysis using method =  minres
Call: fa(r = HolzingerSwineford1939[7:15],
        nfactors = 3)
Standardized loadings (pattern matrix)
           based upon correlation matrix
     MR1   MR3   MR2   h2   u2 com
x1  0.19  0.60  0.03 0.49 0.51 1.2
x2  0.04  0.51 -0.12 0.25 0.75 1.1
x3 -0.07  0.69  0.02 0.46 0.54 1.0
x4  0.84  0.02  0.01 0.72 0.28 1.0
x5  0.89 -0.07  0.01 0.76 0.24 1.0
x6  0.81  0.08 -0.01 0.69 0.31 1.0
x7  0.04 -0.15  0.72 0.50 0.50 1.1
x8 -0.03  0.10  0.70 0.53 0.47 1.0
x9  0.03  0.37  0.46 0.46 0.54 1.9

                       MR1  MR3  MR2
SS loadings           2.24 1.34 1.28
Proportion Var        0.25 0.15 0.14
Cumulative Var        0.25 0.40 0.54
Proportion Explained  0.46 0.28 0.26
Cumulative Proportion 0.46 0.74 1.00

 With factor correlations of
     MR1  MR3  MR2
MR1 1.00 0.33 0.22
MR3 0.33 1.00 0.27
MR2 0.22 0.27 1.00
```

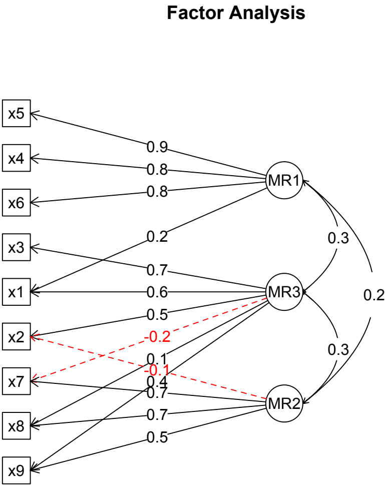
```
Mean item complexity =  1.2
Test of the hypothesis that 3 factors are sufficient.

The degrees of freedom for the null model are  36  and the
               objective function was  3.05 with Chi Square of  90
The degrees of freedom for the model are 12  and the
                          objective function was  0.08

The root mean square of the residuals (RMSR) is  0.02
The df corrected root mean square of the residuals is  0.03

The harmonic number of observations is  301 with the empirical
              chi square  8.03  with prob <  0.78
The total number of observations was  301  with Likelihood Chi
                       Square =  22.38  with prob <  0.034

Tucker Lewis Index of factoring reliability =  0.964
RMSEA index =  0.003  and the 90
              % confidence intervals are  0.003 0.088
BIC =  -46.11
Fit based upon off diagonal values = 1
Measures of factor score adequacy
                                                 MR1  MR3  MR2
Correlation of scores with factors              0.94 0.84 0.85
Multiple R square of scores with factors        0.89 0.71 0.72
Minimum correlation of possible factor scores   0.78 0.42 0.45
```

Holzinger Swineford EFA – compare with CFA



Holzinger Swineford EFA – simple is FALSE

diagram(f3,cut=.1,simple=FALSE)



Fixing parameters to simplify models

- Perhaps the greatest power of SEM type programs is the ability to fix parameters or to force equality constraints.
  - EFA allows all parameters to vary
  - CFA allows only certain parameters to vary
- Can fix covariances to be zero
- Can fix different paths to be equal

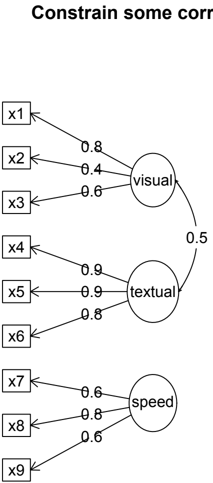
Two ways of fixing the covariances to be 0

```
HS.ortho <- '
# three-factor model
visual =~ x1 + x2 + x3
textual =~ x4 + x5 + x6
speed =~ NA*x7 + x8 + x9
# orthogonal factors
visual ~~ 0*speed
textual ~~ 0*speed
# fix variance of speed factor
speed ~~ 1*speed'
fit.hs.ortho <- cfa(HS.ortho, data=HolzingerSwineford1939,std.ov=TRUE,std.lv=TRUE)

or (if they are all to be zero)

HS.model <- ' visual =~ x1 + x2 + x3
 textual =~ x4 + x5 + x6
 speed =~ x7 + x8 + x9 '
 fit.HS.ortho <- cfa(HS.model, data=HolzingerSwineford1939, orthogonal=TRUE)
```

```
lavaan (0.5-22) converged normally after  20 iterations

  Number of observations                           301

  Estimator                                         ML
  Minimum Function Test Statistic              117.946
  Degrees of freedom                                26
  P-value (Chi-square)                           0.000
```

Holzinger Swineford EFA – simple is FALSE

diagram(f3,cut=.1,simple=FALSE)



Not as good a fit

```
> summary(fit.hs.ortho,fit.measures=TRUE)

lavaan (0.5-22) converged normally after  20 iterations

  Number of observations                           301

  Estimator                                         ML
  Minimum Function Test Statistic              117.946
  Degrees of freedom                                26
  P-value (Chi-square)                           0.000

Model test baseline model:

  Minimum Function Test Statistic              918.852
  Degrees of freedom                                36
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.896
  Tucker-Lewis Index (TLI)                       0.856

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3438.944
  Loglikelihood unrestricted model (H1)      -3379.971

  Number of free parameters                         19
  Akaike (AIC)                                6915.889
  Bayesian (BIC)                              6986.324
  Sample-size adjusted Bayesian (BIC)         6926.067

Root Mean Square Error of Approximation:

  RMSEA                                          0.108
  90 Percent Confidence Interval          0.089  0.129
  P-value RMSEA <= 0.05                          0.000

Standardized Root Mean Square Residual:

  SRMR                                           0.125

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
    x1                0.777    0.075   10.376    0.000
    x2                0.430    0.067    6.423    0.000
    x3                0.568    0.069    8.270    0.000
  textual =~
    x4                0.851    0.049   17.491    0.000
    x5                0.853    0.049   17.545    0.000
    x6                0.837    0.049   17.079    0.000
  speed =~
    x7                0.607    0.067    9.040    0.000
    x8                0.800    0.073   10.899    0.000
    x9                0.560    0.066    8.509    0.000
```

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)
visual ~~				
speed	0.000			
textual ~~				
speed	0.000			
visual ~~				
textual	0.461	0.064	7.195	0.000

Variances:

	Estimate	Std.Err	z-value	P(>|z|)
speed	1.000			
.x1	0.394	0.095	4.155	0.000
.x2	0.811	0.074	10.965	0.000
.x3	0.675	0.074	9.085	0.000
.x4	0.273	0.035	7.735	0.000
.x5	0.269	0.035	7.662	0.000
.x6	0.297	0.036	8.263	0.000
.x7	0.629	0.073	8.650	0.000
.x8	0.357	0.094	3.794	0.000
.x9	0.683	0.071	9.640	0.000
visual	1.000			
textual	1.000			

Fixing starting values

(If you have a problem with a solution, you can help it if you give it a reasonable starting location.)

```
visual =~ x1 + start(0.8)*x2 + start(1.2)*x3
textual =~ x4 + start(0.5)*x5 + start(1.0)*x6
speed =~ x7 + start(0.7)*x8 + start(1.8)*x9
```

This technique works for all SEM programs (although details vary). The reason to give good starting values is that the search optimization can get bogged down in the wrong part of the parameter space.

Automatic naming

R code

```
model <- '
# latent variable definitions
ind60 =~ x1 + x2 + x3
dem60 =~ y1 + y2 + y3 + y4
dem65 =~ y5 + y6 + y7 + y8
# regressions
dem60 ~ ind60
dem65 ~ ind60 + dem60
# residual (co)variances
y1 ~~ y5
y2 ~~ y4 + y6
y3 ~~ y7
y4 ~~ y8
y6 ~~ y8
'
fit <- sem(model, data=PoliticalDemocracy)
coef(fit)
```

```
coef(fit)
   ind60=~x2   ind60=~x3   dem60=~y2   dem60=~y3   dem60=~y4   dem65=~y6   dem65=~y7
       2.180       1.819       1.257       1.058       1.265       1.186       1.280
   dem65=~y8 dem60~ind60 dem65~ind60 dem65~dem60      y1~~y5      y2~~y4      y2~~y6
       1.266       1.483       0.572       0.837       0.624       1.313       2.153
      y3~~y7      y4~~y8      y6~~y8      x1~~x1      x2~~x2      x3~~x3      y1~~y1
       0.795       0.348       1.356       0.082       0.120       0.467       1.891
      y2~~y2      y3~~y3      y4~~y4      y5~~y5      y6~~y6      y7~~y7      y8~~y8
```

Specifying the name

R code

```
model <- '
 # latent variable definitions
 ind60 =~ x1 + x2 + label("myLabel")*x3
dem60 =~ y1 + y2 + y3 + label("anotherLabel")*y4
dem65 =~ y5 + y6 + y7 + y8
 # regressions
dem60 ~ ind60
dem65 ~ ind60 + dem60
 # residual (co)variances
y1 ~~ y5
y2 ~~ y4 + y6
 y3 ~~ y7
 y4 ~~ y8
 y6 ~~ y8
 '
 fit <- sem(model, data=PoliticalDemocracy)
 coef(fit)
```

```
coef(fit)
   ind60=~x2      myLabel    dem60=~y2    dem60=~y3 anotherLabel    dem65=~y6    dem65=~y7
       2.180        1.819        1.257        1.058        1.265        1.186        1.280
   dem65=~y8  dem60~ind60  dem65~ind60  dem65~dem60       y1~~y5       y2~~y4       y2~~y6
       1.266        1.483        0.572        0.837        0.624        1.313        2.153
      y3~~y7       y4~~y8       y6~~y8       x1~~x1       x2~~x2       x3~~x3       y1~~y1
       0.795        0.348        1.356        0.082        0.120        0.467        1.891
      y2~~y2       y3~~y3       y4~~y4       y5~~y5       y6~~y6       y7~~y7       y8~~y8
```

Using names to specify equality constraints

R code

```
model <- '
visual =~ x1 + x2 + equal("visual =~x2") *x3
textual =~ x4 + x5 + x6
speed =~ x7 + x8 + x9'
 fit <- sem(model, data=HolzingerSwineford1939)
 coef(fit)
```

```
lavaan 0.6-3 ended normally after 36 iterations

  Optimization method                           NLMINB
  Number of free parameters                         21
  Number of equality constraints                     1

  Number of observations                           301

  Estimator                                         ML
  Model Fit Test Statistic                      87.971
  Degrees of freedom                                25
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
```

With loadings

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
    x1                1.000
    x2      (.p2.)    0.649    0.088    7.355    0.000
    x3      (v=~2)    0.649    0.088    7.355    0.000
  textual =~
    x4                1.000
    x5                1.113    0.065   17.019    0.000
    x6                0.926    0.055   16.705    0.000
  speed =~
    x7                1.000
    x8                1.182    0.165    7.150    0.000
    x9                1.075    0.150    7.157    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual ~~
    textual           0.414    0.074    5.613    0.000
    speed             0.259    0.056    4.617    0.000
  textual ~~
    speed             0.173    0.049    3.510    0.000
```

Estimate the means

R code

```
HS.model.means <- '
# three-factor model
visual =~ x1 + x2 + x3
textual =~ x4 + x5 + x6
speed =~ x7 + x8 + x9
# intercepts
x1 ~ 1
x2 ~ 1
x3 ~ 1
x4 ~ 1
x5 ~ 1
x6 ~ 1
x7 ~ 1
x8 ~ 1
x9 ~ 1'
fit.means <- cfa(HS.model.means,data = HolzingerSwineford1939,std.lv=
TRUE)
summary(fit.means,fit.measures=TRUE)
or
fit <- cfa(HS.model, data = HolzingerSwineford1939, std.lv=TRUE,
               meanstructure = TRUE)
summary(fit)
```

Show the variable means (intercepts)

```
 Number of observations                           301

  Estimator                                         ML
  Minimum Function Test Statistic               85.306
  Degrees of freedom                                24
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
    x1                0.900    0.081   11.127    0.000
    x2                0.498    0.077    6.429    0.000
    x3                0.656    0.074    8.817    0.000
  textual =~
    x4                0.990    0.057   17.474    0.000
    x5                1.102    0.063   17.576    0.000
    x6                0.917    0.054   17.082    0.000
  speed =~
    x7                0.619    0.070    8.903    0.000
    x8                0.731    0.066   11.090    0.000
```

```
Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                4.936    0.067   73.473    0.000
   .x2                6.088    0.068   89.855    0.000
   .x3                2.250    0.065   34.579    0.000
   .x4                3.061    0.067   45.694    0.000
   .x5                4.341    0.074   58.452    0.000
   .x6                2.186    0.063   34.667    0.000
   .x7                4.186    0.063   66.766    0.000
   .x8                5.527    0.058   94.854    0.000
   .x9                5.374    0.058   92.546    0.000
    visual            0.000
    textual           0.000
    speed             0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                0.549    0.114    4.833    0.000
   .x2                1.134    0.102   11.146    0.000
   .x3                0.844    0.091    9.317    0.000
   .x4                0.371    0.048    7.778    0.000
   .x5                0.446    0.058    7.642    0.000
   .x6                0.356    0.043    8.277    0.000
   .x7                0.799    0.081    9.823    0.000
   .x8                0.488    0.074    6.573    0.000
   .x9                0.566    0.071    8.003    0.000
    visual            1.000
    textual           1.000
    speed             1.000
```

Show the parameter estimates for the meanstructure=TRUE

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
    x1                0.900    0.081   11.127    0.000
    x2                0.498    0.077    6.429    0.000
    x3                0.656    0.074    8.817    0.000
  textual =~
    x4                0.990    0.057   17.474    0.000
    x5                1.102    0.063   17.576    0.000
    x6                0.917    0.054   17.082    0.000
  speed =~
    x7                0.619    0.070    8.903    0.000
    x8                0.731    0.066   11.090    0.000
    x9                0.670    0.065   10.305    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual ~~
    textual           0.459    0.064    7.189    0.000
    speed             0.471    0.073    6.461    0.000
  textual ~~
    speed             0.283    0.069    4.117    0.000
```

```
Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                4.936    0.067   73.473    0.000
   .x2                6.088    0.068   89.855    0.000
   .x3                2.250    0.065   34.579    0.000
   .x4                3.061    0.067   45.694    0.000
   .x5                4.341    0.074   58.452    0.000
   .x6                2.186    0.063   34.667    0.000
   .x7                4.186    0.063   66.766    0.000
   .x8                5.527    0.058   94.854    0.000
   .x9                5.374    0.058   92.546    0.000
    visual            0.000
    textual           0.000
    speed             0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                0.549    0.114    4.833    0.000
   .x2                1.134    0.102   11.146    0.000
   .x3                0.844    0.091    9.317    0.000
   .x4                0.371    0.048    7.778    0.000
   .x5                0.446    0.058    7.642    0.000
   .x6                0.356    0.043    8.277    0.000
   .x7                0.799    0.081    9.823    0.000
   .x8                0.488    0.074    6.573    0.000
   .x9                0.566    0.071    8.003    0.000
    visual            1.000
    textual           1.000
    speed             1.000
```

More useful if we want to fix some intercepts to be different from others

R code

```
# three-factor model
model <- '
visual =~ x1 + x2 + x3
textual =~ x4 + x5 + x6
speed =~ x7 + x8 + x9
# intercepts with fixed values
x1 ~ 0.5*1
x2 ~ 0.5*1
x3 ~ 0.5*1
x4 ~ 0.5*1
'
fit.means <- cfa(model,data = HolzingerSwineford1939,std.lv=TRUE)
summary(fit.means)
```

Some means are fixed

```
avaan 0.6-3 ended normally after 56 iterations

  Optimization method                           NLMINB
  Number of free parameters                         26

  Number of observations                           301

  Estimator                                         ML
  Model Fit Test Statistic                    1120.542
  Degrees of freedom                                28
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
    x1                4.538    0.189   23.986    0.000
    x2                5.554    0.239   23.209    0.000
    x3                1.837    0.094   19.519    0.000
  textual =~
    x4                2.748    0.117   23.426    0.000
    x5                2.928    0.126   23.275    0.000
    x6                2.446    0.106   23.144    0.000
  speed =~
    x7                1.269    0.075   17.020    0.000
    x8                1.501    0.075   20.124    0.000
    x9                1.332    0.071   18.641    0.000
```

Fixed means, continued

```
Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual ~~
    textual           0.946    0.007  132.571    0.000
    speed             0.895    0.015   57.960    0.000
  textual ~~
    speed             0.859    0.019   45.232    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                0.500
   .x2                0.500
   .x3                0.500
   .x4                0.500
   .x5                1.625    0.050   32.530    0.000
   .x6               -0.083    0.043   -1.932    0.053
   .x7                3.083    0.061   50.440    0.000
   .x8                4.222    0.056   75.567    0.000
   .x9                4.216    0.056   75.037    0.000
    visual            0.000
    textual           0.000
    speed             0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                0.442    0.105    4.214    0.000
   .x2                1.758    0.208    8.439    0.000
   .x3                0.964    0.083   11.677    0.000
   .x4                0.355    0.045    7.915    0.000
   .x5                0.463    0.055    8.479    0.000
   .x6                0.361    0.041    8.891    0.000
   .x7                0.791    0.076   10.380    0.000
   .x8                0.473    0.061    7.731    0.000
```

Analyzing multiple groups

- When studying differences in ages, gender, school, it is useful to be able to model them separately, but to get an overall goodness of fit.
  - Does a basic structure hold in different groups?
- More importantly, we can ask if the parameters in the two groups are the same. That is, we can add equality constraints.
- We can examine equality of the loadings, equality of the covariances, equality of the mean structure.

```
 HS.model <- ' visual =~ x1 + x2 + x3
 textual =~ x4 + x5 + x6
 speed =~ x7 + x8 + x9 '
 fit <- cfa(HS.model, data=HolzingerSwineford1939, group="school")
summary(fit)
```

```
lavaan (0.5-22) converged normally after  57 iterations

  Number of observations per group
  Pasteur                                          156
  Grant-White                                      145

  Estimator                                         ML
  Minimum Function Test Statistic              115.851
  Degrees of freedom                                48
  P-value (Chi-square)                           0.000

Chi-square for each group:

  Pasteur                                       64.309
  Grant-White                                   51.542

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard
```

Group 1 [Pasteur]:

```
                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  visual =~
    x1                1.000
    x2                0.394    0.122    3.220    0.001
    x3                0.570    0.140    4.076    0.000
  textual =~
    x4                1.000
    x5                1.183    0.102   11.613    0.000
    x6                0.875    0.077   11.421    0.000
  speed =~
    x7                1.000
    x8                1.125    0.277    4.057    0.000
    x9                0.922    0.225    4.104    0.000

Covariances:
  visual ~~
    textual           0.479    0.106    4.531    0.000
    speed             0.185    0.077    2.397    0.017
  textual ~~
    speed             0.182    0.069    2.628    0.009

Variances:
    x1                0.298    0.232    1.286    0.198
    x2                1.334    0.158    8.464    0.000
    x3                0.989    0.136    7.271    0.000
    x4                0.425    0.069    6.138    0.000
    x5                0.456    0.086    5.292    0.000
    x6                0.290    0.050    5.780    0.000
    x7                0.820    0.125    6.580    0.000
    x8                0.510    0.116    4.406    0.000
    x9                0.680    0.104    6.516    0.000
    visual            1.097    0.276    3.967    0.000
```

Group 2 [Grant-White]:

```
                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  visual =~
    x1                1.000
    x2                0.736    0.155    4.760    0.000
    x3                0.925    0.166    5.584    0.000
  textual =~
    x4                1.000
    x5                0.990    0.087   11.418    0.000
    x6                0.963    0.085   11.377    0.000
  speed =~
    x7                1.000
    x8                1.226    0.187    6.569    0.000
    x9                1.058    0.165    6.429    0.000

Covariances:
  visual ~~
    textual           0.408    0.098    4.153    0.000
    speed             0.276    0.076    3.639    0.000
  textual ~~
    speed             0.222    0.073    3.022    0.003

Variances:
    x1                0.715    0.126    5.675    0.000
    x2                0.899    0.123    7.339    0.000
    x3                0.557    0.103    5.409    0.000
    x4                0.315    0.065    4.870    0.000
    x5                0.419    0.072    5.812    0.000
    x6                0.406    0.069    5.880    0.000
    x7                0.600    0.091    6.584    0.000
    x8                0.401    0.094    4.248    0.000
    x9                0.535    0.089    6.010    0.000
    visual            0.604    0.160    3.762    0.000
```

Multiple groups, multiple constraints

- Can constrain a single parameter to be equal across groups
  - Use the naming convention and the equal command
  - Or, just use the same name for the two parameters
- Can constrain equivalent parameters across groups to be equal (group.equal)

Equal loadings across groups

```
HS.model <- ' visual =~ x1 + x2 + x3
 textual =~ x4 + x5 + x6
 speed =~ x7 + x8 + x9 '
fit <- cfa(HS.model, data=HolzingerSwineford1939, group="school",
 group.equal=c("loadings"),std.lv=TRUE)
 summary(fit)
llavaan (0.5-22) converged normally after  30 iterations

  Number of observations per group
  Pasteur                                            156
  Grant-White                                        145

  Estimator                                           ML
  Minimum Function Test Statistic                127.834
  Degrees of freedom                                  57
  P-value (Chi-square)                             0.000

Chi-square for each group:

  Pasteur                                         71.064
  Grant-White                                     56.770

Parameter Estimates:
```

```
Group 1 [Pasteur]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
    x1      (.p1.)    0.866    0.078   11.149    0.000
    x2      (.p2.)    0.523    0.076    6.916    0.000
    x3      (.p3.)    0.683    0.071    9.689    0.000
  textual =~
    x4      (.p4.)    0.954    0.056   17.002    0.000
    x5      (.p5.)    1.033    0.061   17.012    0.000
    x6      (.p6.)    0.870    0.052   16.750    0.000
  speed =~
    x7      (.p7.)    0.630    0.066    9.500    0.000
    x8      (.p8.)    0.752    0.065   11.586    0.000
    x9      (.p9.)    0.650    0.064   10.205    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual ~~
    textual           0.485    0.087    5.555    0.000
    speed             0.341    0.109    3.126    0.002
  textual ~~
    speed             0.336    0.094    3.590    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                4.941    0.092   53.661    0.000
   .x2                5.984    0.099   60.420    0.000
   .x3                2.487    0.093   26.734    0.000
   .x4                2.823    0.093   30.400    0.000
   .x5                3.995    0.100   39.756    0.000
   .x6                1.922    0.081   23.732    0.000
   .x7                4.432    0.089   49.903    0.000
```

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
    x1      (.p1.)    0.866    0.078   11.149    0.000
    x2      (.p2.)    0.523    0.076    6.916    0.000
    x3      (.p3.)    0.683    0.071    9.689    0.000
  textual =~
    x4      (.p4.)    0.954    0.056   17.002    0.000
    x5      (.p5.)    1.033    0.061   17.012    0.000
    x6      (.p6.)    0.870    0.052   16.750    0.000
  speed =~
    x7      (.p7.)    0.630    0.066    9.500    0.000
    x8      (.p8.)    0.752    0.065   11.586    0.000
    x9      (.p9.)    0.650    0.064   10.205    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual ~~
    textual           0.536    0.084    6.423    0.000
    speed             0.524    0.096    5.449    0.000
  textual ~~
    speed             0.341    0.093    3.670    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                4.930    0.098   50.330    0.000
   .x2                6.200    0.091   68.092    0.000
   .x3                1.996    0.086   23.284    0.000
   .x4                3.317    0.092   35.898    0.000
   .x5                4.712    0.100   47.104    0.000
   .x6                2.469    0.091   27.206    0.000
   .x7                3.921    0.083   47.100    0.000
```

Equal loadings and means across groups

```
HS.model <- ' visual =~ x1 + x2 + x3
 textual =~ x4 + x5 + x6
 speed =~ x7 + x8 + x9 '
fit <- cfa(HS.model, data=HolzingerSwineford1939,
        group="school",
 group.equal=c("loadings","means"),std.lv=TRUE)
summary(fit,fit.measures=TRUE)
```

```
  Number of observations per group
  Pasteur                                          156
  Grant-White                                      145

  Estimator                                         ML
  Minimum Function Test Statistic              127.834
  Degrees of freedom                                57
  P-value (Chi-square)                           0.000

Chi-square for each group:

  Pasteur                                       71.064
  Grant-White                                   56.770

Model test baseline model:

  Minimum Function Test Statistic              957.769
  Degrees of freedom                                72
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.920
  Tucker-Lewis Index (TLI)                       0.899

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3688.189
  Loglikelihood unrestricted model (H1)      -3624.272

  Number of free parameters                         51
  Akaike (AIC)                                7478.377
  Bayesian (BIC)                              7667.440
  Sample-size adjusted Bayesian (BIC)         7505.697

Root Mean Square Error of Approximation:

  RMSEA                                          0.091
  90 Percent Confidence Interval          0.070  0.112
  P-value RMSEA <= 0.05                          0.001

Standardized Root Mean Square Residual:

  SRMR                                           0.079

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard
```

```
Group 1 [Pasteur]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
    x1      (.p1.)    0.866    0.078   11.149    0.000
    x2      (.p2.)    0.523    0.076    6.916    0.000
    x3      (.p3.)    0.683    0.071    9.689    0.000
  textual =~
    x4      (.p4.)    0.954    0.056   17.002    0.000
    x5      (.p5.)    1.033    0.061   17.012    0.000
    x6      (.p6.)    0.870    0.052   16.750    0.000
  speed =~
    x7      (.p7.)    0.630    0.066    9.500    0.000
    x8      (.p8.)    0.752    0.065   11.586    0.000
    x9      (.p9.)    0.650    0.064   10.205    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual ~~
    textual           0.485    0.087    5.555    0.000
    speed             0.341    0.109    3.126    0.002
  textual ~~
    speed             0.336    0.094    3.590    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                4.941    0.092   53.661    0.000
   .x2                5.984    0.099   60.420    0.000
   .x3                2.487    0.093   26.734    0.000
   .x4                2.823    0.093   30.400    0.000
   .x5                3.995    0.100   39.756    0.000
   .x6                1.922    0.081   23.732    0.000
   .x7                4.432    0.089   49.903    0.000
```

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
    x1      (.p1.)    0.866    0.078   11.149    0.000
    x2      (.p2.)    0.523    0.076    6.916    0.000
    x3      (.p3.)    0.683    0.071    9.689    0.000
  textual =~
    x4      (.p4.)    0.954    0.056   17.002    0.000
    x5      (.p5.)    1.033    0.061   17.012    0.000
    x6      (.p6.)    0.870    0.052   16.750    0.000
  speed =~
    x7      (.p7.)    0.630    0.066    9.500    0.000
    x8      (.p8.)    0.752    0.065   11.586    0.000
    x9      (.p9.)    0.650    0.064   10.205    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual ~~
    textual           0.536    0.084    6.423    0.000
    speed             0.524    0.096    5.449    0.000
  textual ~~
    speed             0.341    0.093    3.670    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                4.930    0.098   50.330    0.000
   .x2                6.200    0.091   68.092    0.000
   .x3                1.996    0.086   23.284    0.000
   .x4                3.317    0.092   35.898    0.000
   .x5                4.712    0.100   47.104    0.000
   .x6                2.469    0.091   27.206    0.000
   .x7                3.921    0.083   47.100    0.000
```

Do the measures measure the same construct across groups?

- Is the configuration the same?
  - Most abstract level of invariance – "are the arrows the same"
- Weak invariance – are the loadings the same?
- Strong invariance – equal loadings + intercepts

Use the *semTools* package and the `measurementInvariance` function. (Although `measurementInvariance` has been *deprecated* and we are told to use `measEq.syntax` instead. See the very thoughtful helppage for `measEq.syntax` a great deal of detail.

Testing for measurement invariance

```
measurementInvariance(HS.model, data = HolzingerSwineford1939,
 group = "school")
Measurement invariance models:

Model 1 : fit.configural
Model 2 : fit.loadings
Model 3 : fit.intercepts
Model 4 : fit.means

Chi Square Difference Test

               Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
fit.configural 48 7484.4 7706.8 115.85
fit.loadings   54 7480.6 7680.8 124.04      8.192       6     0.2244
fit.intercepts 60 7508.6 7686.6 164.10     40.059       6  4.435e-07 ***
fit.means      63 7543.1 7710.0 204.61     40.502       3  8.338e-09 ***
---
Signif. codes:  0 ?***? 0.001 ?**? 0.01 ?*? 0.05 ?.? 0.1 ? ? 1


Fit measures:

                 cfi rmsea cfi.delta rmsea.delta
fit.configural 0.923 0.097        NA          NA
fit.loadings   0.921 0.093     0.002       0.004
fit.intercepts 0.882 0.107     0.038       0.015
fit.means      0.840 0.122     0.042       0.015
```

Data.growth: A toy data set

1. t1 Measured value at time point 1
2. t2 Measured value at time point 2
3. t3 Measured value at time point 3
4. t4 Measured value at time point 4
5. x1 Predictor 1 influencing intercept and slope
6. x2 Predictor 2 influencing intercept and slope
7. c1 Time-varying covariate time point 1
8. c2 Time-varying covariate time point 2
9. c3 Time-varying covariate time point 3
10. c4 Time-varying covariate time point 4

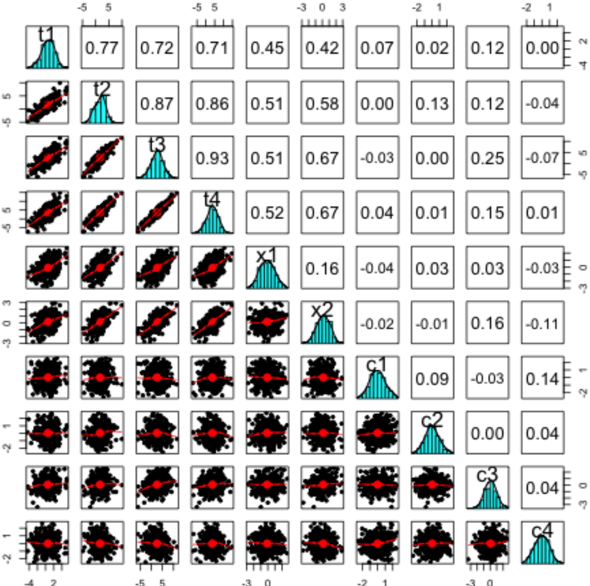
Demo.growth : A toy data set

A toy dataset containing measures on 4 time points (t1,t2, t3 and t4), two predictors (x1 and x2) influencing the random intercept and slope, and a time-varying covariate (c1, c2, c3 and c4).

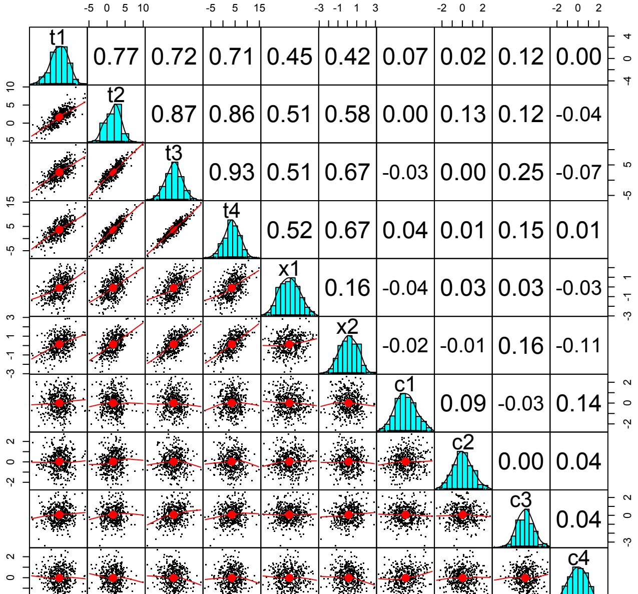
```
data(Demo.growth)
describe(Demo.growth)
pairs.panels(Demo.growth)
```

	var	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
t1	1	400	0.59	1.58	0.67	0.64	1.50	-4.35	5.21	9.56	-0.18	0.23	0.08
t2	2	400	1.67	2.13	1.88	1.69	2.10	-4.86	9.95	14.81	-0.02	0.48	0.11
t3	3	400	2.59	2.72	2.73	2.62	2.62	-6.02	11.53	17.55	-0.14	0.46	0.14
t4	4	400	3.64	3.38	3.72	3.69	3.14	-7.34	14.72	22.06	-0.13	0.44	0.17
x1	5	400	-0.09	1.03	-0.08	-0.11	1.11	-2.82	2.72	5.54	0.08	-0.33	0.05
x2	6	400	0.14	0.96	0.13	0.15	1.01	-2.83	2.88	5.71	-0.12	0.01	0.05
c1	7	400	0.01	0.99	-0.03	-0.01	0.98	-2.58	2.57	5.14	0.11	-0.28	0.05
c2	8	400	0.03	0.95	0.01	0.02	0.87	-2.54	2.71	5.25	0.13	-0.07	0.05
c3	9	400	0.07	0.93	0.07	0.06	0.93	-3.40	2.61	6.01	-0.02	0.38	0.05
c4	10	400	-0.02	0.92	-0.01	-0.02	0.95	-2.45	2.65	5.11	0.02	-0.21	0.05

Growth: SPLOM



The Demo.growth data set - A cleaner graphic



Fitting a growth model to the toy problem

Intercepts and slopes

```
model <- ' i =~ 1*t1 + 1*t2 + 1*t3 + 1*t4
  s =~ 0*t1 + 1*t2 + 2*t3 + 3*t4 '
 fit <- growth(model, data=Demo.growth)
summary(fit)
```

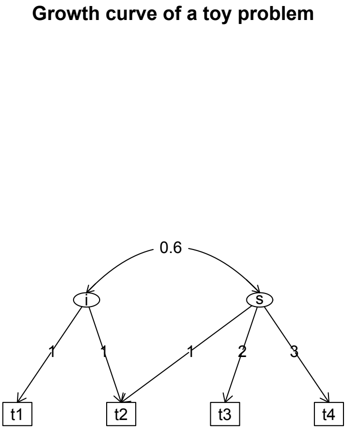
```
lavaan 0.6-3 ended normally after 29 iterations

  Optimization method                           NLMINB
  Number of free parameters                          9

  Number of observations                           400

  Estimator                                         ML
  Model Fit Test Statistic                       8.069
  Degrees of freedom                                 5
  P-value (Chi-square)                           0.152

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard
```

Growth model with parameter values

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  i =~
    t1                1.000
    t2                1.000
    t3                1.000
    t4                1.000
  s =~
    t1                0.000
    t2                1.000
    t3                2.000
    t4                3.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  i ~~
    s                 0.618    0.071    8.686    0.000
```

```
Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .t1                0.000
   .t2                0.000
   .t3                0.000
   .t4                0.000
    i                 0.615    0.077    8.007    0.000
    s                 1.006    0.042   24.076    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .t1                0.595    0.086    6.944    0.000
   .t2                0.676    0.061   11.061    0.000
   .t3                0.635    0.072    8.761    0.000
   .t4                0.508    0.124    4.090    0.000
    i                 1.932    0.173   11.194    0.000
    s                 0.587    0.052   11.336    0.000
```

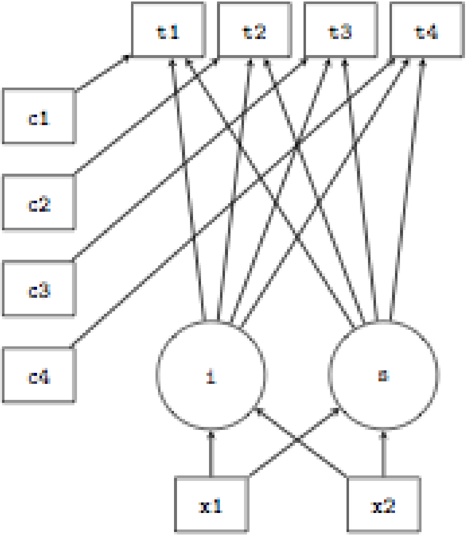
A toy growth problem



From the lavaan manual

- "Technically, the growth function is almost identical to the sem function. But a meanstructure is automatically assumed, and the observed intercepts are fixed to zero by default, while the latent variable intercepts and means are freely estimated.
- A slightly more complex model adds two regressors (x1 and x2) that influence the latent growth factors.
- In addition, a time-varying covariate that influences the outcome measure at the four time points has been added to the model.
- A graphical representation of this model together with the corresponding lavaan syntax is presented".

A growth model



Linear growth with time-varying covariates

```
# a linear growth model with a time-varying covariate
model <- '
# intercept and slope with fixed coefficients
i =~ 1*t1 + 1*t2 + 1*t3 + 1*t4
s =~ 0*t1 + 1*t2 + 2*t3 + 3*t4
# regressions
i ~ x1 + x2
s ~ x1 + x2
# time-varying covariates
t1 ~ c1
t2 ~ c2
t3 ~ c3
t4 ~ c4
'
fit <- growth(model, data=Demo.growth)
summary(fit,fit.measures=TRUE)
lavaan (0.5-22) converged normally after  31 iterations

  Number of observations                           400

  Estimator                                         ML
  Minimum Function Test Statistic               26.059
  Degrees of freedom                                21
  P-value (Chi-square)                           0.204
```

Parameters of the growth model

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  i =~
    t1                1.000
    t2                1.000
    t3                1.000
    t4                1.000
  s =~
    t1                0.000
    t2                1.000
    t3                2.000
    t4                3.000

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  i ~
    x1                0.608    0.060   10.134    0.000
    x2                0.604    0.064    9.412    0.000
  s ~
    x1                0.262    0.029    9.198    0.000
    x2                0.522    0.031   17.083    0.000
  t1 ~
    c1                0.143    0.050    2.883    0.004
  t2 ~
    c2                0.289    0.046    6.295    0.000
  t3 ~
    c3                0.328    0.044    7.361    0.000
  t4 ~
    c4                0.330    0.058    5.655    0.000
```

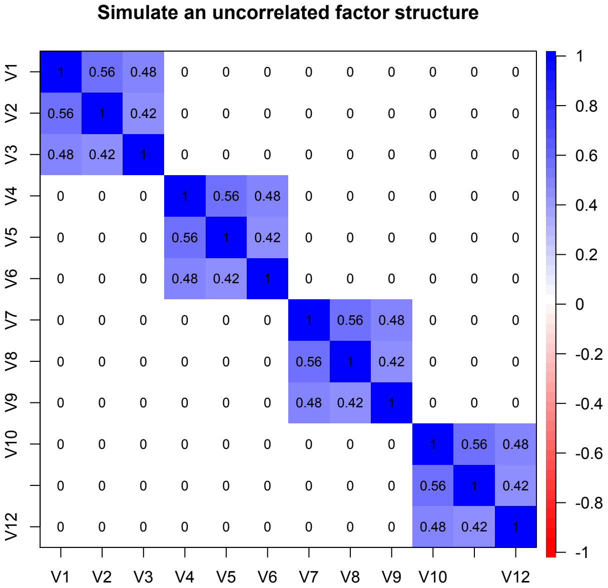
```
Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
 .i ~~
   .s                 0.075    0.040    1.855    0.064

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .t1                0.000
   .t2                0.000
   .t3                0.000
   .t4                0.000
   .i                 0.580    0.062    9.368    0.000
   .s                 0.958    0.029   32.552    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .t1                0.580    0.080    7.230    0.000
   .t2                0.596    0.054   10.969    0.000
   .t3                0.481    0.055    8.745    0.000
   .t4                0.535    0.098    5.466    0.000
   .i                 1.079    0.112    9.609    0.000
   .s                 0.224    0.027    8.429    0.000
```

State Trait Auto Regressive Structure

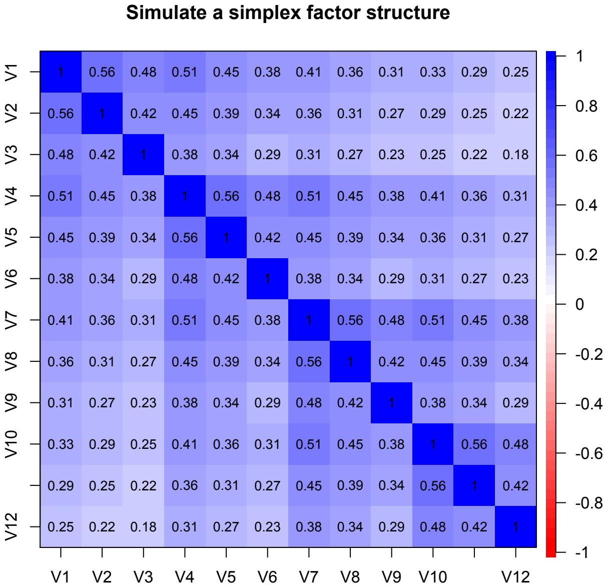
The `sim()` function (in *psych*)

- **fx** The structure of the x factors (defaults to 1 factor with loadings of .8, .7, .6
- **Phi** Inter factor correlation matrix
- **fy** Structure of the y factors (defaults to fx) item[fy]
- **alpha** The autoregressive correlation over time
- **lambda** The stability of the traits over time
- **mu** The means structure
- **n** Number of simulated cases item [raw] If TRUE and $n > 0$, report the data

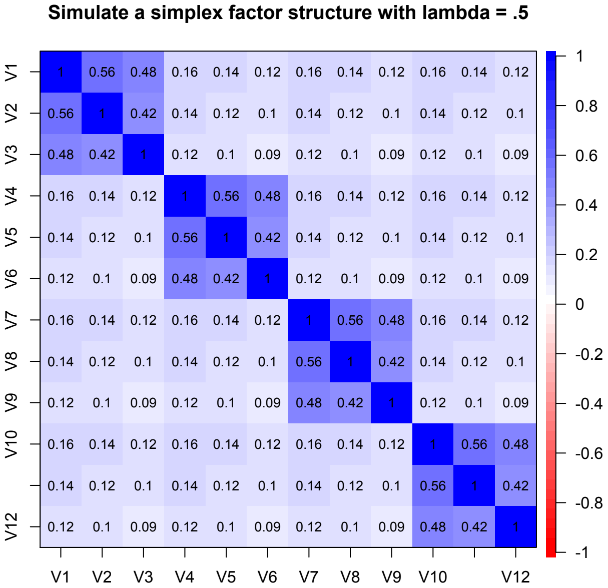
A (non)simplex factor structure, $\alpha = .0\lambda = 0$



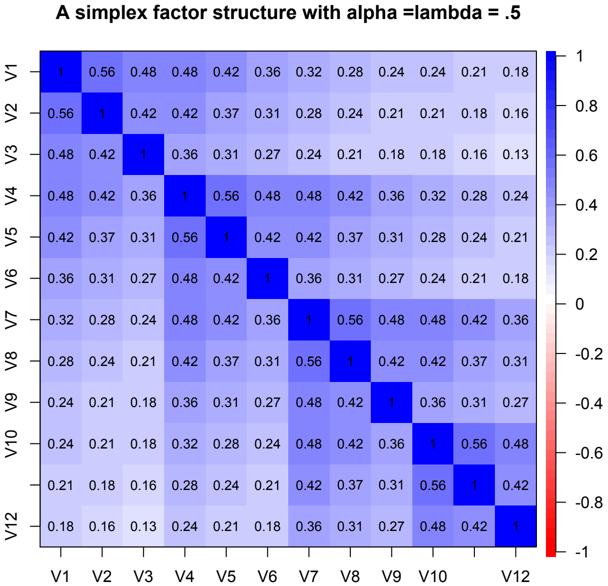
An autoregressive (simplex) factor structure, $\alpha = .8 \lambda = 0$



A stable factor structure, $\alpha = .0 \lambda = .5$



A simplex factor structure, $\alpha = .5\lambda = 5$



Testing various STAR models

R code

```
stars <- sim(alpha=.5,lambda=.5)
starsMod <- '
F1 =~ a*V1 + b*V2 +c * V3
F2 =~  a*V4 + b* V5 + c * V6
F3 =~ a * V7 + b* V8  + c* V9
F4 =~ a * V10 + b*V11 + c * V12
'
fitstars <- cfa(starsMod,sample.cov=stars$model,sample.nobs=1000,std.lv=TRUE)
summary(fitstars)
```

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  F1 =~
    V1         (a)    0.800    0.018   45.282    0.000
    V2         (b)    0.700    0.017   40.199    0.000
    V3         (c)    0.600    0.017   34.558    0.000
  ...
Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  F1 ~~
    F2                0.750    0.025   29.605    0.000
    F3                0.500    0.034   14.911    0.000
    F4                0.375    0.037   10.160    0.000
  F2 ~~
    F3                0.750    0.025   29.616    0.000
    F4                0.500    0.034   14.911    0.000
  F3 ~~
    F4                0.750    0.025   29.605    0.000
```

Testing a reduced model STAR models

R code

```
stars55 <- sim(alpha=.5,lambda=.5)
starsMod55<- '
F1 =~ a*V1 + b*V2 +c * V3
F2 =~  a*V4 + b* V5 + c * V6
F3 =~ a * V7 + b* V8  + c* V9
F4 =~ a * V10 + b*V11 + c * V12
'
fitstars <- cfa(starsMod55,sample.cov=stars55$model,sample.nobs=1000,std.lv=TR
summary(fitstars)
```

```
lavaan (0.5-22) converged normally after  20 iterations

  Number of observations                          1000

  Estimator                                         ML
  Minimum Function Test Statistic                0.000
  Degrees of freedom                                57
  P-value (Chi-square)                           1.000
```

Testing various STAR models

R code

```
stars55 <- sim(alpha=.5,lambda=.5)
starsMod55<- '
F1 =~ a*V1 + b*V2 +c * V3
F2 =~  a*V4 + b* V5 + c * V6
F3 =~ a * V7 + b* V8  + c* V9
F4 =~ a * V10 + b*V11 + c * V12
g =~ L* F1 + L*F2 + L*F3 + L*F4
F2 ~~ al *F1
F3 ~~ al * F2
F4 ~~al* F3
F3 ~~ al2 * F1
F4 ~~ F1
F4 ~~ al2* F2

'
fitstars <- cfa(starsMod55,sample.cov=stars55$model,sample.nobs=1000,std.lv=TR
summary(fitstars)
```

```
lavaan (0.5-22) converged normally after  18 iterations

  Number of observations                          1000

  Estimator                                         ML
  Minimum Function Test Statistic               20.473
  Degrees of freedom                                61
  P-value (Chi-square)                           1.000
```

Modifying a model

- There are many reasons a model does not fit.
  - In particular, some paths may be badly fit (usually because they were ignored).
  - How much will a parameter change (Expected Parameter Change) if a parameter is adjusted.

Modification indices for the HS problem

```
fit <- cfa(HS.model, data = HolzingerSwineford1939)
 mi <- modindices(fit)
mi[mi$op == "=~", ]
```

```
       lhs op rhs     mi    epc sepc.lv sepc.all
1   visual =~  x1     NA     NA      NA       NA
2   visual =~  x2  0.000  0.000   0.000    0.000
3   visual =~  x3  0.000  0.000   0.000    0.000
4   visual =~  x4  1.211  0.077   0.069    0.059
5   visual =~  x5  7.441 -0.210  -0.189   -0.147
6   visual =~  x6  2.843  0.111   0.100    0.092
7   visual =~  x7 18.631 -0.422  -0.380   -0.349
8   visual =~  x8  4.295 -0.210  -0.189   -0.187
9   visual =~  x9 36.411  0.577   0.519    0.515
10 textual =~  x1  8.903  0.350   0.347    0.297
11 textual =~  x2  0.017 -0.011  -0.011   -0.010
12 textual =~  x3  9.151 -0.272  -0.269   -0.238
13 textual =~  x4     NA     NA      NA       NA
14 textual =~  x5  0.000  0.000   0.000    0.000
15 textual =~  x6  0.000  0.000   0.000    0.000
16 textual =~  x7  0.098 -0.021  -0.021   -0.019
17 textual =~  x8  3.359 -0.121  -0.120   -0.118
18 textual =~  x9  4.796  0.138   0.137    0.136
19   speed =~  x1  0.014  0.024   0.015    0.013
20   speed =~  x2  1.580 -0.198  -0.123   -0.105
21   speed =~  x3  0.716  0.136   0.084    0.075
22   speed =~  x4  0.003 -0.005  -0.003   -0.003
23   speed =~  x5  0.201 -0.044  -0.027   -0.021
24   speed =~  x6  0.273  0.044   0.027    0.025
25   speed =~  x7     NA     NA      NA       NA
26   speed =~  x8  0.000  0.000   0.000    0.000
27   speed =~  x9  0.000  0.000   0.000    0.000
```

The fitted model

```
fit <- cfa(HS.model, data = HolzingerSwineford1939)
fitted(fit)
```

```
$cov
   x1    x2    x3    x4    x5    x6    x7    x8    x9
x1 1.358
x2 0.448 1.382
x3 0.590 0.327 1.275
x4 0.408 0.226 0.298 1.351
x5 0.454 0.252 0.331 1.090 1.660
x6 0.378 0.209 0.276 0.907 1.010 1.196
x7 0.262 0.145 0.191 0.173 0.193 0.161 1.183
x8 0.309 0.171 0.226 0.205 0.228 0.190 0.453 1.022
x9 0.284 0.157 0.207 0.188 0.209 0.174 0.415 0.490 1.015

$mean
x1 x2 x3 x4 x5 x6 x7 x8 x9
 0  0  0  0  0  0  0  0  0
```

Examine the raw residuals

```
fit <- cfa(HS.model, data = HolzingerSwineford1939)
 resid(fit)
```

```
$cov
   x1     x2     x3     x4     x5     x6     x7     x8     x9
x1  0.000
x2 -0.041  0.000
x3 -0.010  0.124  0.000
x4  0.097 -0.017 -0.090  0.000
x5 -0.014 -0.040 -0.219  0.008  0.000
x6  0.077  0.038 -0.032 -0.012  0.005  0.000
x7 -0.177 -0.242 -0.103  0.046 -0.050 -0.017  0.000
x8 -0.046 -0.062 -0.013 -0.079 -0.047 -0.024  0.082  0.000
x9  0.175  0.087  0.167  0.056  0.086  0.062 -0.042 -0.032  0.000

$mean
x1 x2 x3 x4 x5 x6 x7 x8 x9
 0  0  0  0  0  0  0  0  0
```

Examine the standardized residuals

```
fit <- cfa(HS.model, data = HolzingerSwineford1939)
 resid(fit, type = "standardized")
```

```
$cov
   x1     x2     x3     x4     x5     x6     x7     x8     x9
x1  0.000
x2 -2.196  0.000
x3 -1.199  2.692  0.000
x4  2.465 -0.283 -1.948     NA
x5 -0.362 -0.610 -4.443  0.856     NA
x6  2.032  0.661 -0.701     NA  0.633  0.000
x7 -3.787 -3.800 -1.882  0.839 -0.837 -0.321     NA
x8 -1.456 -1.137 -0.305 -2.049 -1.100 -0.635  3.804  0.000
x9  4.062  1.517  3.328  1.237  1.723  1.436 -2.771     NA  0.000
```

Many more examples

- The MPlus manual has data sets that may be explored with lavaan code.
    - Chapter 3: Regression and Path Analysis
    - Chapter 5: Confirmatory factor analysis and structural equation modeling
    - Chapter 6: Growth modeling
- LISREL manual also has suitable examples

SEM-AMOS wiki a warning

- It may seem odd to begin with a warning, but the popular misuse and misinterpretation of Structural Equation Modeling is so widespread that users of this wiki should be aware of some of the issues involved before they begin. While this warning is overly brief, you can follow-up these issues and more in the Further Reading section of this article.
- A number of these issues also apply to Confirmatory Factor Analysis. While Structural Equation Modeling has been popular in recent years to test the degree of fit between a proposed structural model and the emergent structure of the data, the perceived superiority of the technique is waning.
- Aside from the fact that the results of Structural Equation Modeling are often poorly reported, the conclusions drawn do not typically grasp the limitations of the technique.

SEM-AMOS wiki a warning page 2

- The most obvious, and some ways the most critical issue is that of incorrectly inferring a particular configuration of causal relationships from correlational data. This mistake can be illustrated with the simplest of all structural examples – that of 2 variables (variable A and B). If we ignore the additional complexity of latent structure, the number of possible causal structures is 4. Clearly, the number of possible models grows exponentially as the number of variables grows. In this example, the 4 possible causal models in this example are:
    - A causes B;
    - B causes A;
    - A and B cause each other (a recursive model);
    - finally, A and B are unrelated.

SEM-AMOS wiki warning page 3

- If A and B are indeed significantly correlated, it is likely that the first 3 models will be supported by significant fit statistics. If this is the case, what has been proven?
- Which of the 3 supported models is the correct model? What makes matters worse is that we have not even conclusively ruled out the last model. It is still possible that the correlation between A and B was spurious.
- To reinforce a maxim that most people know, but fail to apply to Structural Equation Modeling – you can not determine causation from correlation.
- Yet in most cases, researchers only test one or two models out of all the myriad of potential models, poorly report their results, then proclaim confirmation of their model (implying the exclusion of all other possible models).

SEM-AMOS wiki warning page 4

- So what is the value of Structural Equation Modeling?
- If large correlational datasets are already available, and a large range of plausible models are assessed, the results can be valuable in conceiving an experimental study that can test the proposed causal relationships.

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Holzinger, K. & Swineford, F. (1939). A study in factor analysis: the stability of a bi-factor solution. Department of Education, University of Chicago, Chicago, Il.

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