Psychology 350: An introduction to R for Psychological Research
Week 4 Multidimensional analysis

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https://personality-project.org/courses/350

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Outline

Fitting functions as way of testing theory

1. Closed form versus open form
2. Much of classical statistics is "closed form". That is, it is a matter of solving some equations using basic algebra.
   - Examples include the
   - t.test
   - F. test
   - basic linear regression
3. However, other functions are "open form" and have to be estimated using the process of iteration.
4. Although computers are useful for solving closed form equations, they are particularly useful for solving open forms.
5. They do this by optimizing some function over successive iterations.
6. This technique was actually developed by Isaac Newton (see Newton's method

The basic concept of fitting

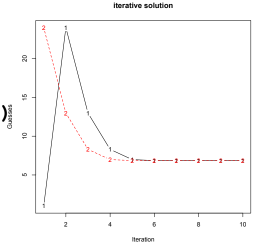
- Given some fit statistic, try to find the optimal values for that fit
  - Consider the case of the square root of 47
    - Guess X
    - find $X' = 47/Guess$
    - $\bar{X} = \frac{X+X'}{2}$use this in the next step
    - try a new $Guess = 47/(\bar{X})$
    - do it again
- The next example is a baby function to do this
- It is the concept, not the programming that is important

Iterative fitting to find a square root

R code

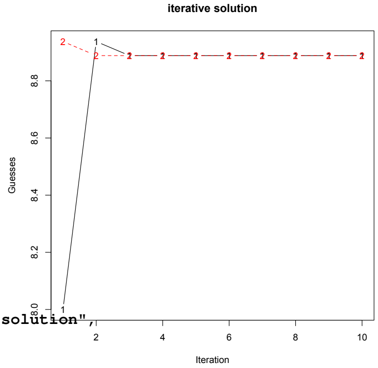
```
iterative.sqrt <- function(X,guess) {
#a dumb guess

if(missing(guess)) guess <- 1
 iterate <- function(X,guess) {
    low <- guess
    high <- X/guess
    guess <-((high+low)/2)
    guess}
Iter <- matrix(NA,ncol=3,nrow=10)
colnames(Iter) <- c("Guess",
      "X/Guess","Error")
for (i in 1:10) {  # a small loop
Iter[i,1] <- guess
Iter[i,2] <- guess <- iterate(X,guess)
Iter[i,3] <- Iter[i,1]- Iter[i,2]
}
Iter   } #this returns the value
```

```
> X <- 47
> iter <- iterative.sqrt(47)
iterative.sqrt(47)
          Guess    X/Guess         Error
 [1,]  1.000000 24.000000 -2.300000e+01
 [2,] 24.000000 12.979167  1.102083e+01
 [3,] 12.979167  8.300177  4.678989e+00
 [4,]  8.300177  6.981353  1.318824e+00
 [5,]  6.981353  6.856786  1.245673e-01
 [6,]  6.856786  6.855655  1.131507e-03
 [7,]  6.855655  6.855655  9.337605e-08
 [8,]  6.855655  6.855655  8.881784e-16
 [9,]  6.855655  6.855655  0.000000e+00
[10,]  6.855655  6.855655  0.000000e+00
 matplot(iter,typ="b",main="iterative solut
```



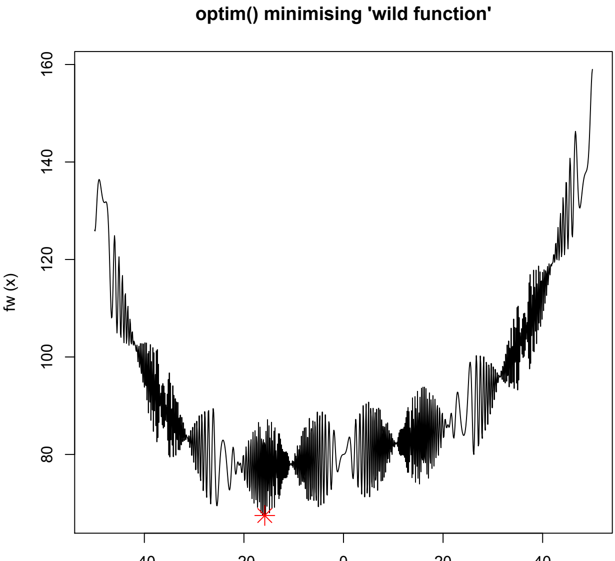
Iterative fitting to find a square root – a better guess

```
> X <- 79
> iter <- iterative.sqrt(X,8)
iter
         Guess  X/Guess         Error
 [1,] 8.000000 8.937500 -9.375000e-01
 [2,] 8.937500 8.888330  4.916958e-02
 [3,] 8.888330 8.888194  1.360012e-04
 [4,] 8.888194 8.888194  1.040501e-09
 [5,] 8.888194 8.888194  0.000000e+00
 [6,] 8.888194 8.888194  0.000000e+00
 [7,] 8.888194 8.888194  0.000000e+00
 [8,] 8.888194 8.888194  0.000000e+00
 [9,] 8.888194 8.888194  0.000000e+00
[10,] 8.888194 8.888194  0.000000e+00
```




```
> matplot(iter,typ="b",main="iterative solution",
xlab="Iteration",ylab="Guesses")
```

The optim function is one way to fit complex models



Iterative fitting has several steps

- Specifying a model and loss function
  - Ideally supplying a first derivative to the loss function
  - Can be done empirically
- Some way to evaluate the quality of the fit
- Minimization of function, but then how good is this minimum

Multivariate analysis

- Many procedures use this concept of iterative fitting
- Some, such as principal components are "closed form" and can just be solved directly
- Others, such as "factor analysis" need to find solutions by successive approximations
- The issue of factor or component rotation is an iterative procedure.
- Principal Components and Factor analysis are all the result of some basic matrix equations to approximate a matrix
- Conceptually, we are just taking the square root of a correlation matrix:
- $R \approx CC'$ or $R \approx FF' + U^2$ (u^2 is a diagonal matrix
- For any given correlation matrix, R, can we find a C or an F matrix which approximates it?

Consider the following matrix

What would be its "square root"? That is to say, what simpler matrix, times itself is equal to R?

```
R
     V1   V2   V3   V4
V1 1.00 0.56 0.48 0.40
V2 0.56 1.00 0.42 0.35
V3 0.48 0.42 1.00 0.30
V4 0.40 0.35 0.30 1.00
```

Find $R \approx FF' + U^2$

We create this matrix using the `sim.congeneric` function

First, create an example matrix

R code

```
"sim.congeneric" <-  function{
 (loads = c(0.8, 0.7, 0.6, 0.5),...)  {
..
n <- length(loads)
 loading <- matrix(loads, nrow = n)
error <- diag(1, nrow = n)
diag(error) <- sqrt(1 - loading^2)

 model <- pattern %*% t(pattern)

...
 result <- model
 return(result)
 }
```

1. Give some default values
2. Create a correlation matrix using matrix algebra
   - $model = \mathbf{FF'}$
   - $\text{diag(model)} = 1$
3. Return the value

A congeneric matrix is one with just one factor

R code

```
R <- sim.congeneric()
R
 f1 <- fa(R)
```

```
> R
     V1   V2   V3   V4
V1 1.00 0.56 0.48 0.40
V2 0.56 1.00 0.42 0.35
V3 0.48 0.42 1.00 0.30
V4 0.40 0.35 0.30 1.00
> f1 <- fa(R)
> f1
Factor Analysis using method =  minres
Call: fa(r = R)
Standardized loadings (pattern matrix) based upon correlation matrix
   MR1   h2   u2 com
V1 0.8 0.64 0.36   1
V2 0.7 0.49 0.51   1
V3 0.6 0.36 0.64   1
V4 0.5 0.25 0.75   1

                MR1
SS loadings    1.74
Proportion Var 0.43
```

But does this really fit?

Think about the residuals (Data - model)

R code

```
R
round(f1$model,2)
residuals(f1)
```

```
> R
     V1   V2   V3   V4
V1 1.00 0.56 0.48 0.40
V2 0.56 1.00 0.42 0.35
V3 0.48 0.42 1.00 0.30
V4 0.40 0.35 0.30 1.00
> round(f1$model,2)
     V1   V2   V3   V4
V1 0.64 0.56 0.48 0.40
V2 0.56 0.49 0.42 0.35
V3 0.48 0.42 0.36 0.30
V4 0.40 0.35 0.30 0.25
> residuals(f1)
   V1   V2   V3   V4
V1 0.36
V2 0.00 0.51
V3 0.00 0.00 0.64
V4 0.00 0.00 0.00 0.75
```

Hence we add the fudge factor U2 = diagonal of residuals and say $R = FF' + U2$

An example correlation matrix

Consider the following correlation matrix

	V1	V2	V3	V4	V5	V6
V1	1.00	0.72	0.63	0.54	0.45	0.36
V2	0.72	1.00	0.56	0.48	0.40	0.32
V3	0.63	0.56	1.00	0.42	0.35	0.28
V4	0.54	0.48	0.42	1.00	0.30	0.24
V5	0.45	0.40	0.35	0.30	1.00	0.20
V6	0.36	0.32	0.28	0.24	0.20	1.00

Is it possible to model these 36 correlations and variances with fewer terms? Yes, of course. The diagonal elements are all 1 and the off diagonal elements are symmetric. Thus, we have $n * (n - 1)$ correlations we want to model.

Eigen vector decomposition

Given a *nxn* matrix **R**, each eigenvector, $\mathbf{x}_i$, solves the equation

$$\mathbf{x}_i\mathbf{R} = \lambda_i\mathbf{x}_i$$

and the set of n eigenvectors are solutions to the equation

$$\mathbf{XR} = \boldsymbol{\lambda}\mathbf{X}$$

where **X** is a matrix of orthogonal eigenvectors and $\boldsymbol{\lambda}$ is a diagonal matrix of the the eigenvalues, λ_i. Then

$$\mathbf{x}_i\mathbf{R} - \lambda_i\mathbf{XI} = 0 <=> \mathbf{x}_i(\mathbf{R} - \lambda_i\mathbf{I}) = 0$$

Finding the eigenvectors and eigenvalues is computationally tedious, but may be done using the `eigen` function. That the vectors making up **X** are orthogonal means that

$$\mathbf{XX}' = \mathbf{I}$$

and because they form the *basis space* for **R** that

$$\mathbf{R} = \mathbf{X}\boldsymbol{\lambda}\mathbf{X}'.$$

Iterative principal axes factor analysis

Principal components represents a $n * n$ matrix in terms of the first k components. It attempts to reproduce all of the **R** matrix. *Factor analysis* on the other hand, attempts to model just the common part of the matrix, which means all of the off-diagonal elements and the common part of the diagonal (the *communalities*). The non-common part, the *uniquenesses*, are simply that which is left over. An easy to understand procedure is *principal axes* factor analysis. This is similar to principal components, except that it is done with a reduced matrix where the diagonals are the communalities. The communalities can either be specified a priori, estimated by such procedures as multiple linear regression, or found by iteratively doing an eigenvalue decomposition and repeatedly replacing the original 1s on the diagonal with the the value of 1 - u^2 where

$$\mathbf{U}^2 = diag(\mathbf{R} - \mathbf{FF}').$$

Principal axes as eigen values of a reduced matrix

That is, starting with the original correlation or covariance matrix, **R**, find the k largest principal components, reproduce the matrix using those principal components. Find the resulting residual matrix, $\mathbf{R}^*$ and uniqueness matrix, $\mathbf{U}^2$ by

$$\mathbf{R}^* = \mathbf{R} - \mathbf{F}\mathbf{F}' \tag{1}$$

$$\mathbf{U}^2 = diag(\mathbf{R}^*)$$

and then, for iteration i, find $\mathbf{R}_i$ by replacing the diagonal of the original **R** matrix with 1 - diag($\mathbf{U}^2$) found on the previous step. Repeat this process until the change from one iteration to the next is arbitrarily small.

Comparing 1 with 5 iterations

```
> f1 <- fa(R,1,fm='pa',max.iter=1)
> f1
> resid(f1)

Factor Analysis using method =  pa
Call: fa(r = R, nfactors = 1, max.iter = 1, fm = "pa")
Standardized loadings (pattern matrix)
    PA1   h2   u2
V1 0.86 0.74 0.26
V2 0.79 0.62 0.38
V3 0.70 0.48 0.52
V4 0.60 0.36 0.64
V5 0.50 0.25 0.75
V6 0.40 0.16 0.84

                PA1
SS loadings    2.62
Proportion Var 0.44

   V1   V2   V3   V4   V5   V6
V1 0.26
V2 0.04 0.38
V3 0.03 0.01 0.52
V4 0.02 0.01 0.00 0.64
V5 0.02 0.00 0.00 0.00 0.75
V6 0.01 0.00 0.00 0.00 0.00 0.84
```

```
> f1 <- fa(R,1,fm='pa',max.iter=5)
> f1
> resid(f1)

Factor Analysis using method =  pa
Call: fa(r = R, nfactors = 1, max.iter = 5, fm =
Standardized loadings (pattern matrix)
   PA1   h2   u2
V1 0.9 0.81 0.19
V2 0.8 0.64 0.36
V3 0.7 0.49 0.51
V4 0.6 0.36 0.64
V5 0.5 0.25 0.75
V6 0.4 0.16 0.84

                PA1
SS loadings    2.71
Proportion Var 0.45

   V1   V2   V3   V4   V5   V6
V1 0.19
V2 0.00 0.36
V3 0.00 0.00 0.51
V4 0.00 0.00 0.00 0.64
V5 0.00 0.00 0.00 0.00 0.75
V6 0.00 0.00 0.00 0.00 0.00 0.84
```

Several example data sets in psychTools

`ability` 16 items measuring cognitive ability from (Condon & Revelle, 2014) and described more fully in (Revelle, Dworak & Condon, 2020) and (Dworak, Revelle, Doebler & Condon, 2021).

`bfi` 25 personality items and 3 demographic variables from the SAPA project. The 25 items are thought to represent the "Big 5" CANOE dimensions.

`sai` 20 State Anxiety Items and 3 demographics from the PMC lab. Discussed in greater detail in (Revelle & Condon, 2019).

`msqR` 75 mood and 13 condition variables from the Motivational State Questionnaire collected in the PMC lab.

`spi` 135 personality items and 10 demographic variables from the SAPA Personality Questionnaire (Condon, 2018) collected using the SAPA-project.org web site.

The `ability` data set

1. 16 multiple choice ability items 1525 subjects taken from the Synthetic Aperture Personality Assessment (SAPA) web based personality assessment project are saved as iqitems. Those data are shown as examples of how to score multiple choice tests and analyses of response alternatives. When scored correct or incorrect, the data are useful for demonstrations of tetrachoric based factor analysis irt.fa and finding tetrachoric correlations.
2. 16 items were sampled from 80 items given as part of the SAPA project (Revelle, Wilt & Rosenthal, 2010) to develop online measures of ability. These 16 items reflect four lower order factors (verbal reasoning, letter series, matrix reasoning, and spatial rotations. These lower level factors all share a higher level factor ('g').

bfi

1. 25 personality self report items taken from the International Personality Item Pool (ipip.ori.org) were included as part of the Synthetic Aperture Personality Assessment SAPA web based personality assessment project. The data from 2800 subjects are included here as a demonstration set for scale construction, factor analysis, and Item Response Theory analysis. Three additional demographic variables (sex, education, and age) are also included.
2. The first 25 items are organized by five putative factors: Agreeableness, Conscientiousness, Extraversion, Neuroticism, and Opennness. The scoring key is created using make.keys, the scores are found using score.items.
3. These five factors are a useful example of using irt.fa to do Item Response Theory based latent factor analysis of the polychoric correlation matrix. The endorsement plots for each item, as well as the item information functions reveal that the items differ in their quality.
4. The item data were collected using a 6 point response scale: 1 Very Inaccurate 2 Moderately Inaccurate 3 Slightly Inaccurate 4 Slightly Accurate 5 Moderately Accurate 6 Very Accurate.

sai

1. State Anxiety was measured two-three times in 11 studies at the Personality-Motivation-Cognition laboratory. Here are item responses for 11 studies (9 repeated twice, 2 repeated three times). In all studies, the first occasion was before a manipulation. In some studies, caffeine, or movies or incentives were then given to some of the participants before the second and third STAI was given. In addition, Trait measures are available and included in the tai data set (3032 subjects).
2. The standard experimental study at the Personality, Motivation and Cognition (PMC) laboratory (Revelle & Anderson, 1998) was to administer a number of personality trait and state measures (e.g. the epi, msq, msqR and sai) to participants before some experimental manipulation of arousal/effort/anxiety. Following the manipulation (with a 30 minute delay if giving caffeine/placebo), some performance task was given, followed once again by measures of state arousal/effort/anxiety.
3. Here are the item level data on the sai (state anxiety) and the tai (trait anxiety). Scores on these scales may be found using the scoring keys. The affect data set includes pre and post scores for two studies (flat and maps) which manipulated state by using four types of movies.

msqR

1. Emotions may be described either as discrete emotions or in dimensional terms. The Motivational State Questionnaire (MSQ) was developed to study emotions in laboratory and field settings. The data can be well described in terms of a two dimensional solution of energy vs tiredness and tension versus calmness. Alternatively, this space can be organized by the two dimensions of Positive Affect and Negative Affect. Additional items include what time of day the data were collected and a few personality questionnaire scores. 3032 unique participants took the MSQ at least once, 2753 at least twice, 446 three times, and 181 four times. The 3032 participants also took the sai state anxiety inventory at the same time. Some studies manipulated arousal by caffeine, others manipulations included affect inducing movies.
2. The Motivational States Questionnaire (MSQ) is composed of 75 items, which represent the full affective space (Revelle & Anderson, 1998). The MSQ consists of 20 items taken from the Activation-Deactivation Adjective Check List (Thayer, 1989), 18 from the Positive and Negative Affect Schedule (PANAS,0(Watson, Clark & Tellegen, 1988) along with the affective circumplex items used by Larsen & Diener (1992) The response format was a four-point scale that corresponds to Russell & Carroll (1999) "ambiguous–likely-unipolar format" and that asks the respondents to indicate their current standing ("at this moment") with the following rating scale:
   0————————-1————————-2————————-3
   Not at all A little Moderately Very much

spi

1. The SPI (SAPA Personality Inventory) is a set of 135 items primarily selected from International Personality Item Pool (ipip.ori.org). This is an example data set collected using SAPA procedures the sapa-project.org web site. This data set includes 10 demographic variables as well. The data set with 4000 observations on 145 variables may be used for examples in scale construction and validation, as well as empirical scale construction to predict multiple criteria.
2. Using the data contributed by about 125,000 visitors to the https://SAPA-project.org website, David Condon has developed a hierarchical framework for assessing personality at two levels. The higher level has the familiar five factors that have been studied extensively in personality research since the 1980s – Conscientiousness, Agreeableness, Neuroticism, Openness, and Extraversion. The lower level has 27 factors that are considerably more narrow. These were derived based on administrations of about 700 public-domain IPIP items to 3 large samples. Condon describes these scales as being "empirically-derived" because relatively little theory was used to select the number of factors in the hierarchy and the items in the scale for each factor (to be clear, he means relatively little personality theory though he relied on quite a lot of sampling and statistical theory). You can read all about the procedures used to develop this framework in his book/manual. If you would like to reproduce these analyses, you can download the data files from Dataverse (links are also provided in the manual) and compile this script in R (he used knitR). Instructions are provided in the Preface to the manual.
3. This small subset of the data is provided for demonstration purposes.

The ability data set

R code

```
dim(ability)
describe(ability)
lowerCor(ability)
```

```
dim(ability)
[1] 1525   16
> dim(ability)
[1] 1525   16
> describe(ability)
          vars    n mean   sd median trimmed mad min max range  skew kurtosis   se
reason.4     1 1442 0.68 0.47      1    0.72   0   0   1     1 -0.75    -1.44 0.01
reason.16    2 1463 0.73 0.45      1    0.78   0   0   1     1 -1.02    -0.96 0.01
reason.17    3 1440 0.74 0.44      1    0.80   0   0   1     1 -1.08    -0.84 0.01
reason.19    4 1456 0.64 0.48      1    0.68   0   0   1     1 -0.60    -1.64 0.01
letter.7     5 1441 0.63 0.48      1    0.67   0   0   1     1 -0.56    -1.69 0.01
letter.33    6 1438 0.61 0.49      1    0.63   0   0   1     1 -0.43    -1.82 0.01
letter.34    7 1455 0.64 0.48      1    0.68   0   0   1     1 -0.59    -1.65 0.01
letter.58    8 1438 0.47 0.50      0    0.46   0   0   1     1  0.12    -1.99 0.01
matrix.45    9 1458 0.55 0.50      1    0.56   0   0   1     1 -0.20    -1.96 0.01
matrix.46   10 1470 0.57 0.50      1    0.59   0   0   1     1 -0.28    -1.92 0.01
matrix.47   11 1465 0.64 0.48      1    0.67   0   0   1     1 -0.57    -1.67 0.01
matrix.55   12 1459 0.39 0.49      0    0.36   0   0   1     1  0.45    -1.80 0.01
rotate.3    13 1456 0.20 0.40      0    0.13   0   0   1     1  1.48     0.19 0.01
rotate.4    14 1460 0.22 0.42      0    0.15   0   0   1     1  1.34    -0.21 0.01
rotate.6    15 1456 0.31 0.46      0    0.27   0   0   1     1  0.80    -1.35 0.01
rotate.8    16 1460 0.19 0.39      0    0.12   0   0   1     1  1.55     0.41 0.01
```

ability: descriptives, continued

```
 lowerCor(ability)
            rsn.4 rs.16 rs.17 rs.19 ltt.7 lt.33 lt.34 lt.58 mt.45 mt.46 mt.47 mt.55 rtt.3 rtt.4
reason.4    1.00
reason.16   0.28  1.00
reason.17   0.40  0.32  1.00
reason.19   0.30  0.25  0.34  1.00
letter.7    0.28  0.27  0.29  0.25  1.00
letter.33   0.23  0.20  0.26  0.25  0.34  1.00
letter.34   0.29  0.26  0.29  0.27  0.40  0.37  1.00
letter.58   0.29  0.21  0.29  0.25  0.33  0.28  0.32  1.00
matrix.45   0.25  0.18  0.20  0.22  0.20  0.20  0.21  0.19  1.00
matrix.46   0.25  0.18  0.24  0.18  0.24  0.23  0.27  0.21  0.33  1.00
matrix.47   0.24  0.24  0.27  0.23  0.27  0.23  0.30  0.23  0.24  0.23  1.00
matrix.55   0.16  0.15  0.16  0.15  0.14  0.17  0.14  0.23  0.21  0.14  0.21  1.00
rotate.3    0.23  0.16  0.17  0.18  0.18  0.17  0.19  0.24  0.16  0.15  0.20  0.18  1.00
rotate.4    0.25  0.20  0.20  0.21  0.23  0.21  0.21  0.27  0.17  0.17  0.20  0.18  0.53  1.00
rotate.6    0.25  0.20  0.27  0.19  0.20  0.21  0.19  0.26  0.15  0.20  0.18  0.17  0.43  0.45
rotate.8    0.21  0.16  0.18  0.16  0.13  0.14  0.15  0.22  0.16  0.15  0.17  0.19  0.43  0.44
>
```

How many factors – no right answer, one wrong answer

1. Statistical
   - Extracting factors until the χ^2 of the residual matrix is not significant.
   - Extracting factors until the change in χ^2 from factor n to factor n+1 is not significant.
2. Rules of Thumb
   - Parallel Extracting factors until the eigenvalues of the real data are less than the corresponding eigenvalues of a random data set of the same size (*parallel analysis*)
   - Plotting the magnitude of the successive eigenvalues and applying the *scree test*.
3. Interpretability
   - Extracting factors as long as they are interpretable.
   - Using the *Very Simple Structure* Criterion (VSS)
   - Using the Minimum Average Partial criterion (MAP).
4. Eigen Value of 1 rule

One way to test techniques it simulation

1. With real data, we do not know ‘truth’
2. With simulated data, we can know the right answer
3. Lets examine the sim item function
   - defaults to certain parameter values
   - processes the data (make it up)
   - returns values
4. Simulations can default to certain values but allow you to specify other values

sim.item

```
 sim.item <-       #define the function and the default parameters
function (nvar = 72, nsub = 500, circum = FALSE, xloading = 0.6,
    yloading = 0.6, gloading = 0, xbias = 0, ybias = 0, categorical = FALSE,
    low = -3, high = 3, truncate = FALSE, cutpoint = 0)
{
    avloading <- (xloading + yloading)/2
    errorweight <- sqrt(1 - (avloading^2 + gloading^2))
    g <- rnorm(nsub)    #create a general factor random vector
    truex <- rnorm(nsub) * xloading + xbias  #create 2 true scores
    truey <- rnorm(nsub) * yloading + ybias
    if (circum) {   #do we want circumplex variables or simple structured?
        radia <- seq(0, 2 * pi, len = nvar + 1)
        rad <- radia[which(radia < 2 * pi)]
    }
    else rad <- c(rep(0, nvar/4), rep(pi/2, nvar/4), rep(pi,
        nvar/4), rep(3 * pi/2, nvar/4))
    error <- matrix(rnorm(nsub * (nvar)), nsub)   #make up some errors
    trueitem <- outer(truex, cos(rad)) + outer(truey, sin(rad))
    item <- gloading * g + trueitem + errorweight * error
    if (categorical) {                #process alternative options
        item = round(item)
        item[(item <= low)] <- low
        item[(item > high)] <- high
    }
    if (truncate) {
        item[item < cutpoint] <- 0
    }
    colnames(item) <- paste("V", 1:nvar, sep = "")
    return(item)
}
```

Simulate 2 factor data

Using the sim.item function

```
>  set.seed(42)  #to generate a reproducible example
>  my.data <- sim.item(12)
>  my.cor <- cor(my.data)
> round(my.cor,2)
       V1    V2    V3    V4    V5    V6    V7    V8    V9   V10   V11   V12
V1   1.00  0.36  0.38 -0.01  0.05  0.03 -0.35 -0.40 -0.41  0.06  0.02  0.01
V2   0.36  1.00  0.37 -0.04 -0.02  0.01 -0.37 -0.34 -0.36  0.07  0.03  0.01
V3   0.38  0.37  1.00 -0.01  0.01  0.01 -0.38 -0.39 -0.32  0.01  0.05 -0.11
V4  -0.01 -0.04 -0.01  1.00  0.34  0.37 -0.09  0.00  0.00 -0.33 -0.37 -0.31
V5   0.05 -0.02  0.01  0.34  1.00  0.35 -0.01  0.08  0.02 -0.32 -0.35 -0.30
V6   0.03  0.01  0.01  0.37  0.35  1.00 -0.05  0.11 -0.03 -0.39 -0.32 -0.33
V7  -0.35 -0.37 -0.38 -0.09 -0.01 -0.05  1.00  0.34  0.32 -0.04  0.02  0.08
V8  -0.40 -0.34 -0.39  0.00  0.08  0.11  0.34  1.00  0.39 -0.11 -0.12 -0.02
V9  -0.41 -0.36 -0.32  0.00  0.02 -0.03  0.32  0.39  1.00 -0.06 -0.01  0.00
V10  0.06  0.07  0.01 -0.33 -0.32 -0.39 -0.04 -0.11 -0.06  1.00  0.41  0.36
V11  0.02  0.03  0.05 -0.37 -0.35 -0.32  0.02 -0.12 -0.01  0.41  1.00  0.39
V12  0.01  0.01 -0.11 -0.31 -0.30 -0.33  0.08 -0.02  0.00  0.36  0.39  1.00
```

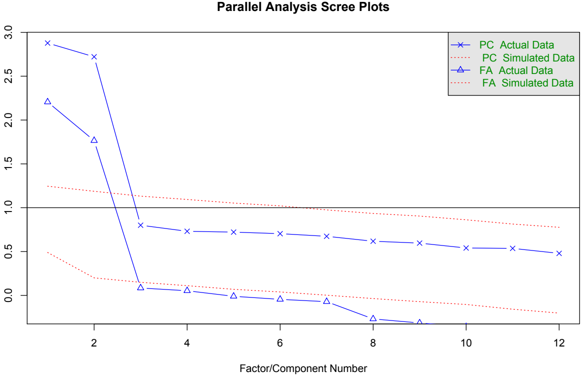
Multiple ways to determine how many factors are in the data

No one answer. Many are good, one should be avoided.

1. Statistical tests
   - χ^2 test of residuals (sensitive to sample size and non-normality of data)
   - χ^2 test of change from nf=n to nf=n+1 (sensitive to sample size)
   - RMSEA, BIC, AIC, SABIC are not as sensitive to sample size, but are to non-normality
2. Rules of Thumb
   - Scree Test of eigen values (Cattell, 1966)
   - Minimum Average Partial (MAP) (Velicer, 1976)
   - Very Simple Structure (Revelle & Rocklin, 1979)
   - Parallel Analysis of random data (Horn, 1965)
   - As many as can be interpreted
3. One test to avoid: Eigen value of 1 (Many programs default to this)

How many factors in my.cor

```
> fa.parallel(my.cor,n.obs=500)
Parallel analysis suggests that the number of factors =  2
                and the number of components =  2
```



Take out 2 factors from my.cor

R code

```
f2 <- fa(my.cor,nfactors =2)
```

```
Factor Analysis using method =  minres
Call: fa(r = my.cor, nfactors = 2)
Standardized loadings (pattern matrix) based upon correlation matrix
      MR1   MR2   h2   u2 com
V1   0.64 -0.02 0.41 0.59 1.0
V2   0.59  0.02 0.35 0.65 1.0
V3   0.61 -0.04 0.37 0.63 1.0
V4   0.03 -0.58 0.34 0.66 1.0
V5   0.01 -0.55 0.30 0.70 1.0
V6   0.03 -0.60 0.36 0.64 1.0
V7  -0.58  0.08 0.34 0.66 1.0
V8  -0.62 -0.10 0.39 0.61 1.1
V9  -0.59  0.00 0.35 0.65 1.0
V10  0.07  0.61 0.38 0.62 1.0
V11  0.03  0.63 0.39 0.61 1.0
V12 -0.06  0.57 0.33 0.67 1.0

                       MR1  MR2
SS loadings           2.21 2.12
Proportion Var        0.18 0.18
Cumulative Var        0.18 0.36
Proportion Explained  0.51 0.49
Cumulative Proportion 0.51 1.00

 With factor correlations of
    MR1  MR2
MR1 1.00 0.04
```

2 factors of my.cor, continued

```
With factor correlations of
     MR1  MR2
MR1 1.00 0.04
MR2 0.04 1.00

Mean item complexity =  1
Test of the hypothesis that 2 factors are sufficient.

The degrees of freedom for the null model are  66  and the objective function was  2.52
The degrees of freedom for the model are 43  and the objective function was  0.11

The root mean square of the residuals (RMSR) is  0.03
The df corrected root mean square of the residuals is  0.03

Fit based upon off diagonal values = 0.99
Measures of factor score adequacy
                                                  MR1  MR2
Correlation of (regression) scores with factors   0.88 0.88
Multiple R square of scores with factors          0.78 0.77
Minimum correlation of possible factor scores     0.56 0.53
```

1 factor of ability, how well does it fit?

R code

```
f1 <- fa(ability)
```

```
> f1
Factor Analysis using method =  minres
Call: fa(r = ability)
Standardized loadings (pattern matrix) based upon correlation matrix
           MR1   h2   u2 com
reason.4  0.55 0.30 0.70   1
reason.16 0.45 0.20 0.80   1
reason.17 0.54 0.29 0.71   1
reason.19 0.47 0.22 0.78   1
letter.7  0.52 0.27 0.73   1
letter.33 0.48 0.23 0.77   1
letter.34 0.54 0.29 0.71   1
letter.58 0.53 0.28 0.72   1
matrix.45 0.41 0.17 0.83   1
matrix.46 0.43 0.18 0.82   1
matrix.47 0.47 0.22 0.78   1
matrix.55 0.35 0.12 0.88   1
rotate.3  0.50 0.25 0.75   1
rotate.4  0.55 0.30 0.70   1
rotate.6  0.53 0.28 0.72   1
rotate.8  0.46 0.21 0.79   1

                MR1
SS loadings    3.81
Proportion Var 0.24

Mean item complexity =  1
```

1 factor (continued)

```
The degrees of freedom for the null model are  120  and the objective function was
              3.28 with Chi Square of  4973.83
The degrees of freedom for the model are 104  and the objective function was  0.7

The root mean square of the residuals (RMSR) is  0.07
The df corrected root mean square of the residuals is  0.07

The harmonic number of observations is  1426 with the empirical chi square  1476.62
                with prob <  3e-241
The total number of observations was  1525  with Likelihood Chi Square =  1063.25
           with prob <  9.5e-159

Tucker Lewis Index of factoring reliability =  0.772
RMSEA index =  0.078  and the 90 % confidence intervals are  0.074 0.082
BIC =  300.96
Fit based upon off diagonal values = 0.93
Measures of factor score adequacy
                                                   MR1
Correlation of (regression) scores with factors   0.91
Multiple R square of scores with factors          0.84
Minimum correlation of possible factor scores     0.67
```

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