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Psychology 205: Research Methods in Psychology Methods in Differential Psychology

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Outline

The two disciplines of psychology

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Two fundamentally different approaches to psychological research

- Experimental versus differential
 - Experimental approaches associated with cognitive, neurobiology, social.
 - Correlational primarily with personality/clinical/life span developmental.
- Some combine both approaches:
 - Experimental clinical/personality examines interactions of situation with individual differences.
 - Experimental developmental examines changes in response time capabilities with age.
- Should not confuse lab based data collection with experimental experimental requires random assignment to conditions.

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Differential Psychology

- 1. Emphasis on dispersion (variance and covariance) rather than central tendency (mean and median)
- 2. Studies of Ability, Interests/Values, Temperament
- 3. Prediction of performance in college/universities, professions, military
- 4. Correlations with marriage, divorce, employment, health and mortality
- 5. Correlations with psychopathology
 - Understanding the risk for depression in terms of cognitive processing
 - Understanding the genetic and environmental basis of schizophrenia

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All studies have three questions:

- 1. What are the data?
 - Collected experimentally
 - Collected observationally
- 2. How do we model them?
- 3. How large is the residual variance?

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Theory of Data + the Data Box

- Theory of data Coombs (1964)
 - Sets of Objects and of People
 - Two types of comparisons (Proximity and order)
 - Comparisons of preferences: Do you like X? is a proximity measure.
 - $|o_i p_j| < \epsilon$ (Do you like something?)
 - $|o_i p_j| < |o_k p_l|$ (do you like something more than I like something else?)
 - Comparisons of order: Can you do X?
 - $o_i < p_j$ Are you better than an item?
 - $o_i < o_j$ Is this object more than that object?
- The Data Box: comparisons across time and space (Cattell, 1946, 1966b)
 - People over tests, people over time
 - Tests over people, tests over time
 - etc.

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The data box: Subjects x Measures x Time



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Cattell and the Data Box

- 1. One occasion
 - R: Correlate measures across persons : standard personality traits
 - Q: Correlate Persons acros measures: Personality typology
- 2. One Person
 - P: Correlate Measures across Occasions; Individual personality structure
 - O: Correlate Occasions across measures: Individual psychological environment
- 3. One Measure
 - T: Correlate Occasions across Persons: Anxiety arousing situations
 - S: Correlate Persons across Occasions: Anxious person types

Cattell (1946, 1966b,a); Revelle (2009, 2015) (Note that Cattell changed his notation from paper to paper).

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Differential Psychology

- 1. Emphasis on variance and covariance rather than mean
- 2. Studies of Ability, Interests/Values, Temperament
- 3. Prediction of performance in college/universities, professions, military
- 4. Correlations with health and mortality

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The basic problem is the same

- $1. \ \mathsf{Data} = \mathsf{Model} + \mathsf{Residual}$
- 2. What are the data?
- 3. How do we model them?

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Methods of data collection

- Self report how do you normally behave traits/states/narratives
- 2. Ability tests what is the best you can do
- 3. Behavioral observation
- 4. Physiology EEG/MRI/fMRI/PET
- 5. Remote data web/big EAR,websites
- 6. National and International Surveys
- 7. Animal lesion/drug/observation

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Types of designs

- 1. Experimental
 - lab based
 - field based
 - Quasi-experimental designs
- 2. Observational
 - Cross sectional
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A variety of longitudinal designs

- 1. Brief within subject
 - Momentary Experience Samplng (ESM)
 - Studying the dynamics of mood (Wilt, Funkhouser & Revelle, 2011; Revelle & Wilt, 2021; Beck & Jackson, 2020, Beck & Jackson)
- 2. Long term within subject
 - Terman and the "termites" (Terman & Baldwin, 1926; Terman & Oden, 1959; Oden, 1968)
 - Berkeley/Oakland growth studies
 - Scottish Mental Health study (Deary, Whiteman, Starr, Whalley & Fox, 2004; Deary, 2009; Deary, Pattie & Starr, 2013)
 - Seattle Longitudinal Study (Schaie, 2005)
 - Dunedin Longitudinal Study (Reuben, Caspi, Belsky, Broadbent, Harrington,

Sugden, Houts, Ramrakha, Poulton & Moffitt, 2017)

- Study of Mathematically Precocious Youth (Lubinski, Webb, Morelock & Benbow, 2001; Lubinski & Benbow, 2006; Lubinski, 2016)
- Project Talent

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- 1. Issues in scaling what do the numbers mean?
 - Do the measures have linear or just ordinal properties?
 - Are we measuring the same thing at age 11 as age 77?
 - Can we adapt prior measures for current theory?
- 2. The correlation coefficient and its many forms
 - Y varies as X
 - Is Y a continuous variable?
 - is X a continuous variable?
 - what is the underlying relationship?

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Alternative versions of the correlation coefficient

Table: A number of correlations are Pearson r in different forms, or with particular assumptions. If $r = \frac{\sum x_i y_i}{\sqrt{\sum x_i^2 \sum y_i^2}}$, then depending upon the type of data being analyzed, a variety of correlations are found.

Coefficient	symbol	Х	Y	Assumptions
Pearson	r	continuous	continuous	
Spearman	rho (ρ)	ranks	ranks	
Point bi-serial	r _{pb}	dichotomous	continuous	
Phi	ϕ	dichotomous	dichotomous	
Bi-serial	r _{bis}	dichotomous	continuous	normality
Tetrachoric	r _{tet}	dichotomous	dichotomous	bivariate normality
Polychoric	r _{pc}	categorical	categorical	bivariate normality

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Many other statistics (F, d, b) may be expressed in terms of r

Table: Alternative Estimates of effect size. Using the correlation as a scale free estimate of effect size allows for combining experimental and correlational data in a metric that is directly interpretable as the effect of a standardized unit change in x leads to r change in standardized y.

Statistic	Estimate	r equivalent	as a function of r
Pearson correlation	$r_{xy} = \frac{C_{xy}}{\sigma_x \sigma_y}$	r _{xy}	
Regression	$b_{y.x} = \frac{Cxy}{\sigma_x^2}$	$r = b_{y.x} \frac{\sigma_y}{\sigma_x}$	$b_{y.x} = r \frac{\sigma_x}{\sigma_y}$
Cohen's d	$d = \frac{X_1 - \hat{X}_2}{\sigma_x}$	$r = \frac{d}{\sqrt{d^2+4}}$	$d = \frac{2r}{\sqrt{1-r^2}}$
Hedge's g	$g = \frac{X_1 - X_2}{s_x}$	$r = rac{g}{\sqrt{g^2 + 4(df/N)}}$	$g=rac{2r\sqrt{df/N}}{\sqrt{1-r^2}}$
t - test	$t = rac{d\sqrt{df}}{2}$	$r=\sqrt{t^2/(t^2+df)}$	$t = \sqrt{\frac{r^2 df}{1 - r^2}}$
F-test	$F = \frac{d^2 df}{4}$	$r = \sqrt{F/(F + df)}$	$F = \frac{r^2 df}{1 - r^2}$
Chi Square		$r = \sqrt{\chi^2/n}$	$\chi^2 = r^2 n$
Odds ratio	$d = \frac{\ln(OR)}{1.81}$	$r = \frac{\ln(OR)}{1.81\sqrt{(\ln(OR)/1.81)^2 + 4}}$	$ln(OR) = rac{3.62r}{\sqrt{1-r^2}}$
r _{equivalent}	r with probability p	$r = r_{equivalent}$	

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Multiple Regression: decomposing correlations



 $r_{x_1y} = \overrightarrow{\beta_{y.x_1}} + \overbrace{r_{x_1x_2}\beta_{y.x_2}}^{indirect}$

$$r_{x_2y} = \underbrace{\beta_{y,x_2}}_{direct} + \underbrace{r_{x_1x_2}\beta_{y,x_1}}_{indirect}$$

$$\beta_{y.x_1} = \frac{r_{x_1y} - r_{x_1x_2}r_{x_2y}}{1 - r_{x_1x_2}^2}$$

$$\beta_{y.x_2} = \frac{r_{x_2y} - r_{x_1x_2}r_{x_1y}}{1 - r_{x_1x_2}^2}$$

 $R^2 = r_{x_1y}\beta_{y.x_1} + r_{x_2y}\beta_{y.x_2}$

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3 special cases of regression Orthogonal predictors Correlated predictors





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Three basic cases



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3 special cases of regression Orthogonal predictors Correlated predictors







$$\beta_{y.x_2} = \frac{r_{x_2y} - r_{x_1x_2} r_{x_1y}}{1 - r_{x_1x_2}^2}$$

 $R^{2} = r_{x_{1}y}\beta_{y.x_{1}} + r_{x_{2}y}\beta_{y.x_{2}}$

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Three basic cases: Theoretical examples



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Data quality

1. Estimates of reliability reliability (Revelle & Condon, 2019)

- parallel tests,
- alternate form,
- stability
- internal consistency $\omega_h, \alpha, \omega_t$
- 2. Validity does the arrow hit the target

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Advanced methods

- 1. Factor analysis, cluster analysis, multidimensional scaling (see Revelle (prep) for a discussion of many of these concepts)
- 2. Structural Equation modeling Rosseel (2012)
- 3. Multi-level modeling of emotion (Revelle & Wilt, 2019)
- 4. Computer modeling (Revelle & Condon, 2015; Revelle, Elleman & Hall, 2020)
- 5. Machine (Statistical) learning: Old ideas in new software.

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Multivariate analysis

- Many procedures use the concept of iterative fitting
- Some, such as principal components are "closed form" and can just be solved directly
- Others, such as "factor analysis" need to find solutions by successive approximations
- The issue of factor or component rotation is an iterative procedure.
- Principal Components and Factor analysis are all the result of some basic matrix equations to approximate a matrix
- Conceptually, we are just taking the square root of a correlation matrix:
- $R \approx CC'$ or $R \approx FF' + U^2$ (u^2 is a diagonal matrix
- For any given correlation matrix, R, can we find a C or an F matrix which approximates it?

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Consider the following matrix

What would be its "square root"? That is to say, what simpler matrix, times itself is equal to R?

Find $R \approx FF' + U^2$

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A congeneric matrix is one with just one factor

R code R <- sim.congeneric()</pre> R. f1 < -fa(R)> R. V1 V2 V3 V4 V1 1.00 0.56 0.48 0.40 V2 0.56 1.00 0.42 0.35 V3 0.48 0.42 1.00 0.30 V4 0.40 0.35 0.30 1.00 > f1 <- fa(R) > f1 Factor Analysis using method = minres Call: fa(r = R)Standardized loadings (pattern matrix) based upon correlation matrix MR1 h2 u2 com V1 0.8 0.64 0.36 1 V2 0.7 0.49 0.51 1 V3 0.6 0.36 0.64 1 V4 0 5 0 25 0 75 1 MR1 SS loadings 1.74 Proportion Var 0.43

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Using multivariate statistics to simplify our data

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- 2. Approximate complex data with simpler models
- 3. Scale construction tales multiple items to make one or a few scales.

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